Today, we will:

- Discuss conservation of energy for a control volume
- Do an example problem – energy equation with a comresssor
- Discuss the kinetic energy correction factor

C. Conservation of Energy

1. Equations and definitions

From previous lecture…**the conservation of energy equation for a fixed control volume:**

(from the RTT)

$$\boxed{\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS} e\rho\left(\vec{V}\cdot\vec{n}\right)dA} \qquad (2)$$

net rate of heat transfer into the CV + net rate of work done on the CV = rate of change of total energy inside the CV + net rate of energy flow out of the CV

Power

Internal energy U → per unit mass = u (specific internal en.)

k.e. = $\frac{mV^2}{2}$ → per mass = $\frac{V^2}{2}$ (k.e. per unit mass)

p.e. = mgz → per mass = gz (p.e. per unit mass)

But from thermodynamics, specific total energy $\boxed{e = u + \frac{V^2}{2} + gz}$. Thus, (2) becomes

$$\boxed{\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS}\left(u + \frac{V^2}{2} + gz\right)\rho\left(\vec{V}\cdot\vec{n}\right)dA} \qquad (3)$$

let's work on this power term

(move this pressure power term into this term)

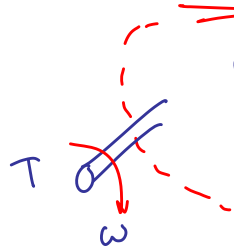
$$\dot{W}_{\text{net in}} = \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} + \dot{W}_{\text{viscous, net in}} + \dot{W}_{\text{other, net in}}$$

$$\dot{W}_{\text{pressure, net in}} = -\int_{CS} \frac{P}{\rho}\rho\left(\vec{V}\cdot\vec{n}\right)dA$$

(+) inlet (push fluid in)

(−) outlet (push fluid out)

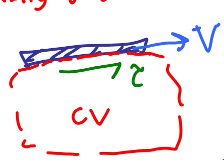
$\boxed{\dot{W}_{shaft} = \omega T}$



(+) for a pump (adds power)

(−) for a turbine (extracts power)

usually zero



(zero when wall is stationary)

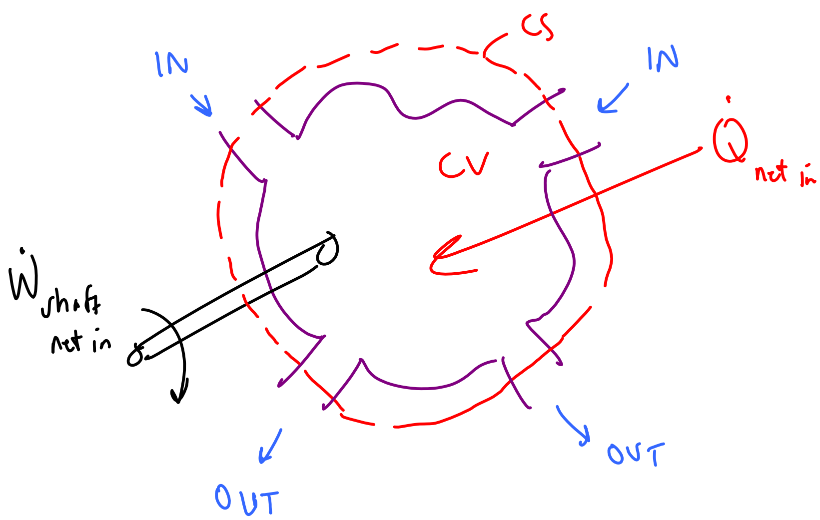
always zero in our course (electromagnetic etc.)

Eq (3) becomes [put pressure term inside the CS term]

$$\boxed{\dot{Q}_{net\ in} + \dot{W}_{shaft,\,net\ in} = \frac{d}{dt}\int_{CV} e\rho\, d\forall + \int_{CS}\Big(\underbrace{\Big(u+\frac{P}{\rho}\Big)}_{\text{specific enthalpy, } h}+\frac{V^2}{2}+gz\Big)\rho\big(\vec{V}\cdot\vec{n}\big)dA}$$

This eq is exact for a fixed CV with no viscous or "other" power terms

- Simplify for well-defined inlets & outlets



Approximate the CS integral at inlets & outlets as follows:

$$\star\ \boxed{\dot{Q}_{net\ in} + \dot{W}_{shaft,\,net\ in} = \frac{d}{dt}\int_{CV}\rho e\, d\forall + \sum_{out}\Big[\dot{m}\Big(h+\frac{V^2}{2}+gz\Big)\Big] - \sum_{in}\Big[\dot{m}\Big(h+\frac{V^2}{2}+gz\Big)\Big]}$$

these $\left(h+\frac{V^2}{2}+gz\right)$ terms are average over inlets & outlets

For steady flow, we drop the $\cancel{\frac{d}{dt}\int_{CV} \rho e \, d\forall}$ term

0

We now have the steady-state, steady flow form of the 1st law of thermo for a C.V.

SSSF energy eq for a fixed CV

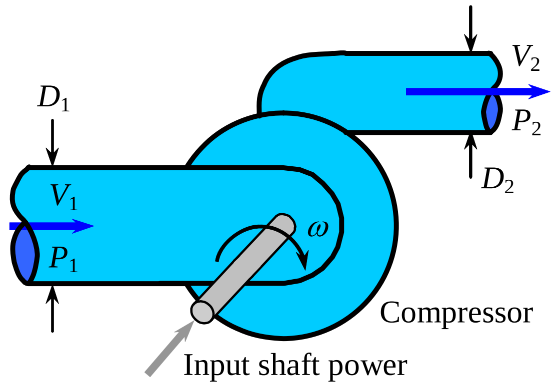
You should be familiar with this eq. from thermodynamics class

Steady-state, steady-flow (SSSF) form of the 1st Law of Thermo. for a fixed CV:

$$\boxed{\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)} \qquad (4)$$

Example: Control volume energy equation applied to an air compressor

Given: A large air compressor takes in air at absolute pressure P_1 = 14.0 psia, at temperature T_1 = 80°F (539.67 R), and with mass flow rate $\dot{m}$ = 20.0 lbm/s. The diameter of the compressor inlet is D_1 = 24.5 inches. At the outlet, P_2 = 70.0 psia and T_2 = 500°F (959.67 R). The diameter of the compressor outlet is D_2 = 7.50 inches. The shaft driving the compressor supplies 3100 horsepower to the compressor.



(a) **To do**: Calculate the average velocity of the air entering the compressor.

Solution: At the inlet, $\dot{m} \approx \rho_{1,\text{ avg}} V_{1,\text{ avg}} A_1 = \rho_1 V_1 A_1$ where the subscripts "avg" have been dropped for convenience. Thus, $V_1 = \dfrac{\dot{m}}{\rho_1 A_1} = \dfrac{4\dot{m}}{\pi \rho_1 D_1^2} = \dfrac{4RT_1\dot{m}}{\pi P_1 D_1^2}$, where we have used the ideal gas law $P = \rho RT$ to calculate the density of the air. Substitution of the values yields

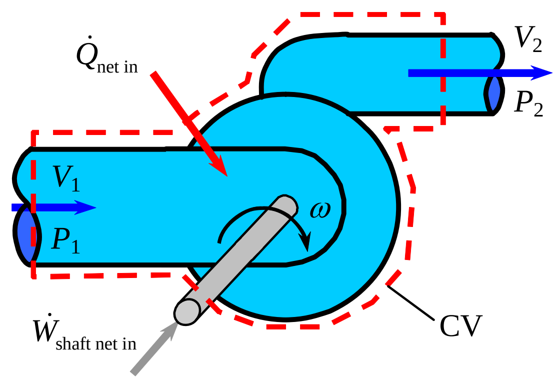
$$V_1 = \frac{4RT_1\dot{m}}{\pi P_1 D_1^2} = \frac{4\left(53.34\,\dfrac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot\text{R}}\right)(539.67\text{ R})\left(20.0\,\dfrac{\text{lbm}}{\text{s}}\right)}{\pi\left(14.0\,\dfrac{\text{lbf}}{\text{in}^2}\right)(24.5\text{ in})^2} = 87.229\,\frac{\text{ft}}{\text{s}} \approx \mathbf{87.2\,\frac{ft}{s}}$$

(b) **To do**: Calculate the average velocity of the air leaving the compressor.

Solution: Similarly, using the pressure, temperature, and diameter at the compressor outlet, we get V_2 = 331.051 ft/s, or $\mathbf{V_2 = 331.\ ft/s}$ (to three significant digits of precision)

(c) **To do**: Calculate the net rate of heat transfer from the air compressor into the room in units of Btu/hr.

Solution: First we choose a control volume. We draw the control volume around the entire compressor, cutting through the shaft, and cutting through the inlet and outlet, as sketched. Note that we draw the net rate of heat transfer $\dot{Q}_{\text{net in}}$ *into* the control volume to keep the signs straight. We expect a negative value since the compressor will actually give off heat into the room.



Next, we apply the approximate form of the control volume energy equation,

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} = \underbrace{\cancel{\frac{d}{dt}\int_{CV} e\rho\, dV}}_{\text{steady}} + \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$ SSSF energy eq.

and we solve for $\dot{Q}_{\text{net in}}$.

Solution to be completed in class.

$$\dot{Q}_{net\ in} = -\dot{W}_{shaft\ net\ in} + \dot{m}\left(h_2 + \frac{V_2^2}{2} + g\cancel{z_2}\right) - \dot{m}\left(h_1 + \frac{V_1^2}{2} + g\cancel{z_1}\right)$$

Outlet ② gas (neglect) inlet ① gas

$\dot{m}$ = same at ① & ②

$$\boxed{\dot{Q}_{net\ in} = \text{''} + \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right)}$$

ideal gas → $\boxed{h_2 - h_1 = C_p(T_2 - T_1)}$

avg. between T_1 & T_2

Numbers:

(unity conversion factors)

$$\dot{Q}_{net\ in} = -\underbrace{(3100\ hp)}_{\dot{W}_{shaft}}\left(\frac{2544.5\ Btu}{hp\cdot hr}\right) + \left(20.0\ \frac{lbm}{s}\right)\left[\underbrace{\left(6019\ \frac{ft^2}{s^2 R}\right)}_{C_p}(\underbrace{959.67}_{T_2} - \underbrace{539.67}_{T_1})R + \frac{(\underbrace{331.051\ ft/s}_{V_2})^2 - (\underbrace{87.229\ ft/s}_{V_1})^2}{2}\right]\left(\frac{lbf\cdot s^2}{32.174\ lbm\ ft}\right)\left(\frac{1\ Btu}{778.169\ lbf\cdot ft}\right)\left(\frac{3600\ s}{hr}\right)$$

$$\boxed{\dot{Q}_{net,in} = -4.71\times 10^5\ Btu/hr}$$

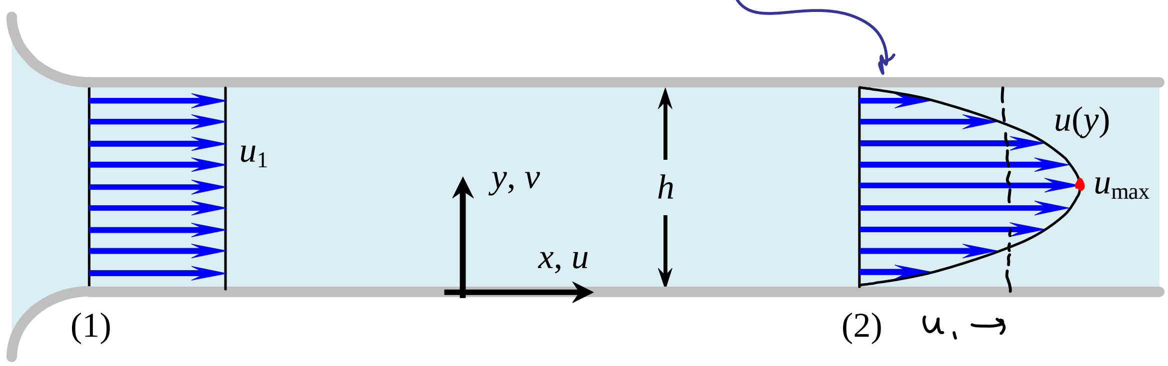
≈ 94 room air conditioners (5000 Btu/hr each)

That's a lot of heat!

Example from previous lecture: Velocity profiles in 2-D channel flow

Given: Consider steady, incompressible, two-dimensional flow of a liquid between two very long parallel plates as sketched. At the inlet (1) there is a nice bell mouth, and the velocity is nearly uniform (except for a very thin boundary layer, not shown).

- At (1), $u = u_1$ = constant, $v = 0$, and $w = 0$.
- At (2), the flow is fully developed, and $u = ay(h - y)$, $v = 0$, and $w = 0$, where a is a constant.



To do: Generate expressions for constant a and speed u_{max} in terms of the given variables.

Solution:

We used conservation of mass, and integrated to solve for constant a,

$$\boxed{a = \frac{6u_1}{h^2}}$$

Now, since we also know that at location 2, $\boxed{u = ay(h - y)}$, we can solve for u_{max}.

@ $y = h/2 = u_{max}$

$$u_{max} = \frac{6u_1}{\cancel{h^2}} \cdot \frac{\cancel{h}}{2} \cdot \frac{\cancel{h}}{2} = \boxed{\frac{3u_1}{2} = u_{max}}$$

If u_1 = 3.0 m/s, calculate u_{max} in m/s. $\boxed{4.5 \text{ m/s} = u_{max}}$

NOTICE: 1) Stay in variable form – h drops out

2) u_{max} is > u_1 since near the walls, the speed is < u_1 → must be > u_1 in middle to ensure that avg. speed = u_1