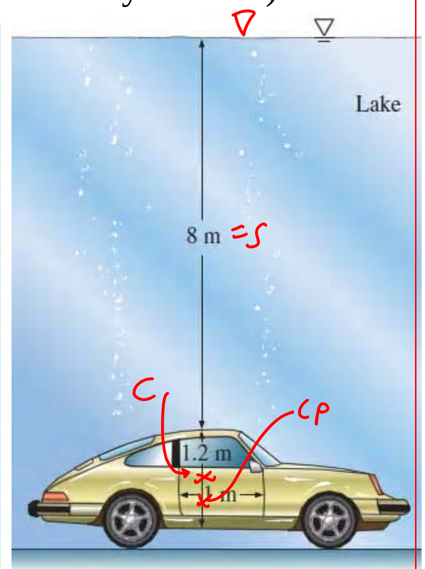
Today, we will:

- Do a review problem – hydrostatic forces on submerged bodies
- Discuss buoyancy and stability in hydrostatics
- Discuss fluids in rigid-body motion (linear acceleration and solid-body rotation)



EXAMPLE 3–8 **Hydrostatic Force Acting on the Door of a Submerged Car**

A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels (Fig. 3–34). The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.

SOLUTION A car is submerged in water. The hydrostatic force on the door is to be determined, and the likelihood of the driver opening the door is to be assessed.

Assumptions **1** The bottom surface of the lake is horizontal. **2** The passenger cabin is well-sealed so that no water leaks inside. **3** The door can be approximated as a vertical rectangular plate. **4** The pressure in the passenger cabin remains at atmospheric value since there is no water leaking in, and thus no compression of the air inside. Therefore, atmospheric pressure cancels out in the calculations since it acts on both sides of the door. **5** The weight of the car is larger than the buoyant force acting on it. → it sinks

Properties We take the density of lake water to be 1000 kg/m³ throughout.

Analysis The average (gage) pressure on the door is the pressure value at the centroid (midpoint) of the door and is determined to be

(use gage pressure everywhere)

y_c = depth of centroid = $s + b/2$ → @ C, @ $y = y_c$, $P_{avg} = P_c = \rho g y_c = P_{avg,\,gage}$

force $\boxed{F_R = P_{avg,\,gage} \cdot A = \rho g (s + b/2)(b)(a)}$ = answer in variables. Plug in #s $\boxed{F_R = 101.\ \text{kN}}$

· CP → where F_R acts, at CP $y = y_p$

rectangle $I_{xx,c} = ab^3/12$ compare

Eq (3-22b) → $y_p = y_c + \dfrac{I_{xx,c}}{y_c A} = \left(s + \dfrac{b}{2}\right) + \dfrac{ab^3}{12(s+b/2)(a)(b)} = \boxed{y_c = s + \dfrac{b}{2} + \dfrac{b^2}{12(s+b/2)}}$ = center of pressure

#s $\boxed{y_p = 8.61\ \text{m}}$

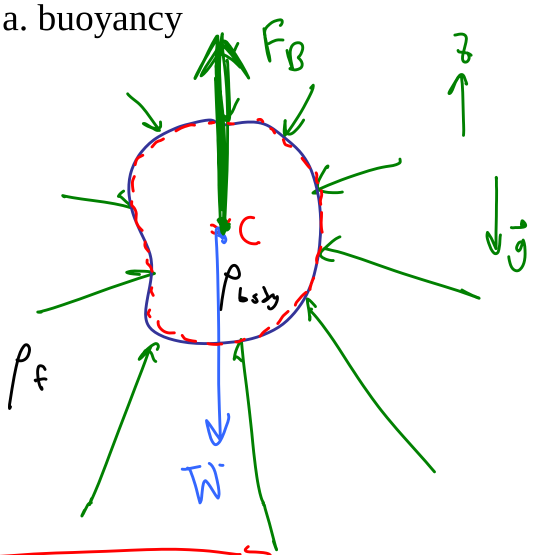
(just a little below the centroid, which is at $y_c = 8.60$ m)

Discussion A strong person can lift 100 kg, which is a weight of 981 N or about 1 kN. Also, the person can apply the force at a point farthest from the hinges (1 m farther) for maximum effect and generate a moment of 1 kN·m. The resultant hydrostatic force acts under the midpoint of the door, and thus a distance of 0.5 m from the hinges. This generates a moment of 50.6 kN·m, which is about 50 times the moment the driver can possibly generate. Therefore, it is impossible for the driver to open the door of the car. The driver's best bet is to let some water in (by rolling the window down a little, for example) and to keep his or her head close to the ceiling. The driver should be able to open the door shortly before the car is filled with water since at that point the pressures on both sides of the door are nearly the same and opening the door in water is almost as easy as opening it in air.

E. Hydrostatic Forces on Submerged Surfaces (continued)

3. Buoyancy and Stability

a. buoyancy



Since $\boxed{P_{below} = P_{above} + \rho g |\Delta z|}$

$$\boxed{W = \rho_{body}\, g\, \forall_{sub}}$$

Integrate these pressures $\times$ area $\rightarrow F_B$

F_B = buoyant force = resultant force from all the pressures acts @ centroid

$\boxed{\text{Archimedes' Principle}}$ $\rightarrow$ F_B = weight of the fluid displaced by the body

$$\boxed{F_B = \text{buoyant force} = \rho_f\, g\, \forall_{sub}}$$

$\boxed{\forall_{sub} = \text{submerged volume}}$

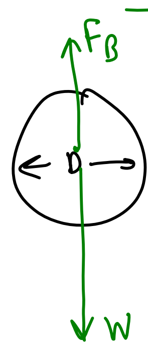
(and it acts upward through the centroid of the displaced volume)

Example: Buoyancy

Given: A sphere of diameter D = 0.0550 m and density ρ_{body} = 1700 kg/m^3 falls into a tank of water (ρ_f = 1000 kg/m^3).

To do: Calculate the net downward body force on the sphere due to gravity. in N

Solution:



Hint: $\forall_{sphere} = \dfrac{\pi D^3}{6}$

$F_B = \rho_f\, g\, \dfrac{\pi D^3}{6}$ (upward force)

$W = \rho_{body}\, g\, \dfrac{\pi D^3}{6}$ (downward force)

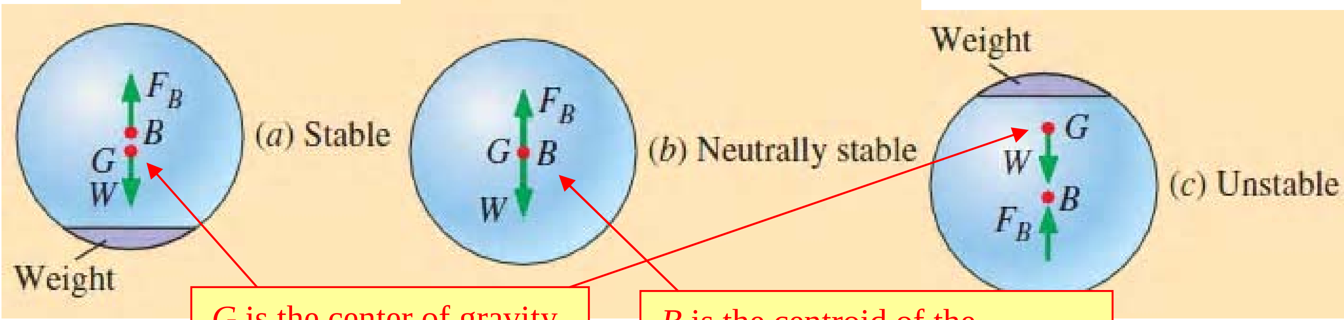
Net downward force = $\boxed{F_{down} = W - F_B = (\rho_{body} - \rho_f)\, g\, \dfrac{\pi D^3}{6}}$ = $\boxed{0.598\text{ N}}$

$$\left[\#'s \rightarrow F_{down} = (1700-1000)\frac{\cancel{kg}}{\cancel{m^3}}\left(9.807\frac{\cancel{m}}{\cancel{s^2}}\right)\frac{\pi (0.055\ \cancel{m})^3}{6}\left(\frac{N}{\cancel{kg \cdot m/s^2}}\right) = \underline{0.598\text{ N}}\right]$$

(down)

b. stability

Stability of a symmetric submerged body



G is the center of gravity (depends on how the weight is distributed inside the body)

B is the centroid of the submerged portion of the body = center of buoyancy, and also = center of rotation.

Bottom line: **The body is *unstable* if center of buoyancy *B* is *below* center of gravity *G*.**

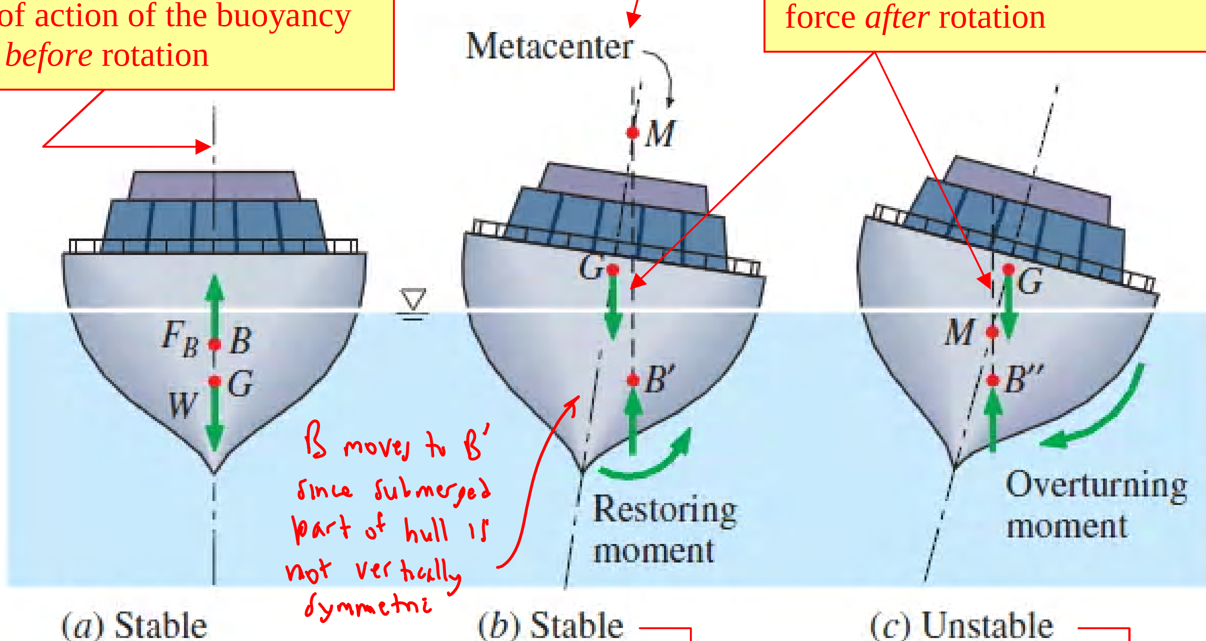
Stability of a boat (partially submerged)

Define *M* = the **metacenter** = the point where the line of action of the buoyancy force *before* rotation and the line of action of the buoyancy force *after* rotation intersect.

Line of action passes through B, the centroid

Line of action of the buoyancy force *before* rotation

Line of action of the buoyancy force *after* rotation



B moves to B′ since submerged part of hull is not vertically symmetric

Point *M* is *above* point *G*

Point *M* is *below* point *G*

Bottom line: **The boat is *unstable* if metacenter *M* is *below* center of gravity *G*.**

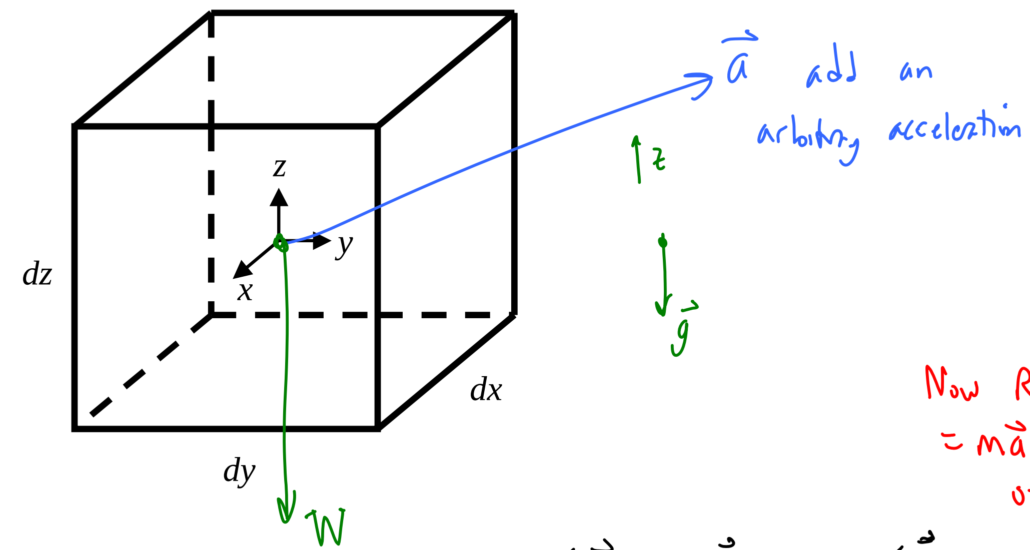
F. Fluids in Rigid-Body Motion (Section 3-7)

1. Equations

Key: In rigid-body motion, the fluid moves and accelerates as a rigid or solid body – no distortion, and therefore no shear stresses on a fluid element – only pressure and weight.

Consider a small fluid element of dimensions *dx*, *dy*, and *dz* as sketched previously.

For statics, we set $\sum \vec{F} = 0$. Here $\boxed{\sum \vec{F} = m\vec{a}}$



Now RHS $= m\vec{a}$ instead of 0

recall from notes for hydrostatics, $\sum \vec{F} = \sum \vec{F}_{pressure} + \sum \vec{F}_{gravity} = m\vec{a}$

$$\sum \vec{F} = -\underbrace{\left(\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}\right)}_{\vec{\nabla} P}\cancel{dx\,dy\,dz} + \rho\vec{g}\,\cancel{dx\,dy\,dz} = \rho\,\cancel{dx\,dy\,dz}\,\vec{a}$$

$\vec{\nabla} P$ = gradient of P

$\vec{\nabla}$ = del = gradient vector operator

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)$$

$$\rho\vec{g} - \vec{\nabla} P = \rho\vec{a}$$

$$\boxed{\vec{\nabla} P = \rho\left(\underbrace{\vec{g} - \vec{a}}_{\vec{G}}\right)} \quad (1)$$

$\boxed{\text{Eq. for a fluid in rigid-body acceleration}}$

I like to call $\boxed{\vec{G} = \vec{g} - \vec{a}}$ → $\boxed{\vec{\nabla} P = \rho \vec{G}}$ rigid body motion

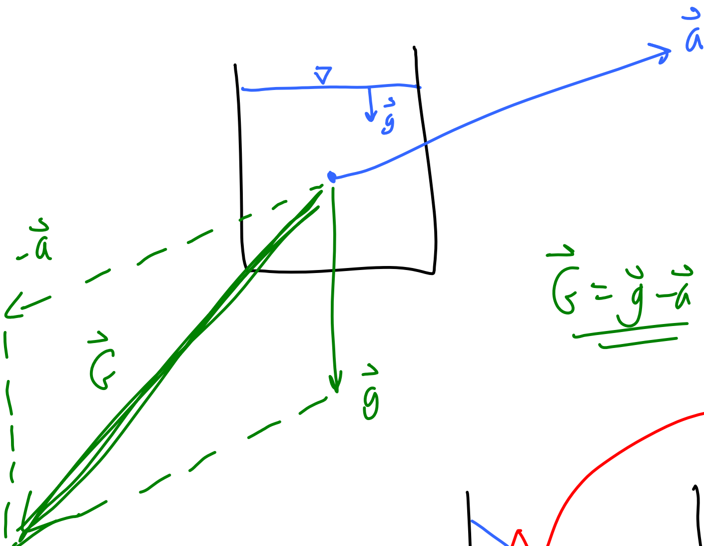
Compare to hydrostatics → $\boxed{\vec{\nabla} P = \rho \vec{g}}$ hydrostatics

if $\vec{a} = 0$ → $\vec{G} = \vec{g} - \cancel{\vec{a}}$ $\quad \underline{\underline{\vec{G} = \vec{g}}}$

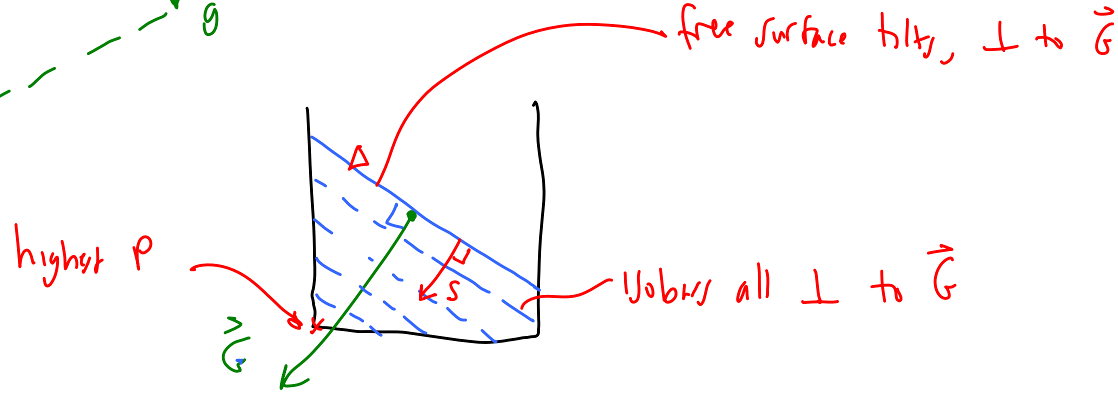
Pretend $\vec{G}$ replaces $\vec{g}$ → $\vec{G}$ is a pretend gravitational accel.

$$\boxed{\vec{G} = \vec{g} - \vec{a}} \ \star$$

2. Uniform linear acceleration



$\underline{\underline{\vec{G} = \vec{g} - \vec{a}}}$

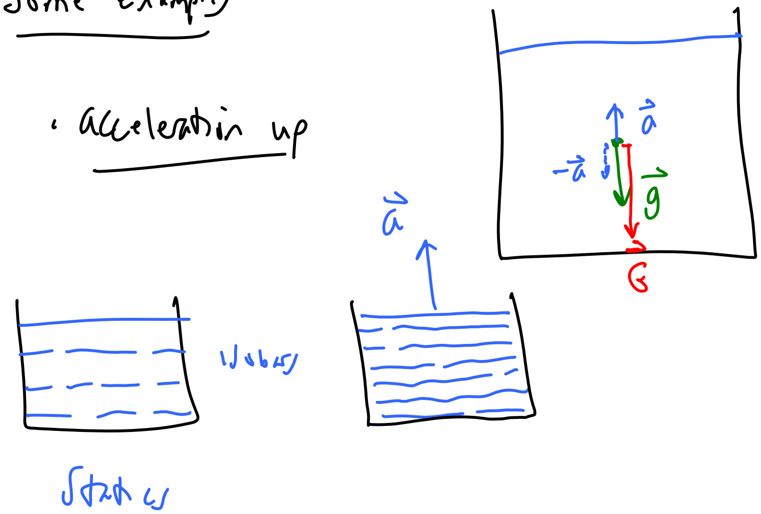


$$\boxed{P_{\text{"below"}} = P_{\text{"above"}} + \rho G |\Delta s|}$$

— same as hydrostatics, but replace $\vec{g}$ by $\vec{G}$ & replace $|\Delta z|$ by $|\Delta s|$

Some examples

· acceleration up



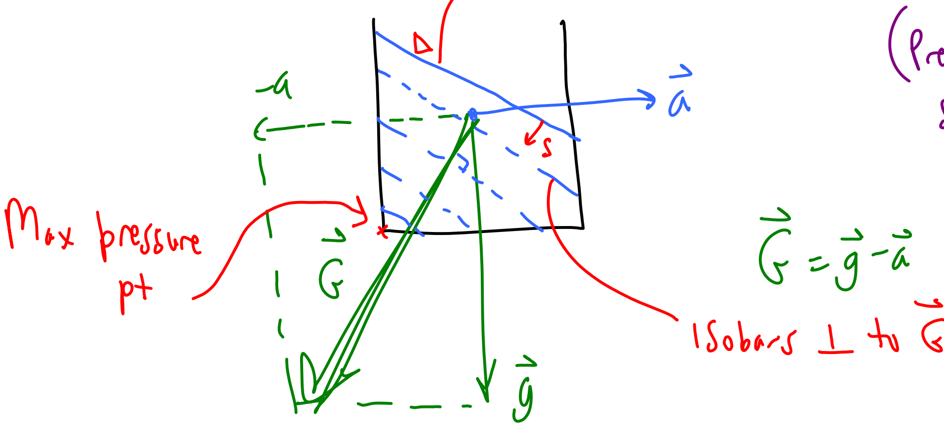
$\vec{G} = \vec{g} - a$

$\vec{G}$ acts down

$|\vec{G}| > |\vec{g}|$

(pretend you are on Jupiter, with a stronger gravity)

· accel to right



(Pretend you are on a strange planet with gravity acting at an angle)

$\vec{G} = \vec{g} - \vec{a}$

Bottom line → $\boxed{\text{Treat problems with rigid-body acceleration exactly the same as hydrostatics, but with } \vec{g} \text{ replaced by } \vec{G} = \vec{g} - \vec{a}}$