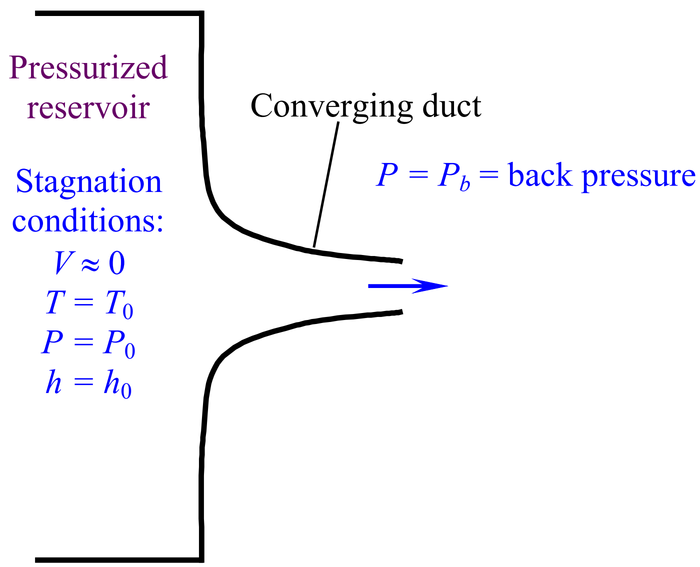
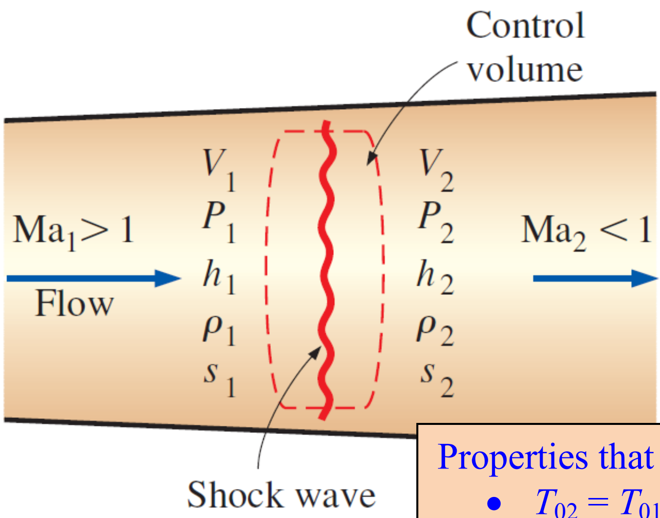
Today, we will:

- Finish discussing choked flow and its implications
- Discuss shock waves, particularly *normal* shock waves

Consider a large tank with a converging duct that *ends at the throat* (no diverging part):



Consider a *stationary* normal shock wave (as in a supersonic wind tunnel)



Properties that ***increase*** across the shock:

- $P_2 > P_1$
- $T_2 > T_1$, thus:
  - $c_2 > c_1$
  - $h_2 > h_1$
- $\rho_2 > \rho_1$
- $s_2 > s_1$
- $A_2^* > A_1^*$

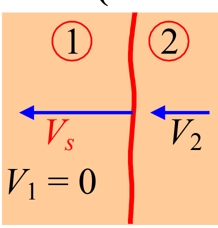
Properties that ***decrease*** across the shock:

- $Ma_2 < Ma_1$
- $P_{02} < P_{01}$
- $\rho_{02} < \rho_{02}$
- $V_2 < V_1$

Properties that ***stay the same*** across the shock:

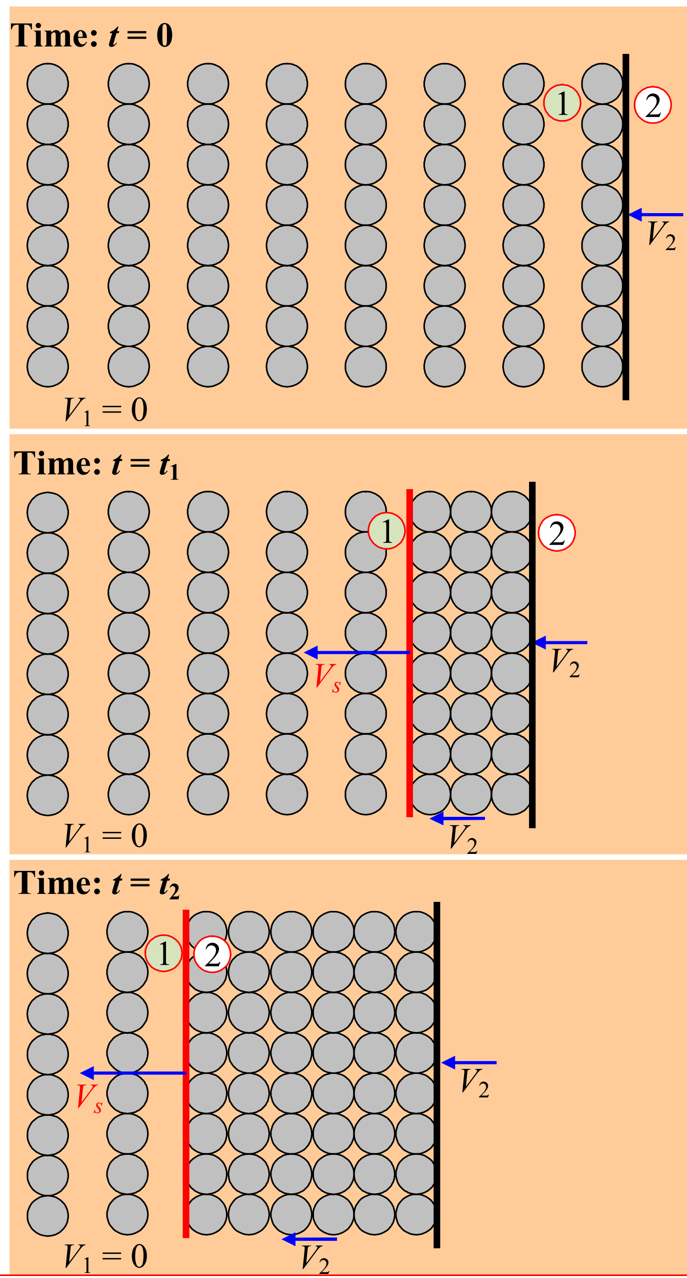
- $T_{02} = T_{01}$
- $h_{02} = h_{01}$

Consider instead a *moving* normal shock wave (as in a blast wave from an explosion)



- The shock is moving into quiescent air (region 1)
- In this frame of reference we define $Ma_1 = V_s / c_1$
- The shock wave travels into region 1 at supersonic speed ($Ma_1 > 1$)
- The air behind the shock (region 2) follows at a slower speed

The "dime analogy" (model the moving shock as rows of dimes that pile up when pushed by a rod or "piston" as sketched; three sequential times):



Comments:

- The vertical red line is analogous to a shock wave: $V_1 = 0$, $V_s > V_2$, $\rho_2 > \rho_1$ (there is sudden increase in density, and the "wave front" of dimes moves faster than the piston).
- The dimes in region 1 don't "know" anything is happening until the shock hits them.
- Similarly in a shock wave in air, the air in region 1 does not "know" anything is happening until the shock wave hits it.

Normal Shock Equations (1 = upstream, 2 = downstream of stationary shock):

$$T_{01} = T_{02}$$

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - k + 1}}$$

$$\frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{2k\mathrm{Ma}_1^2 - k + 1}{k + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)}$$

$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1 + \mathrm{Ma}_2^2(k-1)/2}{1 + \mathrm{Ma}_1^2(k-1)/2}\right]^{(k+1)/[2(k-1)]}$$

$$\frac{P_{02}}{P_1} = \frac{(1 + k\mathrm{Ma}_1^2)[1 + \mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\mathrm{Ma}_2^2}$$

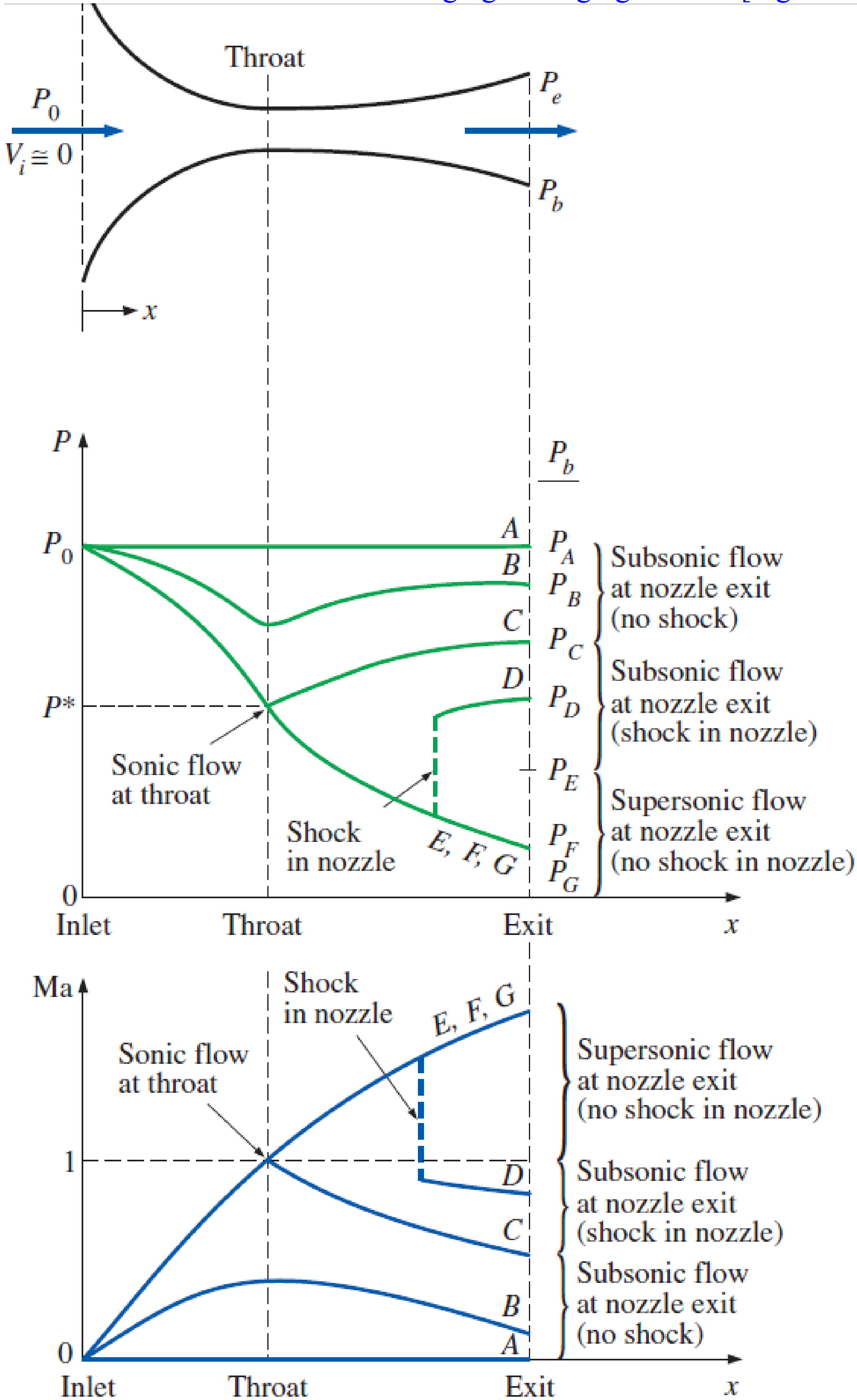
TABLE A–14

One-dimensional normal shock functions for an ideal gas with $k = 1.4$

For air

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.8929
1.1	0.9118	1.2450	1.1691	1.0649	0.9989	2.1328
1.2	0.8422	1.5133	1.3416	1.1280	0.9928	2.4075
1.3	0.7860	1.8050	1.5157	1.1909	0.9794	2.7136
1.4	0.7397	2.1200	1.6897	1.2547	0.9582	3.0492
1.5	0.7011	2.4583	1.8621	1.3202	0.9298	3.4133
1.6	0.6684	2.8200	2.0317	1.3880	0.8952	3.8050
1.7	0.6405	3.2050	2.1977	1.4583	0.8557	4.2238
1.8	0.6165	3.6133	2.3592	1.5316	0.8127	4.6695
1.9	0.5956	4.0450	2.5157	1.6079	0.7674	5.1418
2.0	0.5774	4.5000	2.6667	1.6875	0.7209	5.6404
2.1	0.5613	4.9783	2.8119	1.7705	0.6742	6.1654
2.2	0.5471	5.4800	2.9512	1.8569	0.6281	6.7165
2.3	0.5344	6.0050	3.0845	1.9468	0.5833	7.2937
2.4	0.5231	6.5533	3.2119	2.0403	0.5401	7.8969
2.5	0.5130	7.1250	3.3333	2.1375	0.4990	8.5261
2.6	0.5039	7.7200	3.4490	2.2383	0.4601	9.1813
2.7	0.4956	8.3383	3.5590	2.3429	0.4236	9.8624
2.8	0.4882	8.9800	3.6636	2.4512	0.3895	10.5694
2.9	0.4814	9.6450	3.7629	2.5632	0.3577	11.3022
3.0	0.4752	10.3333	3.8571	2.6790	0.3283	12.0610
4.0	0.4350	18.5000	4.5714	4.0469	0.1388	21.0681
5.0	0.4152	29.000	5.0000	5.8000	0.0617	32.6335
∞	0.3780	∞	6.0000	∞	0	∞

Effect of Back Pressure in a Converging-Diverging Nozzle: [Fig. 12-22 in Ed 2 of textbook]



Overexpanded nozzles:

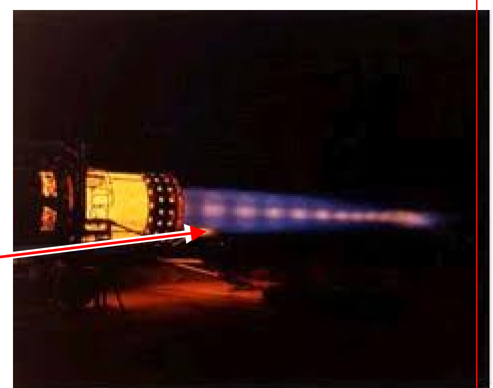
Example – High speed jet aircraft

Given: The SR-71 travels at Ma = 3.2 at 24 km altitude (80,000 ft).

To do: Calculate its air speed.

Solution:

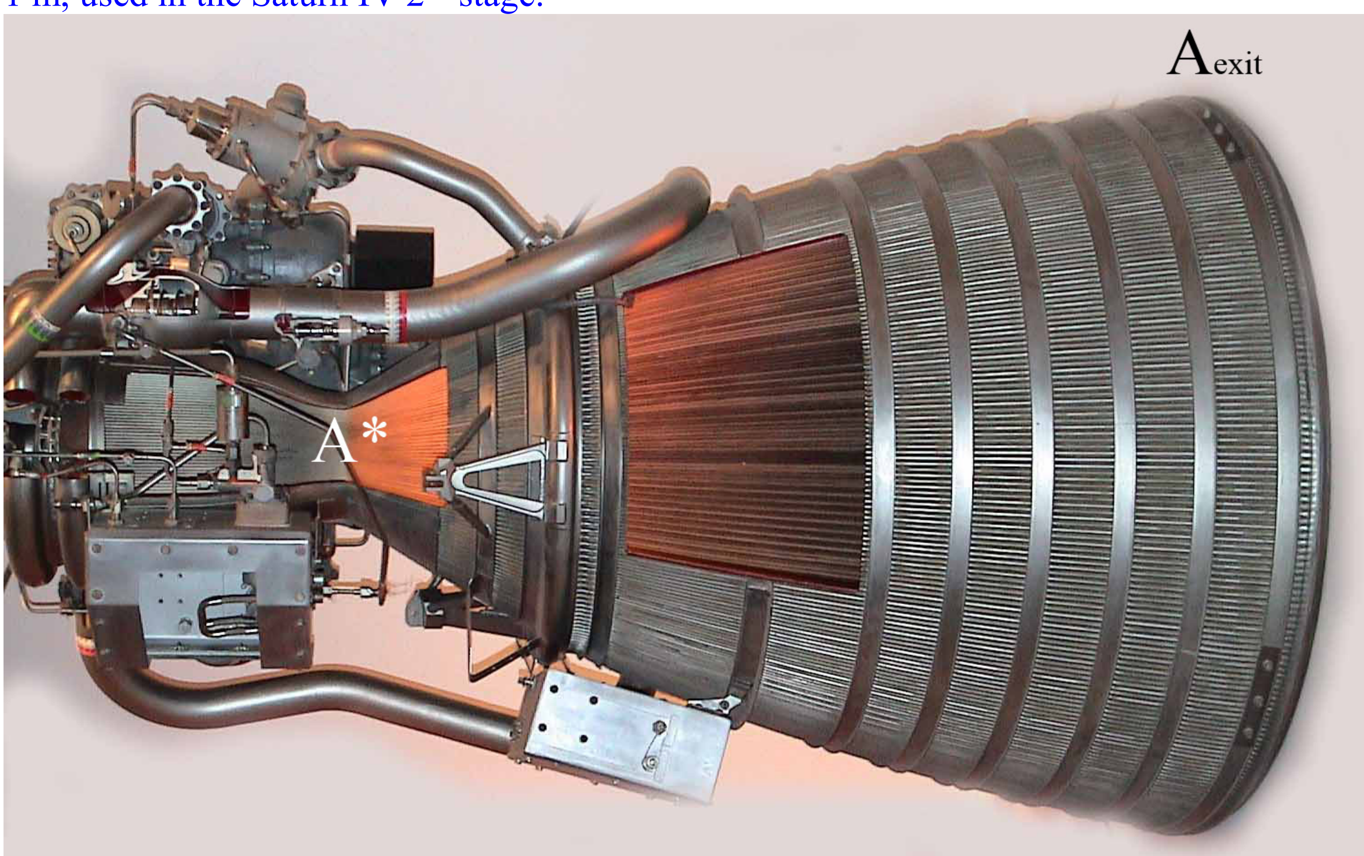
- From Table A-11E, T at 24 km altitude is -69.7ºF = 217 K.
- Using $k = 1.4$ and $R_{air} = 287\ m^2/(s^2 \cdot K)$, $c = (kRT)^{1/2} = 295$ m/s.
- Thus, $V = Ma \cdot c = 3.2(295\ m/s) =$ 944 m/s (= 2110 mph).



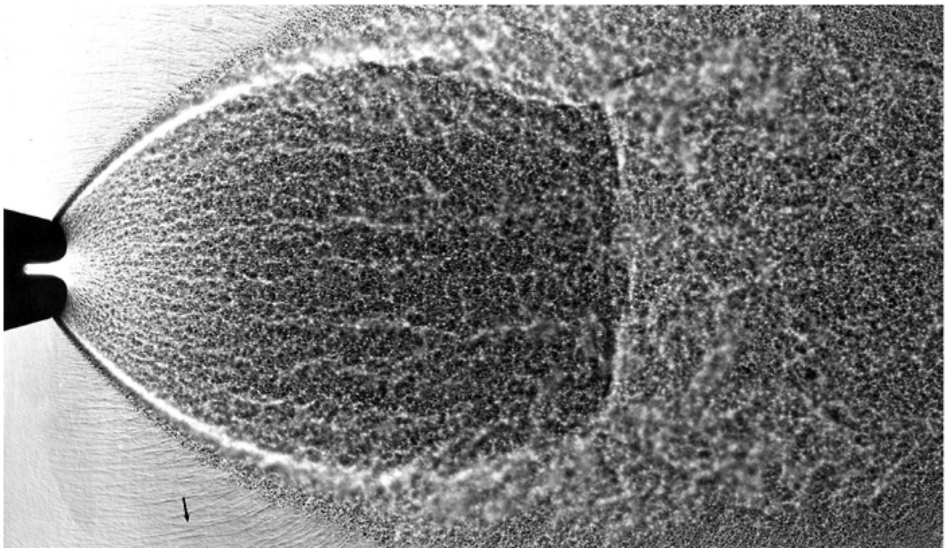
Shock Diamonds ("tiger tail"):

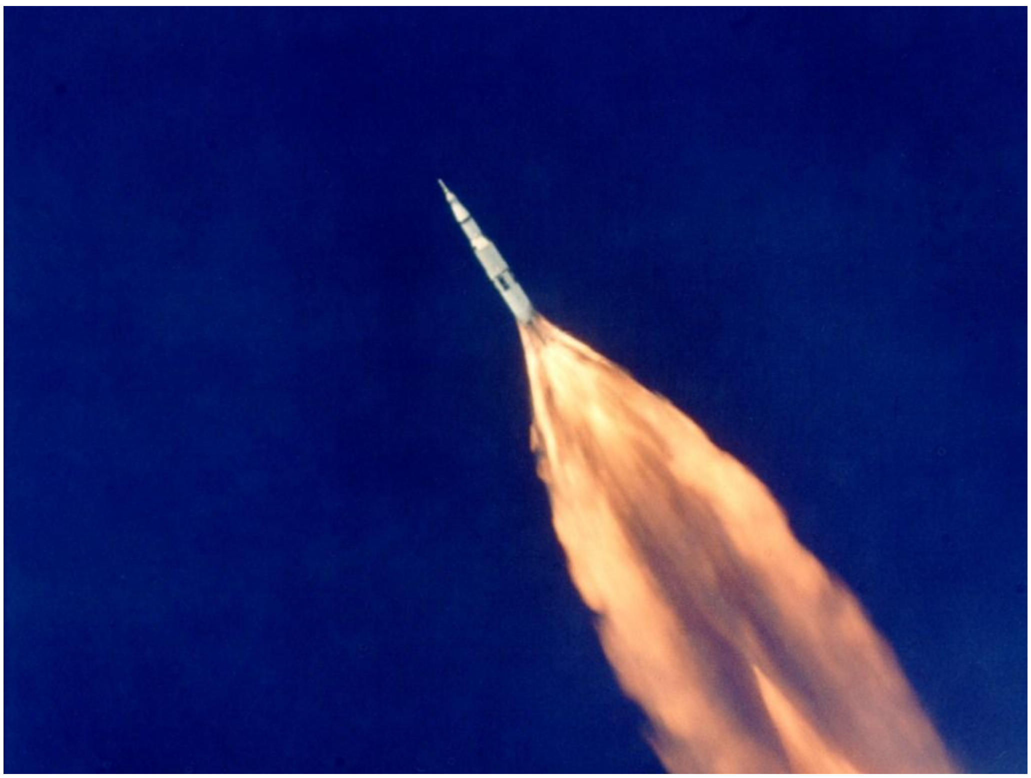
Example of a Rocket Engine:

Pratt &Whitney RL-10 rocket motor designed for a specific Ma_{exit} (photographed at the National Air & Space Museum). 1960-vintage, $Ma_{exit} = 5$, $k = 1.33$, thrust = 15,000 lbf, $D_e \sim$ 1 m, used in the Saturn IV 2nd stage.



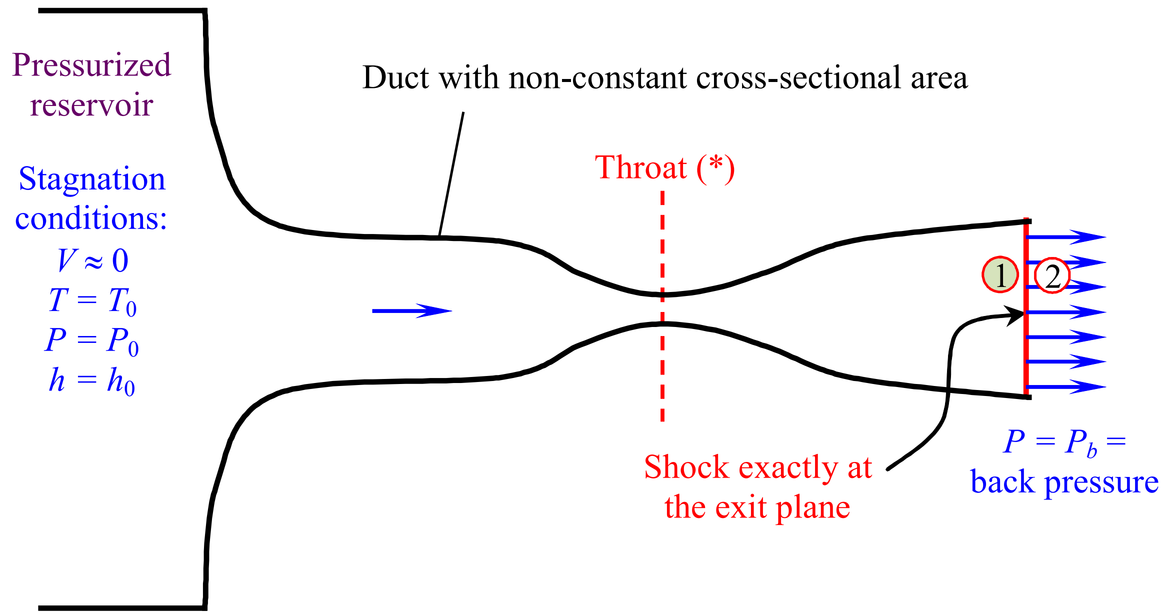
Underexpanded Nozzles:





Example – Normal shock at Ma = 3.0

Given: A large tank has upstream stagnation properties T_0 = 1000 K and P_0 = 1.00 MPa. A converging/diverging nozzle accelerates air isentropically from the tank to Ma = 3.0 just before the exit. Right at the exit plane is a normal shock wave as sketched.



To do: Calculate the pressure, temperature, and density upstream (1) and downstream (2) of the shock.

Solution:

- From Table A-13 at Ma_1 = 3.0,
  A/A^* = 4.2346, P/P_0 = 0.0272, T/T_0 = 0.3571, and ρ/ρ_0 = 0.0760.
- From Table A-14 at Ma_1 = 3.0,
  Ma_2 = 0.4752, P_2/P_1 = 10.3333, T_2/T_1 = 2.679, and ρ_2/ρ_1 = 3.8571.
- The rest of the problem to be completed in class.