Today, we will:

- Continue discussion about capillary action (Chapter 2)
- Begin Chapter 3 – Pressure and Fluid Statics
- Discuss different kinds of pressure measurement (absolute, gage, vacuum)
- Derive the equation of fluid statics (hydrostatic pressure relation)

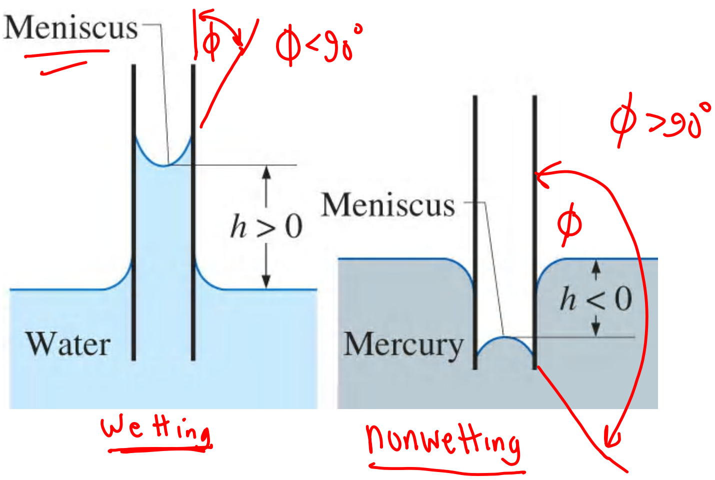
3. Other (miscellaneous) properties (continued)

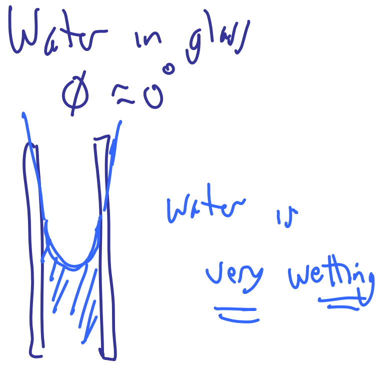
d. surface tension, σ_s (continued) = Force per length

the capillary effect

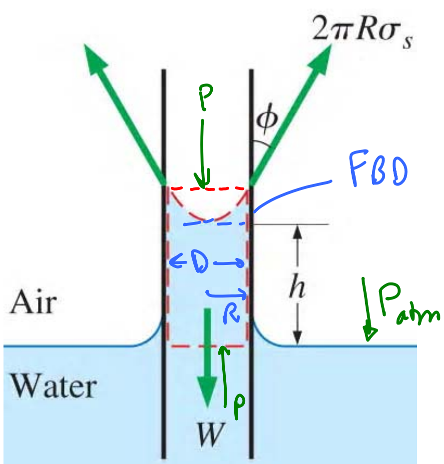
The combined effects of surface tension and contact angle lead to ***capillary action*** – the rise (or fall) of liquids in small-diameter capillary tubes, as illustrated here:

ϕ





We can predict the rise height h as a function of contact angle and surface tension, along with other parameters like inner tube diameter, liquid density, and gravitational constant:



Pressure force on top & bottom of the FBD cancel out (same pressure & same area)

$$\sum \vec{F} = 0$$

in vertical direction

Down: $W = mg = \rho V g = \rho \frac{\pi D^2}{4} h g$

Up: $\sigma_s 2\pi R \cdot \cos\phi = \sigma_s \pi D \cos\phi$

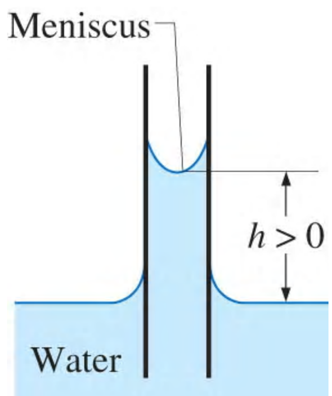
$\sum F = 0$ vertically

$$\rho \frac{\cancel{\pi} D^{\cancel{2}}}{4} h g = \sigma_s \cancel{\pi} \cancel{D} \cos\phi$$

$$\boxed{h = \frac{4\sigma_s \cos\phi}{\rho g D}} = \boxed{\frac{2\sigma_s \cos\phi}{\rho g R}}$$

Example: Prediction of capillary rise in a tube

Given: A glass capillary tube of inner diameter 1.3 mm is pushed vertically into a cup of water. The contact angle between glass and water is nearly 0^o. The surface tension of the water is 0.073 N/m.



The equation for capillary rise is $\boxed{h = \frac{4\sigma_s}{\rho g D}\cos\phi}$.

To do: Calculate capillary rise h in units of cm.

Solution: Note that ρ_{water} = 1000 kg/m^3 and g = 9.807 m/s^2.

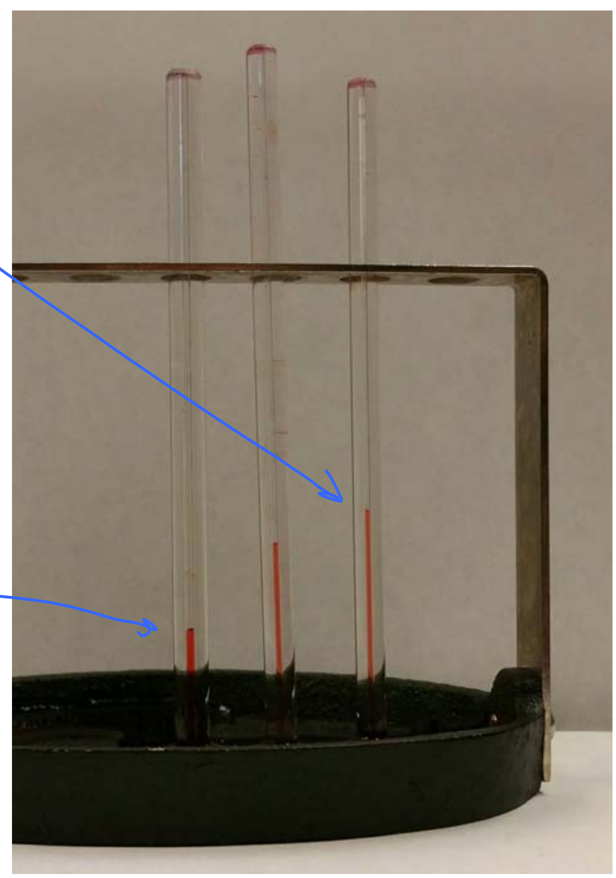
$$h = \frac{4\sigma_s}{\rho g D}\cos\phi = \frac{4(0.073\ N/m)\overbrace{\cos 0^\circ}^{\approx 1}}{(1000\ \frac{kg}{m^3})(9.807\ \frac{m}{s^2})(1.3\times10^{-3}\ m)}\left(\frac{kg\,m/s^2}{N}\right)\left(\frac{100\ cm}{m}\right) = 2.29035\ cm$$

$$\boxed{h = 2.3\ cm}$$

Demo – capillary rise in a glass tube (colored water)

Small D

Large D



The smaller the tube diameter, the larger the capillary rise

II. PRESSURE AND FLUID STATICS (Chapter 3)

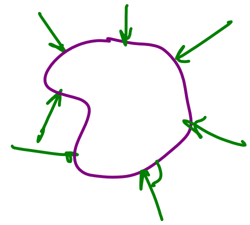
A. Pressure, P

1. Some basics

P is a scalar, not a vector

Pressure always acts normal & inward on any surface (real or "imaginary")

Pressure causes a surface force, not a body force



2. Dimensions and units

P is a normal stress

$\{P\} = \left\{\frac{\text{Force}}{\text{area}}\right\}$

units → SI N/m^2 = Pascal = Pa

Eng lbf/in^2 = psi

B. Types of Pressure Measurement

1 atm = 101,325 Pa = 101.325 kPa

= 14.696 psi

1. Absolute pressure

$\boxed{P \text{ relative to a total vacuum}}$

$P = P_{abs}$

2. Gage pressure

$\boxed{P_{gage} = P \text{ relative to local atm. pressure}}$

$\boxed{P_{gage} = P - P_{atm}}$ ☆

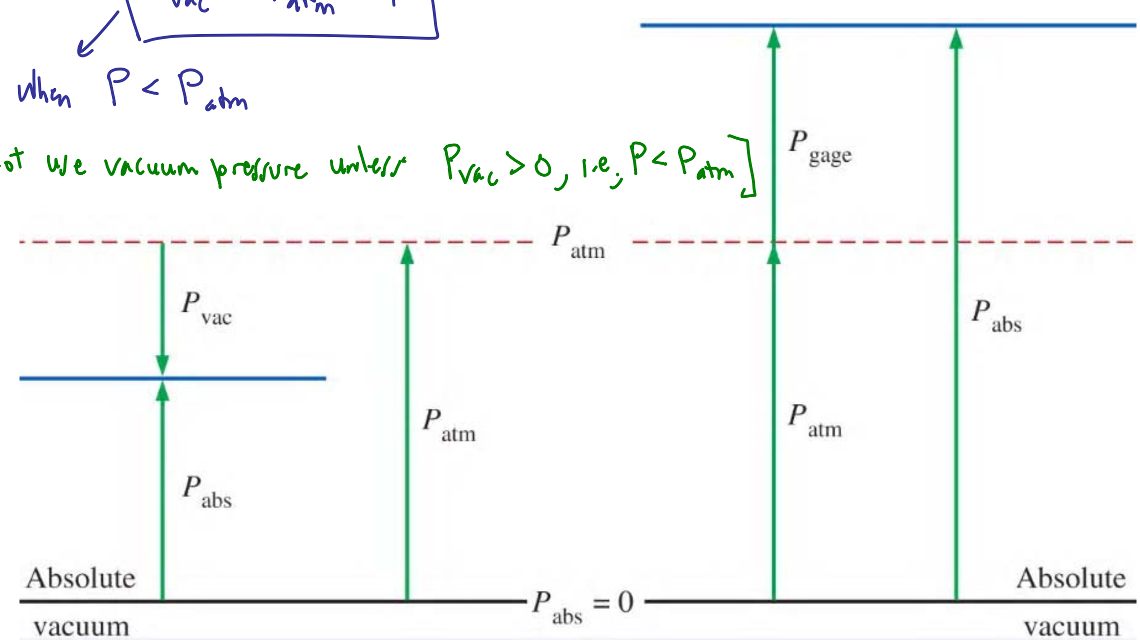
3. Vacuum pressure

$\boxed{P_{vac} = P_{atm} - P}$ ☆

$P_{vac} = -P_{gage}$

Used only when $P < P_{atm}$

[we typically do not use vacuum pressure unless $P_{vac} > 0$, i.e., $P < P_{atm}$]



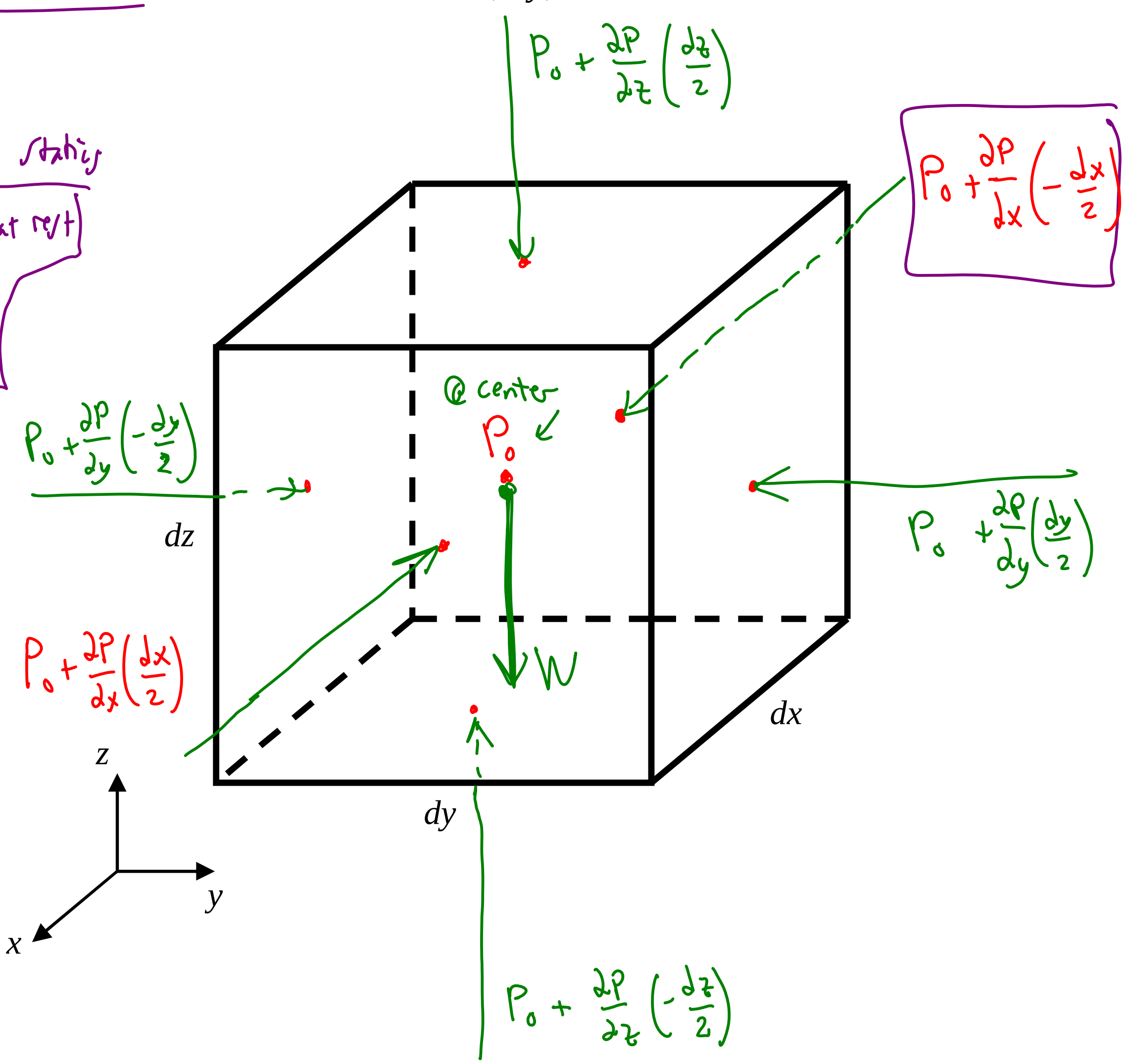
C. Equation of Fluid Statics → Fluid is sitting still (no motion)

Consider a small fluid element of dimensions *dx*, *dy*, and *dz* as sketched here.

$\boxed{\sum \vec{F} = 0}$ for statics

recall - a fluid at rest cannot resist a shear stress

We have only normal stresses here



$\vec{g}$

$$\sum \vec{F} = \sum \vec{F}_{\text{body forces}} + \sum \vec{F}_{\text{surface forces}}$$

1. Body forces → Weight due to gravity, W

$$\vec{F}_{grav} = m\vec{g} = \rho \forall \vec{g} = \rho\, dx\,dy\,dz\, \vec{g}$$ down - z direction

$$\boxed{\vec{g} = -g\vec{k}}$$

$$\boxed{\sum \vec{F}_{\text{body forces}} = -\rho g \, dx\,dy\,dz\, \vec{k}}$$

2. Surface forces (the only one here is pressure)

Let $P = P_o$ @ center of our element

Pressure = Force/area

In x-direction,

$$\sum F_{surface,x} = \underbrace{-}_{\text{acting in } \ominus \text{ x-direction}} \underbrace{\left(\underbrace{P_o + \frac{\partial P}{\partial x}\frac{dx}{2}}_{\text{pressure on front face}}\right) \underbrace{dy\,dz}_{\text{surface area}}}_{\text{front face}} \underbrace{+}_{\text{acts in } \oplus \text{ x-direction}} \underbrace{\left(\underbrace{P_o + \frac{\partial P}{\partial x}\left(-\frac{dx}{2}\right)}_{\text{pressure on back face}}\right) \underbrace{dy\,dz}_{\text{surface area}}}_{\text{back face}}$$

$$\boxed{\sum F_{surface,x} = -\frac{\partial P}{\partial x}\,dx\,dy\,dz} = \text{Pressure force in x-dir}$$

Similarly,

$$\boxed{\sum F_{surface,y} = -\frac{\partial P}{\partial y}\,dx\,dy\,dz} = \text{" } \quad y \text{ "}$$

$$\boxed{\sum F_{surface,z} = -\frac{\partial P}{\partial z}\,dx\,dy\,dz} = \text{" } \quad z \text{ "}$$

Hydrostatic Pressure Relation

$$\sum \vec{F} = 0$$

$$\boxed{\sum \vec{F}_{body} + \sum \vec{F}_{surface} = 0}$$

X-dir:

$$0 - \frac{\partial P}{\partial x}\,dx\,dy\,dz = 0 \rightarrow \boxed{\frac{\partial P}{\partial x} = 0}$$

In fluid statics, P does not vary in the x-direction! ✩

Similarly

y-dir $\quad 0 - \frac{\partial P}{\partial y}\,dx\,dy\,dz = 0 \rightarrow \boxed{\frac{\partial P}{\partial y} = 0}$

In fluid statics, P does not vary in the y-direction!

z-dir: $\quad -\rho g\,\cancel{dx\,dy\,dz} - \frac{\partial P}{\partial z}\,\cancel{dx\,dy\,dz} = 0$

$$\boxed{\frac{\partial P}{\partial z} = -\rho g}$$

In fluid statics, P does vary in the z-direction

Bottom line → In fluid statics, in a continuous fluid, P does not vary horizontally, but it does vary vertically ☆

P goes up as you go down in a fluid

Since $\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = 0 \rightarrow P \neq \text{fnc of } x, y$

$\therefore\ P = P(z)$ only $\qquad \frac{\partial P}{\partial z} = -\rho g \rightarrow \boxed{\frac{dP}{dz} = -\rho g}$

(use total derivatives d instead of partial derivatives ∂)

Integrate $\int_1^2 dz$

$$\boxed{P_2 - P_1 = -\rho g\,(z_2 - z_1)}$$

Hydrostatic Pressure Relation:

$$\boxed{P_{below} = P_{above} + \rho g\,|\Delta z|}$$ ☆ in same liquid in statics

②

①