

Today, we will:

- Continue Chapter 3 – Pressure and Fluid Statics
- Discuss applications of fluid statics (barometers and U-tube manometers)
- Do some example problems (manometers)

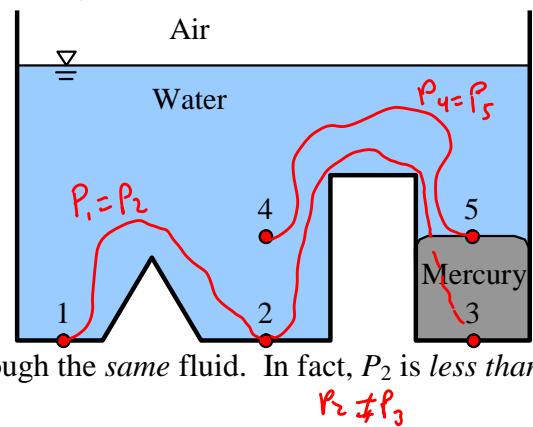
D. Applications of *Fluid Statics* (also called *Hydrostatics*)

Note: The equation for fluid statics is $\frac{dP}{dz} = -\rho g$ and the simplest way to remember and apply the fluid statics equation is $P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z|$. ★

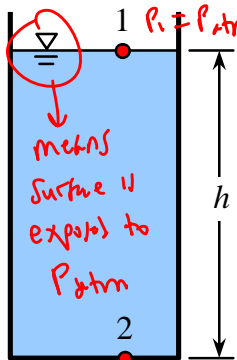
See pdf file on the course website: [Some Rules About Hydrostatics](#) (will review in class).

- There are several “rules” that directly result from the above equation:

1. **If you can draw a continuous line through the same fluid from point 1 to point 2, then $P_1 = P_2$ if $z_1 = z_2$.**
 E.g., consider the oddly shaped container in the sketch. By this rule, $P_1 = P_2$ and $P_4 = P_5$ since these points are at the same elevation in the same fluid. However, P_2 does not equal P_3 even though they are at the same elevation, because one cannot draw a line connecting these points through the same fluid. In fact, P_2 is less than P_3 since mercury is denser than water.



2. **Any free surface open to the atmosphere has atmospheric pressure, P_{atm} .**



(This rule holds not only for hydrostatics, by the way, but for any free surface exposed to the atmosphere, whether that surface is moving, stationary, flat, or curved.) Consider the hydrostatics example of a container of water. The little upside-down triangle indicates a free surface, and means that the pressure there is atmospheric pressure, P_{atm} . In other words, in this example, $P_1 = P_{\text{atm}}$. To find the pressure at point 2, our hydrostatics equation is used:

$$P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z|$$

$$P_2 = P_1 + \rho g h$$

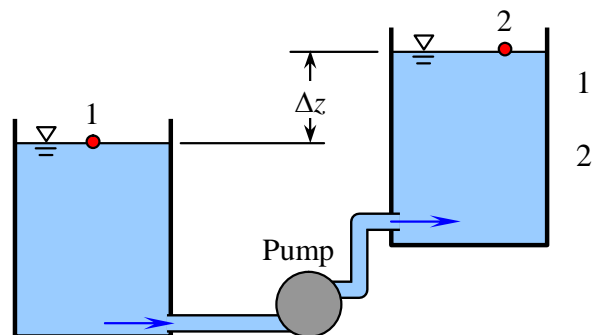
$$\text{Or, } P_{2, \text{gage}} = \rho g h \text{ (gage pressure)}$$

$$P_2 = P_{\text{atm}} + \rho g h \text{ (absolute pressure)}$$

3. **In most practical problems, atmospheric pressure is assumed to be constant at all elevations** (unless the change in elevation is large).

Consider the example shown, in which water is pumped from one large reservoir to another. The pressure at both 1 and 2 is atmospheric. But since point 2 is higher in elevation than point 1, the local atmospheric pressure at 2 is a little lower than that at point 1. To be precise, our hydrostatics equation may be used to account for the difference in elevation between points 1 and 2.

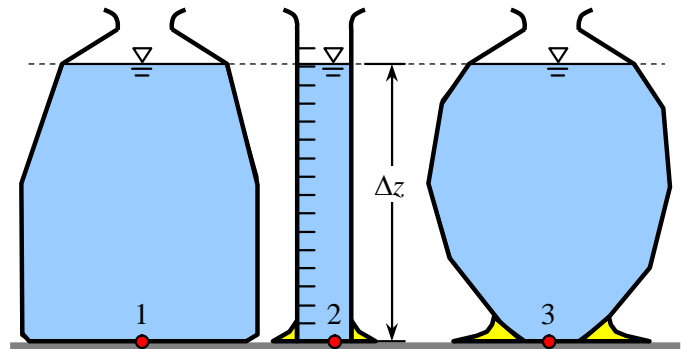
However, since the density of water is so much greater than that of air, it is common to ignore the difference between P_1 and P_2 , and call them both the same value of atmospheric pressure, P_{atm} .



[We will do many flow problems like this later]

4. **The shape of a container does not matter in hydrostatics.** (Except of course for very small diameter tubes, where surface tension and the capillary effect become important.)

Consider the three containers in the figure. At first glance, it may seem that the pressure at point 3 would be greater than that at point 2, since the weight of the water is more “concentrated” on the small area at the bottom, but in reality, all three pressures are identical. Use of our hydrostatics equation confirms this conclusion, i.e.,



$$P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z|$$

$$P_1 = P_2 = P_3 = P_{\text{atm}} + \rho g \Delta z$$

In all three cases, a thin column of water above the point in question at the bottom is identical. Pressure is a force per unit area, and over a small area at the bottom, that force is due to the weight of the water above it, which is the same in all three cases, regardless of the container shape.

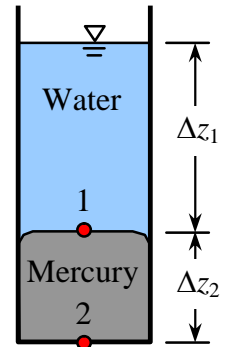
5. **Pressure is constant across a flat fluid-fluid interface.**

For example, consider the container in the figure, which is partially filled with mercury, and partially with water. In this case, our hydrostatics equation must be used *twice*, once in each of the liquids.

$$P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z|$$

$$P_1 = P_{\text{atm}} + \rho_{\text{water}} g \Delta z_1$$

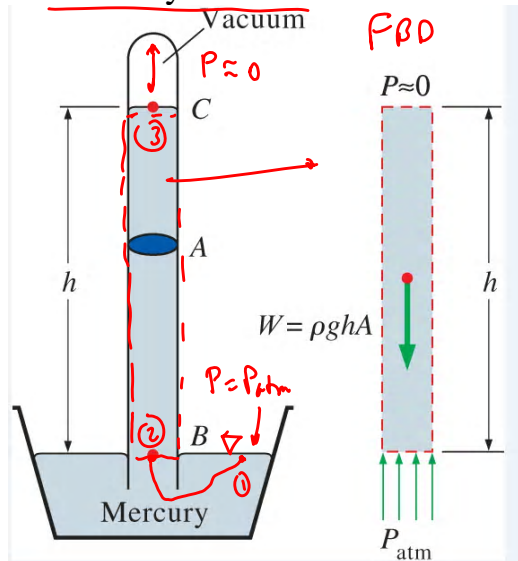
$$P_2 = P_1 + \rho_{\text{mercury}} g \Delta z_2 = P_{\text{atm}} + \rho_{\text{water}} g \Delta z_1 + \rho_{\text{mercury}} g \Delta z_2 = P_2$$



Note that if the interface is not flat, but *curved*, there will be a pressure difference across that interface.

Applications of Hydrostatics:

1. Mercury barometer – a device used to measure atmospheric pressure



$$P_1 = P_2 = P_{\text{atm}} \quad (\text{using our rules; see above})$$

$$P_2 = P_1 + \rho_{\text{Hg}} g h = P_{\text{atm}}$$

$$P_{\text{atm}} = \rho_{\text{Hg}} g h$$

"Head"

"Barometric pressure is 29 inches of Hg"

73.7 cm

$$P_{\text{atm}} = \rho_{\text{Hg}} g h = \left(13580 \frac{\text{kg}}{\text{m}^3}\right) \left(9.807 \frac{\text{m}}{\text{s}^2}\right) (0.737 \text{ m}) \left(\frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}\right) \left(\frac{\text{kPa} \cdot \text{m}^2}{1000 \text{ N}}\right) = \boxed{98.0 \text{ kPa}}$$

2. "Head" as a pressure measurement

$$P = \rho gh \quad \rightarrow \quad h = \text{"head"}$$

H_{eq} = pressure expressed as an equivalent column height of a fluid

Calculate P in Pa when $P = \text{"3.40 inches of water"}$

$$P = \rho gh = (1000 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(3.40 \text{ in}) \left(\frac{0.0254 \text{ m}}{\text{in}} \right) \left(\frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \right) \left(\frac{\text{Pa} \cdot \text{m}^2}{\text{N}} \right) = 846.9 \approx 847 \text{ Pa}$$

3. The U-tube manometer

Purpose:

Example: Pressure measurement with a U-tube manometer

Given: A U-tube manometer is used as an instrument to measure the pressure in a tank. The right leg of the manometer is open to atmospheric pressure.

(a) To do: Calculate the absolute and gage pressure P_A and $P_{A,gage}$ for the general case in which ρ_A is not small compared to ρ_m .

(b) To do: Simplify for the case in which $\rho_A \ll \rho_m$ (e.g., A is air and m is mercury).

Solution:

$$P_{\text{below}} = P_{\text{above}} + \rho g \Delta z$$

(a) $P_2 = P_{atm}$

$P_1 = P_1'$

start @ (1) & work around counterclockwise to (2)

$$P_A \oplus \rho_A g (z_A - z_2) \oplus \rho_A g (z_2 - z_1) \ominus \rho_m g (z_2 - z_1) = P_2 = P_{atm}$$

\downarrow || $P_1 = P_1'$

$$P_A = P_{atm} + (\rho_m - \rho_A) g (z_2 - z_1) - \rho_A g (z_A - z_2)$$

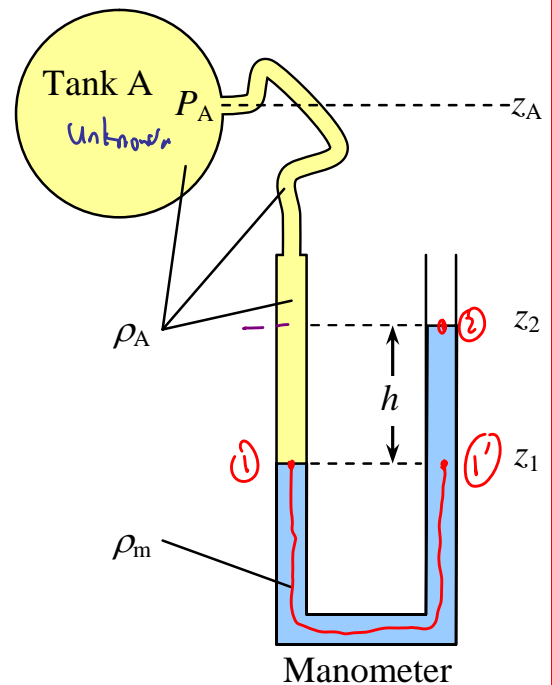
(b) if $\rho_m \gg \rho_A$

neglect compns to ρ_m

Do not neglect

$$P_A \approx P_{atm} + \rho_m g (z_2 - z_1) - \rho_A g (z_A - z_2)$$

It is best to keep all terms - do not approximate



Class Demo: U-tube manometer can measure both \oplus ve & \ominus ve pressure

Example: Pressure measurement with a U-tube manometer

Given: A U-tube manometer is used as a differential pressure measurement instrument to measure the pressure difference between two tanks. The two tanks are at the same elevation.

(a) To do: Calculate the pressure difference $P_B - P_A$ for the general case in which ρ_A is not the same as ρ_B (they are different fluids).

(b) To do: Simplify for the case in which $\rho_A = \rho_B$ (they are the same fluid).

Solution:

(a) start at A, work ccw to B

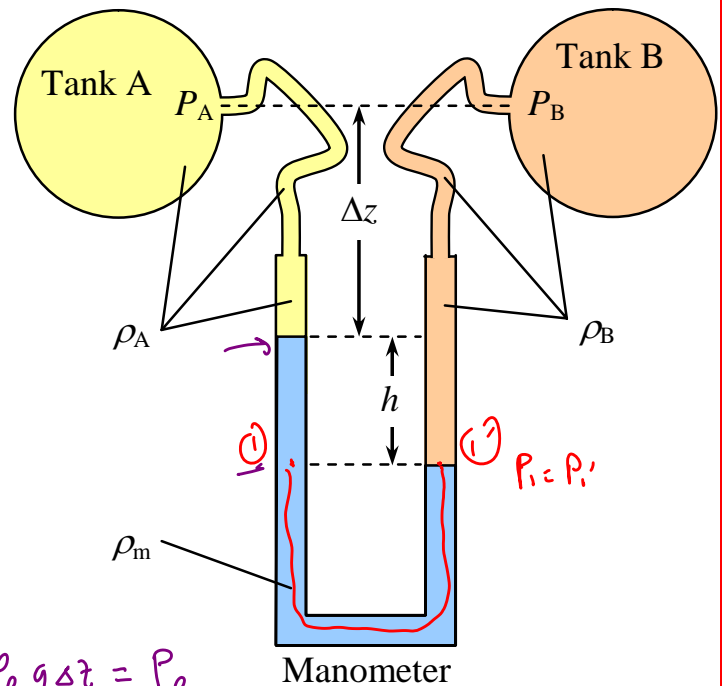
$$P_A + \rho_A g \Delta z + \rho_m g h - \rho_B g h - \rho_B g \Delta z = P_B$$

$$P_B - P_A = (\rho_m - \rho_B) g h + (\rho_A - \rho_B) g \Delta z$$

(b) if $\rho_A = \rho_B$

$$P_B - P_A = (\rho_m - \rho_B) g h$$

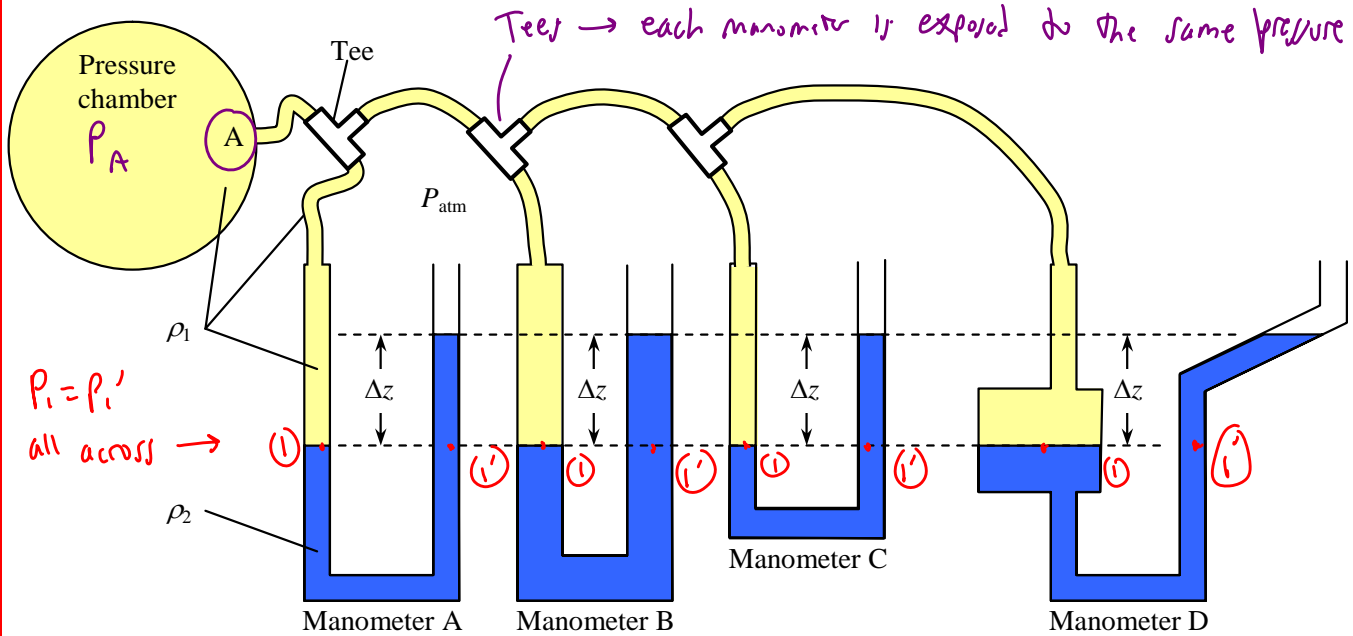
$\left[\Delta z \text{ drops out since both legs of the manometer have the same fluid} \right]$



4. Some Notes about Manometry [from pdf file on website]

The elevation difference Δz in a U-tube manometer does *not* depend on the following:

- U-Tube diameter** (provided that the tube diameter is large enough that capillary effects are negligible). In the sketch below, for a given pressure in the tank, Δz is the same in manometers A and B, even though the tube diameter of manometer B is larger than that of manometer A. Note that the amount of manometer liquid in each of the U-tube manometers has been adjusted such that the level of the interface between fluids 1 and 2 on the left side of each manometer is at the same elevation, for direct horizontal comparison.



Why?

$$P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z| \rightarrow \text{same, diameter is not in eq.}$$

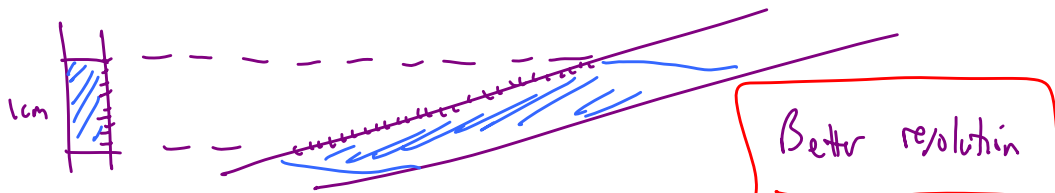
- U-Tube length** (provided that the tubes are long enough to include elevation difference Δz). In the sketch, Δz is the same in manometers A and C, even though manometer C is shorter than manometer A.

Why?

Compare A & C

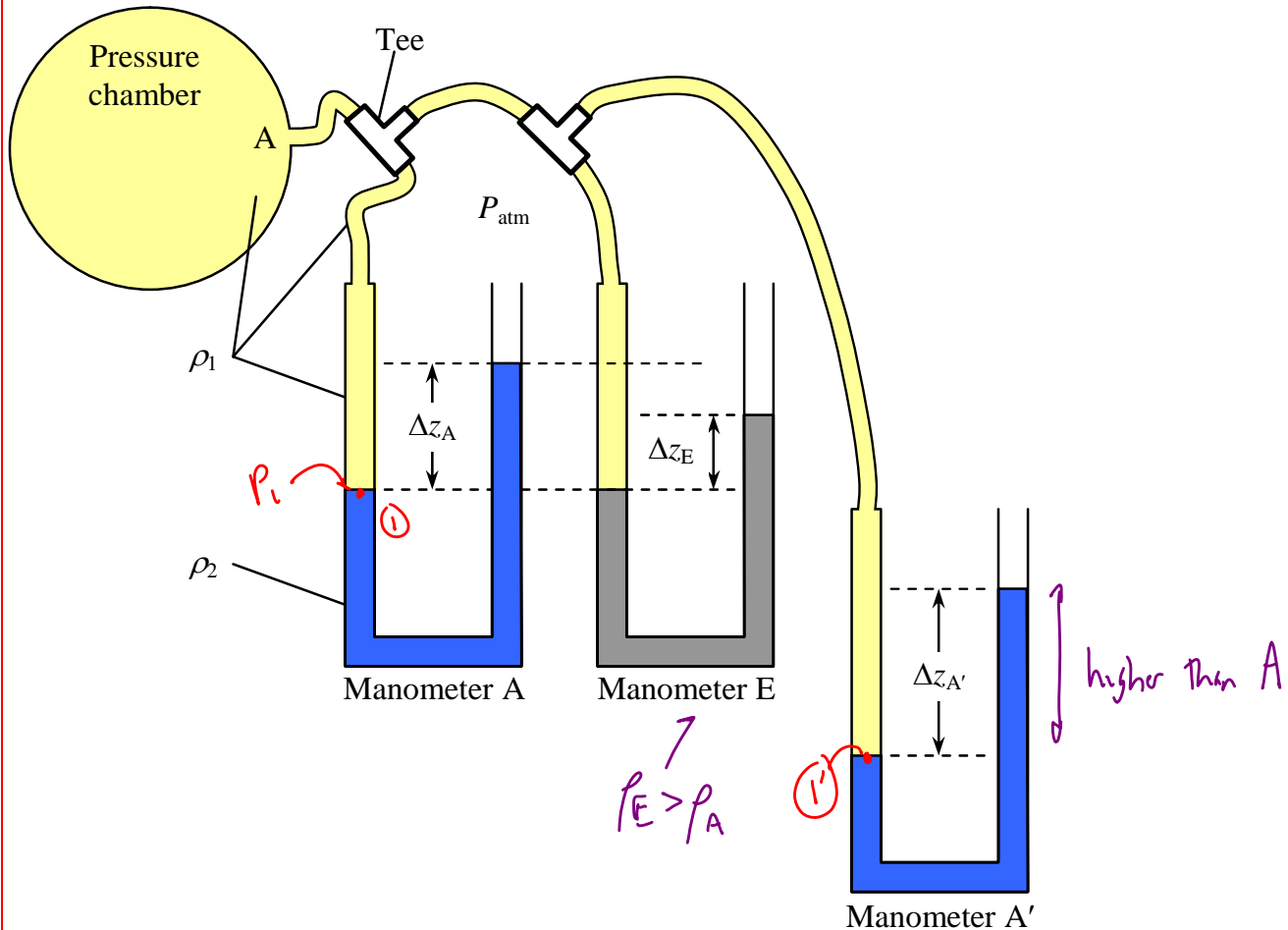
Below interface ①, nothing matters since it is the same fluid below ①

- U-Tube shape** (again provided that capillary effects are not important and the relative elevation is the same). In the sketch, Δz is the same in manometers A and D, even though manometer D is oddly shaped. Can you think of an advantage of the “inclined manometer” configuration of manometer D?

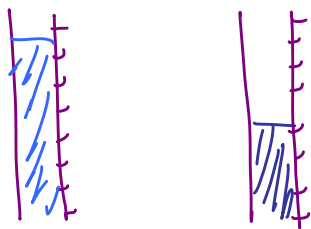


However, the elevation difference Δz in a U-tube manometer *does* depend on the following:

1. **Manometer fluid.** For example, if we replace the blue manometer fluid in the above sketch with a *higher density* (gray colored) fluid, as in the sketch below, Δz would *decrease*. In other words, $\Delta z_E < \Delta z_A$.



Which manometer (A or E) would have better *resolution*?



(A)

For same ΔP , A has a bigger Δz . \therefore better resolution

2. **Vertical location of the manometer.** For example, if we move manometer A to a lower elevation, all else being the same, and ignoring changes in atmospheric pressure (manometer A' in the above sketch), Δz would *increase*, i.e., $\Delta z_{A'} > \Delta z_A$. Why?

$\Delta z_{A'} > \Delta z_A$ since manometer A' is lower in elevation than manometer A.

$\therefore P_1' > P_1$ since (1') is lower than (1)

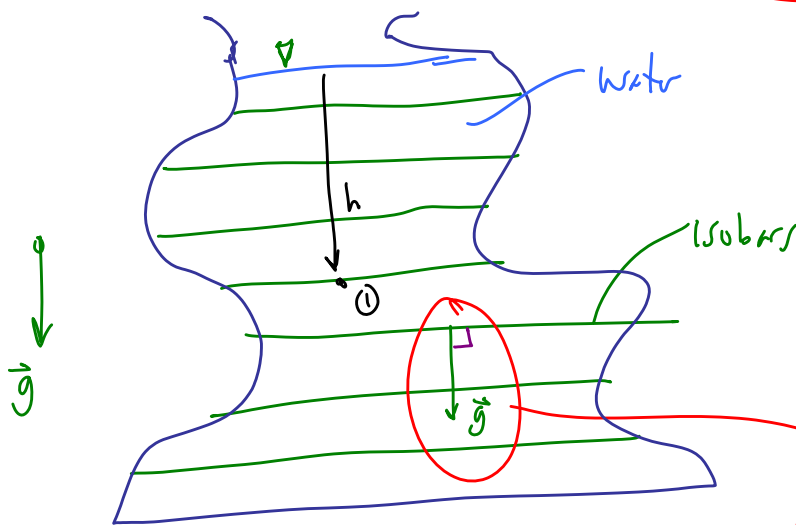
Note: if $\rho_1 \ll \rho_2$, then $\Delta z_{A'} \approx \Delta z_A$, regardless of the vertical location of the manometers. This is usually the case, for example, when fluid 1 is a gas, but the effect can be significant if both fluids are liquids.

5. Isobars

"bar" = pressure, "iso" = same

ISOBAR = a surface of constant pressure ★

In hydrostatics, $P = P(z)$ only \Rightarrow Isobars are horizontal ★



$$P_i = P_{atm} + \rho g h$$

In hydrostatics, ★
Isobars are \perp
(perpendicular) to gravity

(\perp to gravitational acceleration vector)

