Today, we will:

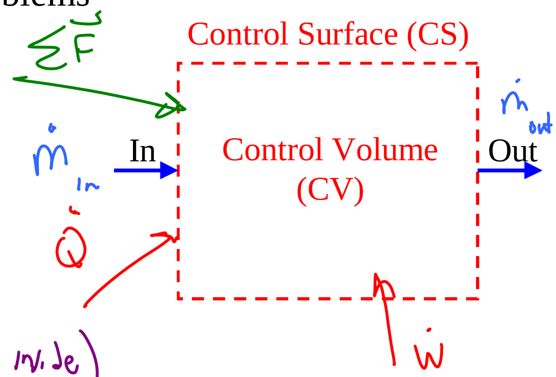
- Begin Chapter 5 – Conservation of mass and energy for control volumes
- Do some example problems, conservation of mass
- If time, begin to discuss conservation of energy

III. Conservation Laws and the Control Volume (Integral) Technique (Chapter 5)

A. Introduction

1. Overview – Techniques for solving fluid flow problems

a. Control volume analysis (Ch. 5, 6, 8)

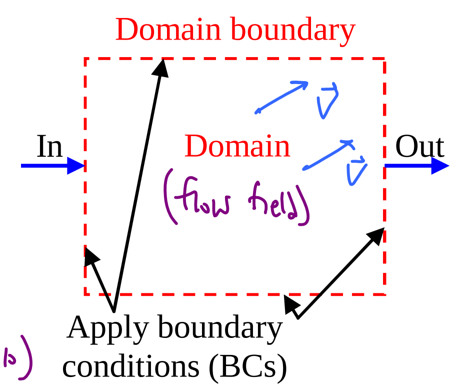


- We solve the integral (CV) equations
- We calculate gross (overall) properties only
- Treat CV as a "black box" (we care about the boundaries, not the details inside)

b. Dimensional analysis and experiment (Ch. 7)

- Don't solve any eq's
- Use Dimensions to form nondimensional parameters – helps us design experiments
- Use wind tunnels, etc. to measure the flow

c. Differential analysis (Ch. 9, 10, 15)



- Solve the differential eqs of motion
- Get details everywhere in the domain
- Can do analytically (simple problems) (Ch 9 & 10)

... computationally CFD (computational fluid dynamics) – Ch. 15

B. Conservation of Mass

1. Equations and definitions

From previous lecture…**the conservation of mass equation for a fixed control volume:**

RTT gives →

$$\boxed{\underbrace{\frac{d}{dt}\int_{CV}\rho dV}_{\substack{\text{rate of change}\\ \text{of mass}\\ \text{within the CV}}} + \underbrace{\int_{CS}\rho\left(\vec{V}\cdot\vec{n}\right) dA}_{\substack{\dot m_{cs}\\ \text{net rate of}\\ \text{mass flow}\\ \text{out of the CV through the CS}}} = 0} \quad \text{Cons. of mass} \qquad (1)$$

rate of change of mass within the CV + net rate of mass flow out of the CV through the CS = 0

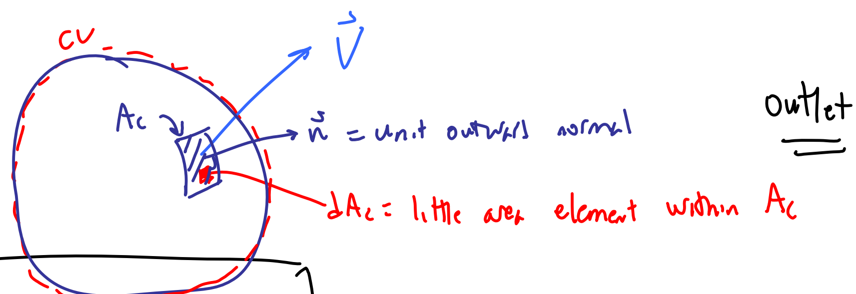
if ⊕ + ⊖ = 0

if ⊖ + ⊕ = 0

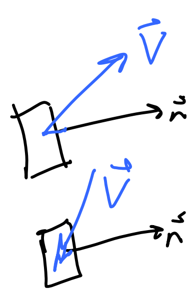
if 0 + 0 = 0

↓ steady; ↓ no net mass flow out of CV

- Mass flow rate



$$\boxed{\dot m_{A_c} = \int_{A_c} \rho\left(\vec V\cdot\vec n\right) dA_c}$$ = mass flow rate out of area A_c



Outlet, $\dot m_{A_c} > 0$

Inlet, $\dot m_{A_c} < 0$

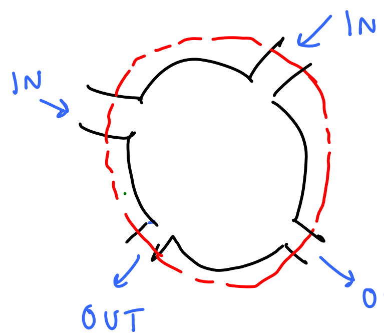
Signs take care of themselves

For the entire CS, we write

$$\boxed{\dot m_{cs} = \int_{CS} \rho\left(\vec V\cdot\vec n\right) dA}$$ = net mass flow rate out of the CV through the CS

It is easier if there are several separate inlets & outlets

Simplification



$$\boxed{\dot{m}_{CS} = \sum_{out} \dot{m} - \sum_{in} \dot{m}}$$

Then Eq (1) becomes

$$\frac{d}{dt}\underbrace{\int_{CV} \rho \, d\forall}_{m_{cv}} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

or

$$\boxed{\underbrace{\frac{dm_{cv}}{dt}}_{\text{rate of change of mass within the CV}} = \underbrace{\sum_{in} \dot{m} - \sum_{out} \dot{m}}_{\text{net rate of mass flow into the CV through inlets \& outlets}}}$$

If flow is steady, then $\frac{dm_{cv}}{dt} = 0 \rightarrow \boxed{\sum_{in} \dot{m} = \sum_{out} \dot{m}}$ Steady CV

"What goes in must go out"

• Volume flow rate (usually used with incompressible flow — const. density)

$$\boxed{\dot{\forall}_{A_c} = \int_{A_c} (\vec{V} \cdot \vec{n}) \, dA_c}$$

same as $\dot{m}_{A_c}$ but without the ρ

[We use $\dot{\forall}$ for volume flow rate. Some other textbooks use Q for $\dot{\forall}$]

• Define Average Velocity through A_c

$$\boxed{V_{avg, A_c} = \frac{1}{A_c} \int_{A_c} \vec{V} \cdot \vec{n} \, dA_c} \rightarrow \boxed{V_{avg, A_c} = \frac{\dot{\forall}}{A_c}}$$

- For a given inlet or outlet, A_c we can make this approximation:

$$\boxed{\dot{m}_{A_c} \approx \rho_{avg} V_{avg} A_c}$$

- we usually drop the "avg" subscript (to be lazy)

$$\boxed{\dot{m}_{A_c} = \rho V A_c} \rightarrow \text{implies } V = V_{avg}, \ \rho = \rho_{avg}$$

or $$\boxed{\dot{m}_{A_c} = \rho \dot{\forall}_{A_c}}$$

Finally, if flow is also incompressible $[\rho \approx \text{const}]$ and steady

$$\sum_{out} \dot{m} = \sum_{in} \dot{m}$$

$$\sum_{out} (\cancel{\rho} \dot{\forall}) = \sum_{in} (\cancel{\rho} \dot{V}) \quad (\rho \approx \text{const})$$

So,

$$\boxed{\sum_{in} \dot{\forall} = \sum_{out} \dot{\forall}}$$

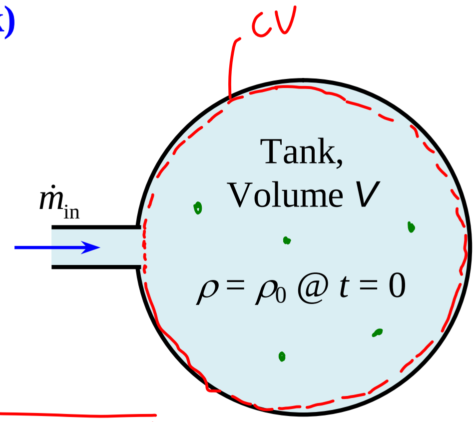
cons. of volume flow rate

Volume flow rate in = volume flow rate out

2. Examples

Example: Unsteady conservation of mass (flow into a tank)

Given: Air is pumped into a rigid tank of volume V. The mass flow rate of the air entering the tank is constant, $\dot{m}_{in}$. We assume that the process is slow enough that the air in the tank remains at the same temperature (isothermal conditions).



To do: Generate an equation for density ρ in the tank as a function of time.

Solution:

$\boxed{\text{• First step in any CV analysis is to pick \& draw a CV. !}}$

• Use cons. of mass   Fixed control volume, but compressible & unsteady

$$\frac{d}{dt}\int_{CV} \rho \, d\forall = \sum_{in} \dot{m} - \cancel{\sum_{out} \dot{m}}$$

0 (no outlets)

only 1 inlet

$$\frac{d\rho}{dt}\int_{CV} d\forall = \dot{m}_{in} - 0$$

$\forall$ of tank

(Approx that $\rho \approx$ constant inside the CV but changes with time)

$[\rho = \rho(t)$ only$] \rightarrow$ $\frac{d\rho}{dt} = \frac{\dot{m}_{in}}{\forall}$ = a constant $\rightarrow$ integrate: from $t=0$ $(\rho = \rho_0)$ to t

$$\rho = \frac{\dot{m}_{in}}{\forall} t + \text{const}$$

When $t=0$, $\rho = \rho_0$

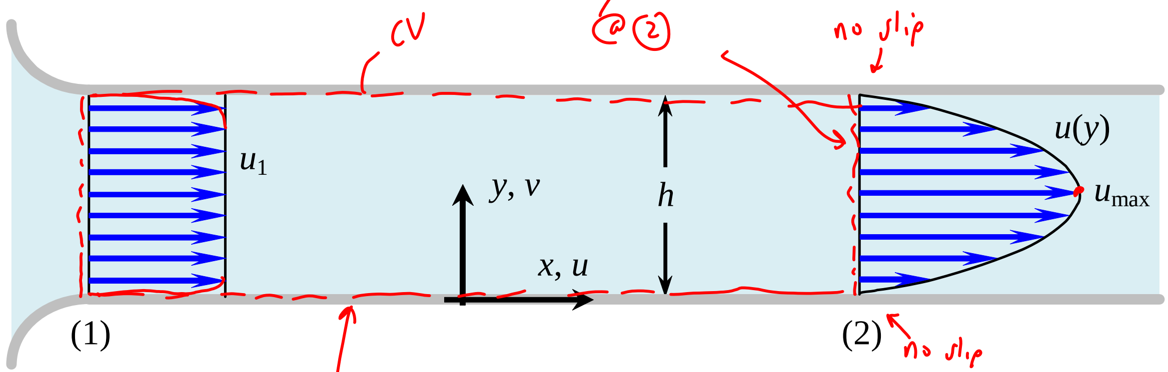
$\therefore$ const $= \rho_0$

$$\boxed{\rho = \rho_0 + \frac{\dot{m}_{in}}{\forall} t}$$

Example: Velocity profiles in 2-D channel flow

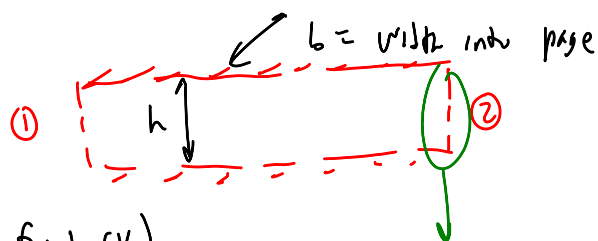
Given: Consider steady, incompressible, two-dimensional flow of a liquid between two very long parallel plates as sketched. At the inlet (1) there is a nice bell mouth, and the velocity is nearly uniform (except for a very thin boundary layer, not shown).

- At (1), $u = u_1$ = constant, $v = 0$, and $w = 0$.
- At (2), the flow is fully developed, and $u = ay(h-y)$, $v = 0$, and $w = 0$, where a is a constant.



To do: Generate expressions for constant a and speed u_{max} in terms of the given variables.

Solution: Flow setup: Draw a CV

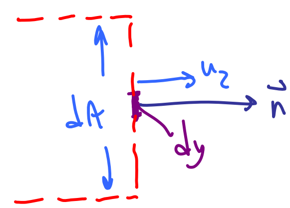


· Cons. of mass (steady, incomp, fixed CV)

$$\sum_{in} \dot{V} = \sum_{out} \dot{V}$$

$$u_1 h \cdot b = \int_{A_2} (\vec{V}\cdot\vec{n})\, d\vec{A}$$

$\vec{V}\cdot\vec{n} = u_2$ $\quad dA = b\,dy$



$$u_1 h b = b \int_{y=0}^{y=h} u_2\, dy$$

☆ $$\boxed{u_1 h \cancel{b} = \cancel{b} \int_{y=0}^{y=h} ay(h-y)\, dy}$$

Integrate this (try it on your own for practice).

Get $\boxed{u_1 h = \frac{ah^3}{6}}$

$$\boxed{a = \frac{6u_1}{h^2}}$$

Answer for const. a