Today, we will:

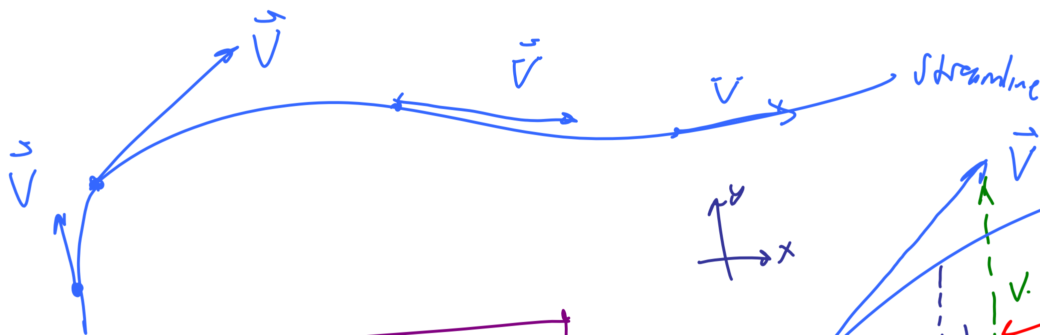
- Discuss various kinds of flow patterns and flow visualization techniques
- Discuss the motion and deformation of fluid particles
- Discuss linear strain, shear strain, and the strain rate tensor

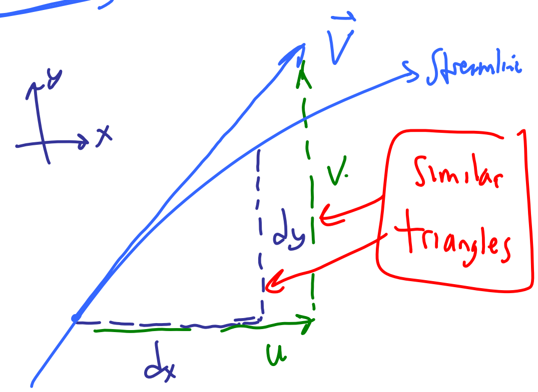
B. Flow Patterns and Flow Visualization (Section 4-2)

1. Streamlines, pathlines, streaklines, and timelines

a. Streamline: A streamline is a curve everywhere parallel to the local velocity field. (tangent)

Instantaneous → at one instant in time





Eq. → for 2-D streamline

$$\boxed{\left.\frac{dy}{dx}\right)_{\text{along a streamline}} = \frac{v}{u}}$$

Example: $\vec{V} = \underbrace{3x}_{u}\,\vec{i} \; \underbrace{-\,3y}_{v}\,\vec{j}$

Calc. eq. for the streamlines

$$\frac{dy}{dx} = \frac{v}{u} \quad \text{along a 2-D streamline}$$

$$\frac{dy}{dx} = \frac{-3y}{3x} = \frac{-y}{x}$$

Sep. variable $\quad \frac{dy}{y} = -\frac{dx}{x}$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\ln y = -\ln x + C_1$$

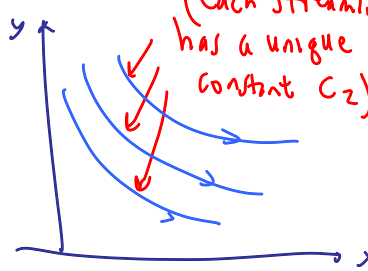
call $-\ln C$

$$\ln y = -\ln(cx) = \ln(cx)^{-1}$$

$e^{()} \rightarrow \quad y = \frac{1}{cx}$

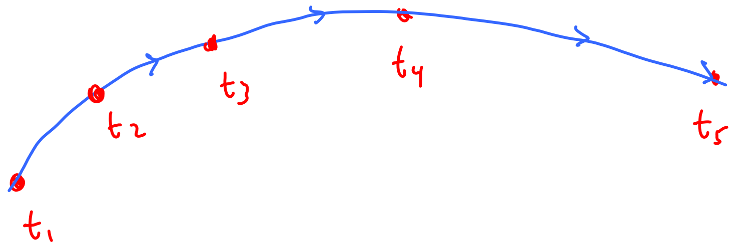
or $\boxed{y = \frac{C_2}{x}}$

[where $C_2 = \frac{1}{c}$]



(each streamline has a unique constant C_2)

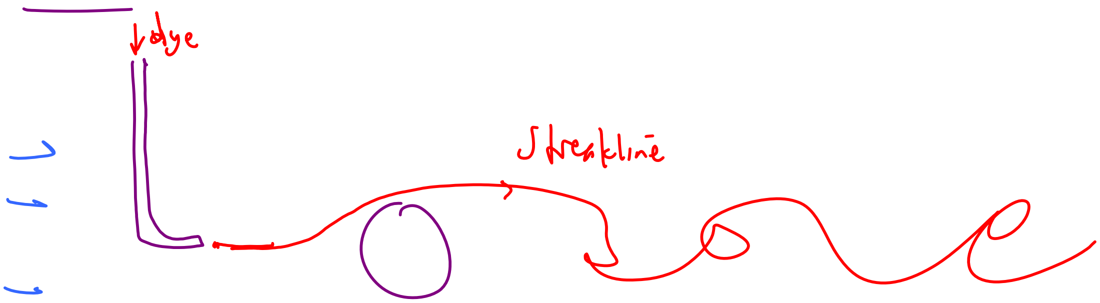
b. Pathline: A pathline is the path traveled by a marked fluid particle over some time period.



Pathline between t_1 & t_5

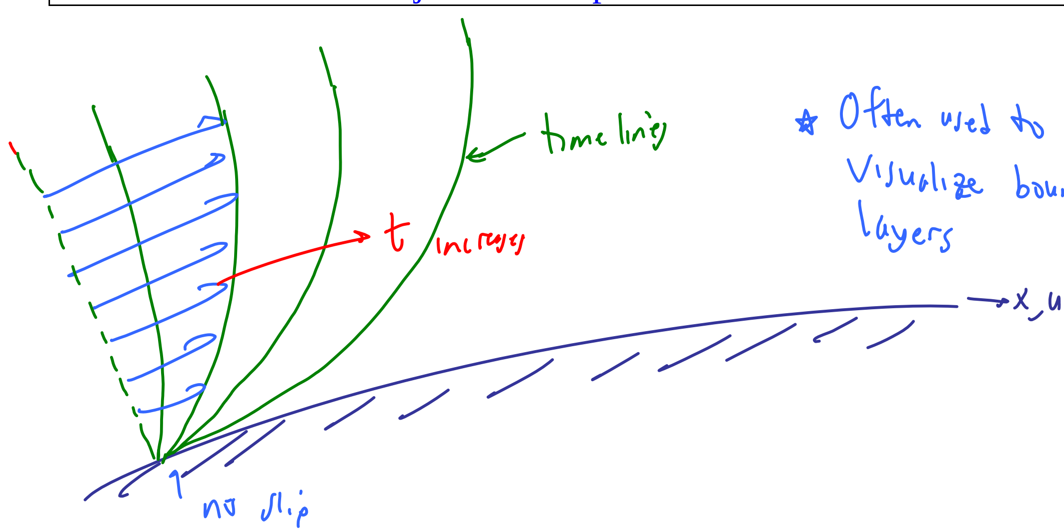
[Imagine → as a long time exposure photograph

c. Streakline: A streakline is the locus of fluid particles introduced at a point.



(Has history associated with it)

d. Timeline: A timeline is a *set* of adjacent fluid particles that were marked at the same time.

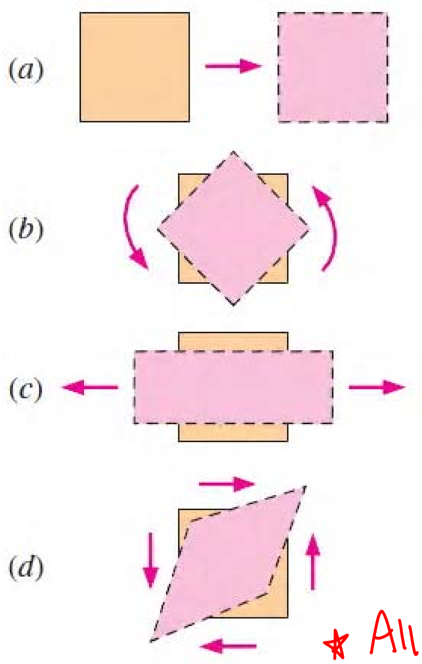


☆ Often used to visualize boundary layers

2. Other flow visualization techniques (Section 4.2) – read on your own.
3. Fluid flow plots (Sec. 4.3) – read on your own (self explanatory).
   Examples: profile plots, vector plots, contour plots.

C. Other Kinematic Descriptions (Section 4-4)

1. Motion and deformation of fluid particles. There are four fundamental types of fluid element motion or deformation:



Translation (fluid particle moves)

Rotation (" " rotates)

no deformation, just motion

Linear strain (fluid particle stretches)

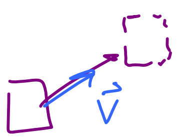
Shear strain (" " shears)

is deformation, no net motion

★ All 4 of these may happen simultaneously in a fluid flow!

In fluid mechanics, we prefer to use rates of motion or deformation

(a) rate of translation = velocity vector, $\vec{V} = \frac{dx_{particle}}{dt}\vec{i} + \frac{dy_{particle}}{dt}\vec{j} + \cdots$



i.e., $\boxed{\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}}$

(b) rate of rotation = angular velocity vector $\vec{\omega}$ (Greek omega)

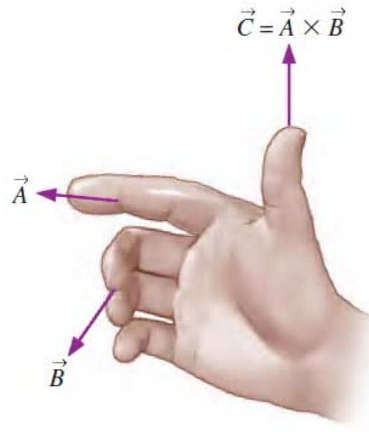
(See text for derivation)

$$\boxed{\text{Rate of rotation: } \vec{\omega} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}}$$ ★

Vorticity vector $\vec{\zeta}$ (Greek zeta) $\boxed{\vec{\zeta} = 2\vec{\omega}}$ ★

Vorticity and Rotationality (Section 4-5)

The **vorticity vector** is defined as the **curl of the velocity vector**, using the **right-hand rule**.



Greek letter zeta

$$\boxed{\vec{\zeta} = \vec{\nabla} \times \vec{V}}$$

It turns out that **vorticity is equal to twice the angular velocity of a fluid particle**,

$$\vec{\zeta} = 2\vec{\omega}$$

Thus, ***vorticity is a measure of rotation of a fluid particle***.

if $\vec{\zeta} = 0$, the flow is irrotational

if $\vec{\zeta} \neq 0$, the flow is rotational

Vorticity vector in Cartesian coordinates:

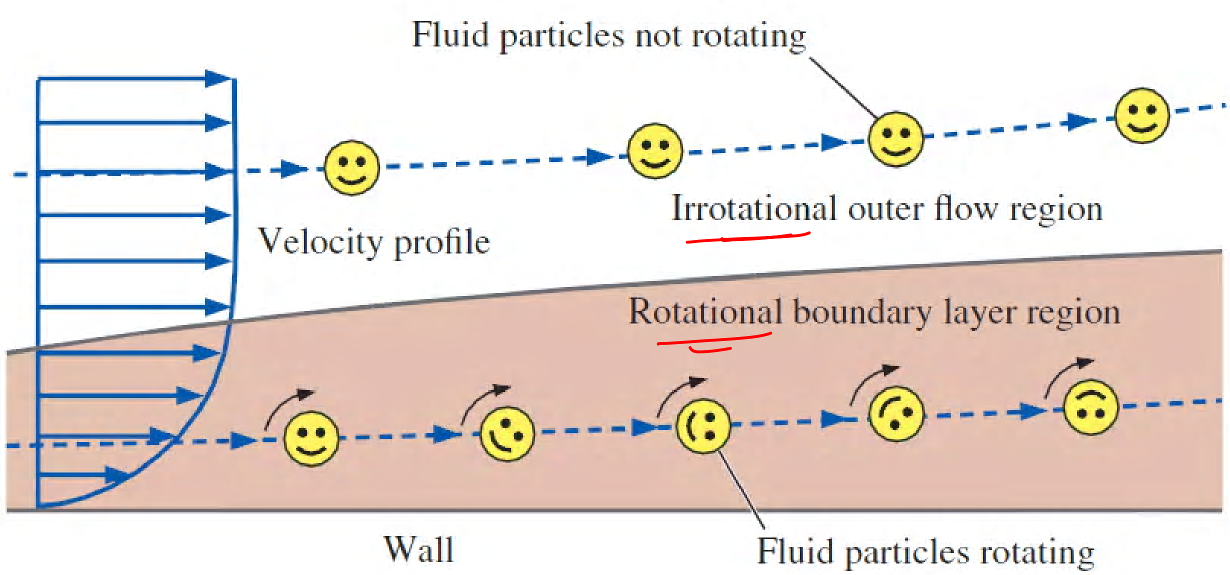
$$\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k} \qquad (4\text{–}30)$$

Vorticity vector in cylindrical coordinates:

$$\vec{\zeta} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\vec{e}_\theta + \frac{1}{r}\left(\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\vec{e}_z \qquad (4\text{–}32)$$

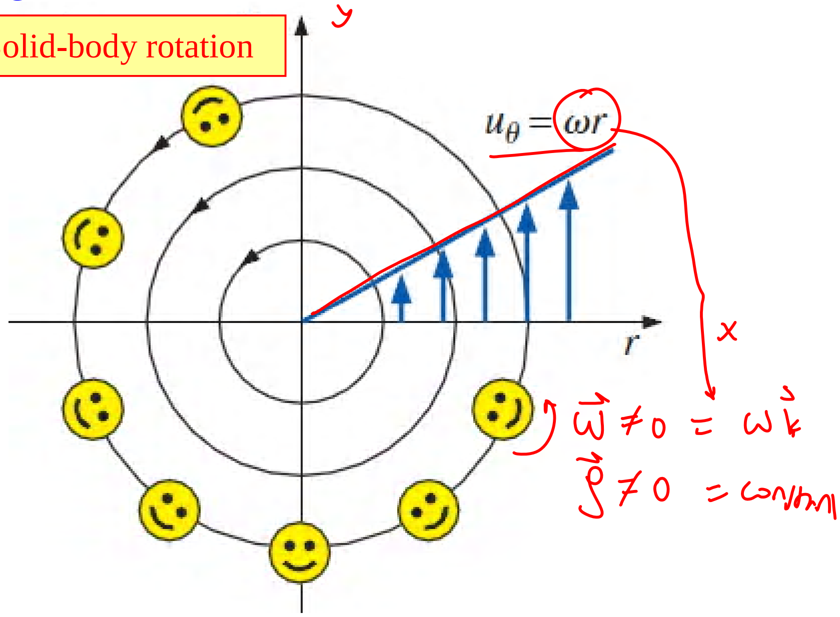
Examples:

1. Inside a **boundary layer**, where viscous forces are important, the flow in this region is *rotational* ($\vec{\zeta} \neq 0$). However, outside the boundary layer, where viscous forces are not important, the flow in this region is *irrotational* ($\vec{\zeta} = 0$).



2. A **solid-body rotation** (rigid-body rotation) flow is *rotational* ($\vec{\zeta} \neq 0$). In fact, since vorticity is equal to twice the angular velocity, $\vec{\zeta} = 2\vec{\omega}$ *everywhere* in the flow field. Fluid particles rotate as they revolve around the center of the flow. This is analogous to a merry-go-round or a roundabout.

Solid-body rotation



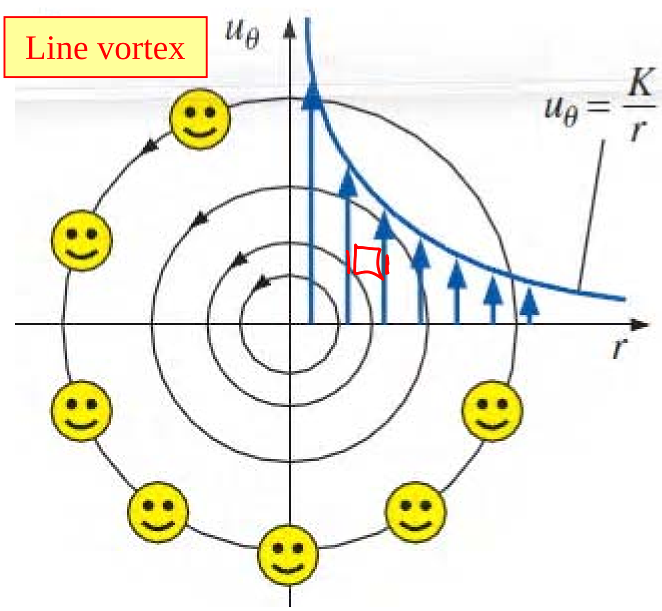
$\vec{\omega} \neq 0 = \omega \hat{k}$

$\vec{\zeta} \neq 0$ = constant

ROTATIONAL

3. A **line vortex** flow, however, is *irrotational* ($\vec{\zeta} = 0$), and fluid particles do not rotate, even though they revolve around the center of the flow. This is analogous to a Ferris wheel.

Line vortex



IRROTATIONAL

See text for details and calculations.

If $\vec{\zeta} = 0$ → flow is irrotational in that region

If $\vec{\zeta} \neq 0$ → " " rotational "

Example: Vorticity and irrotationality

Given: A two-dimensional velocity field in the *x*-*y* plane: $\boxed{\vec{V} = (u, v) = 2xy\vec{i} - y^2\vec{j}}$. ($w = 0$)

u v

To do: Is this flow rotational or irrotational?

Solution:

The rate of rotation is

A B

$$\vec{\omega} = \frac{1}{2}\left(\cancel{\frac{\partial w}{\partial y}} - \cancel{\frac{\partial v}{\partial z}}\right)\vec{i} + \frac{1}{2}\left(\cancel{\frac{\partial u}{\partial z}} - \cancel{\frac{\partial w}{\partial x}}\right)\vec{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$ and vorticity = $\vec{\zeta} = 2\vec{\omega}$.

2-D 2-D 2-D 2-D

0 0

$0 \quad -\frac{\partial}{\partial y}(2xy) \rightarrow = -2x$

$$\boxed{\begin{aligned}\vec{\omega} &= -x\vec{k}\\ \vec{\zeta} &= -2x\vec{k}\end{aligned}}$$

This flow is rotational

Example: Vorticity and irrotationality

Given: A two-dimensional velocity field in the *x*-*y* plane: $\boxed{\vec{V} = (u, v) = 3x\vec{i} - 3y\vec{j}}$. ($w = 0$)

To do: Calculate (**a**) the rate of translation and (**b**) the rate of rotation.

Solution:

(**a**) The rate of translation is simply the velocity vector,

$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

$$\boxed{\vec{V} = 3x\vec{i} - 3y\vec{j}}$$

$u = 3x$ → $\partial u/\partial y = 0$

$v = -3y$ → $\frac{\partial v}{\partial x} = 0$

$w = 0$

(**b**) The rate of rotation is

$$\vec{\omega} = \frac{1}{2}\left(\cancel{\frac{\partial w}{\partial y}} - \cancel{\frac{\partial v}{\partial z}}\right)\vec{i} + \frac{1}{2}\left(\cancel{\frac{\partial u}{\partial z}} - \cancel{\frac{\partial w}{\partial x}}\right)\vec{j} + \frac{1}{2}\left(\cancel{\frac{\partial v}{\partial x}} - \cancel{\frac{\partial u}{\partial y}}\right)\vec{k}$$ and vorticity = $\vec{\zeta} = 2\vec{\omega}$.

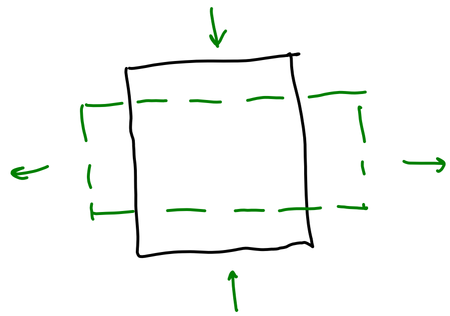
2-D 2-D 2-D 2-D 0 0

$\vec{\omega} = 0$ $\quad \boxed{\vec{\zeta} = 0}$

This flow is irrotational

Fluid particles do not rotate as they move

c. Linear strain rate due to normal stresses



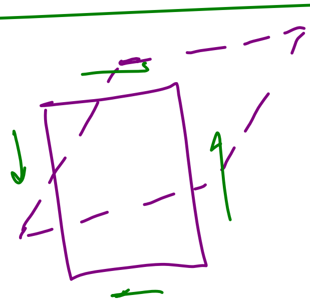
linear Strain = increase in length per unit length

rate of linear strain = linear strain rate

see text

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

d. Shear strain rate



Shear strain rate

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \qquad \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

2. Strain Rate Tensor

We can (conveniently) combine linear strain rates & shear strain rates into one

- 9-component
- 3x3
- tensor

$$\boxed{\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}}$$

* strain rate tensor