Today, we will:

- Discuss the kinetic energy correction factor
- Derive the "head" form of the energy equation ← most useful form (workhorse eq)
- Discuss pumps and turbines and their efficiencies
- Do an example problem – energy equation with pumps and turbines

C. Conservation of Energy (continued)

3. The kinetic energy correction factor

From previous lecture…the Steady-State Steady-Flow (**SSSF**) conservation of energy equation for a fixed control volume (no shear work term and no "other" work terms) for fixed known inlets and outlets:

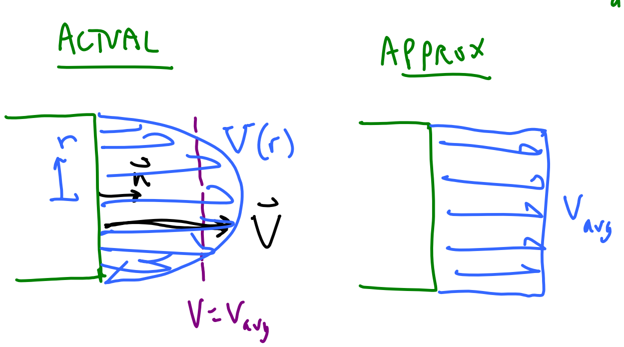
average over outlet avg. over inlet

$$\boxed{\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \sum_{\text{outlets}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{inlets}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)}$$

The summation terms on the right are actually *approximations* of the *exact* integral form of these terms. For steady flow the exact form is

$$\boxed{\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \int_{CS}\left(u + \frac{V^2}{2} + gz\right)\rho\left(\vec{V}\cdot\vec{n}\right)dA}$$

$V = V_{avg}$ at an inlet or outlet



$\frac{1}{2}\rho\int V^2\left(\vec{V}\cdot\vec{n}\right)dA$

$\vec{V}\cdot\vec{n} = V = $ fnc. of r for round pipe

$= \frac{1}{2}\rho\int V^3\,dA \neq \rho V A \frac{V^2}{2}$

in fact " > "

Introduce a kinetic energy correction factor

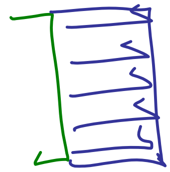
$$\boxed{\alpha = \frac{1}{A}\int\left(\frac{V}{V_{avg}}\right)^3 dA}$$

$\alpha \geq 1$ for any profile

Corrected SSSF conservation of energy equation for a fixed control volume:

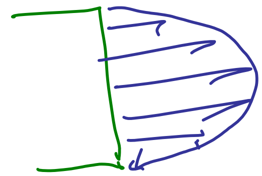
$$\boxed{\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \sum_{\text{outlets}} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right) - \sum_{\text{inlets}} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right)}$$

- Uniform flow



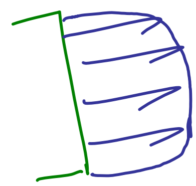
$V = \text{const} \rightarrow \boxed{\alpha = 1}$

- Fully developed laminar pipe flow



$\boxed{\alpha = 2}$

- Fully developed turbulent pipe flow



$\alpha = 1.04$ to 1.11

★ Let's use $\boxed{\alpha = 1.05}$ for turbulent fully dev. pipe flow

We will include α in all our problems

Example → air compressor example from last lecture

– re-do with $\alpha = 1.05$ @ inlet & outlet

get $\dot{Q}_{net\ in} = \boxed{-4.64 \times 10^5\ \text{Btu/hr}}$ ← more accurate answer by including α ★

recall, without α (approx $\alpha = 1$), we had $\dot{Q}_{net\ in} = -4.71 \times 10^5\ \frac{\text{Btu}}{\text{hr}}$

(diff about 1.5%)

For laminar flow, α makes a much bigger difference ($\alpha = 2$)

C. Conservation of Energy (continued)

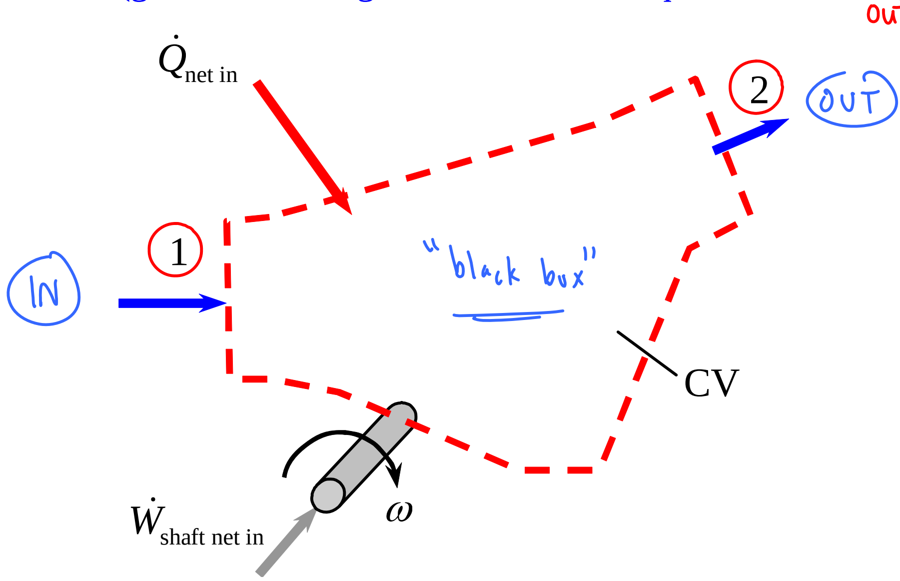
4. The "head" form of the energy equation

Start with the SSSF conservation of energy equation for a fixed control volume:

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \cancel{\sum_{\text{outlets}}} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right) - \cancel{\sum_{\text{inlets}}} \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right) \qquad (1)$$

Assumptions and approximations:

1. steady (we already removed the unsteady term in Eq. 1)
2. only one inlet (get rid of the sigma for inlets in Eq. 1 – call the inlet **1**)
3. only one outlet (get rid of the sigma for outlets in Eq. 1 – call the ~~inlet~~ outlet **2**)



$h = u + \frac{P}{\rho}$

Equation (1) becomes

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right)_{2\text{ (outlet)}} - \dot{m}\left(h + \alpha\frac{V^2}{2} + gz\right)_{1\text{ (inlet)}}$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net shaft in}} = \dot{m}\left(u + \frac{P}{\rho} + \alpha\frac{V^2}{2} + gz\right)_{2} - \dot{m}\left(u + \frac{P}{\rho} + \alpha\frac{V^2}{2} + gz\right)_{1}$$

$\boxed{\dot{m}_1 = \dot{m}_2 = \dot{m}}$ → Cons. of mass

Now divide each term by $\dot{m}g$ and rearrange,

$$\frac{\dot{W}_{\text{net shaft in}}}{\dot{m}g} + \left(\frac{P}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_1 = \left(\frac{P}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_2 + \frac{u_2 - u_1}{g} - \frac{\dot{Q}_{\text{net in}}}{\dot{m}g}$$

$= h_L$ = irreversible head loss $= \{L\}$

The "**head form**" of the energy equation:

$$\frac{\dot{W}_{\text{net shaft in}}}{\dot{m}g} + \left(\frac{P}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_1 = \left(\frac{P}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_2 + h_L \qquad (2)$$

Dimensions of this term $\{L\}$

h_L always > 0

$$\boxed{h_L = \frac{u_2 - u_1}{g} - \frac{\dot{Q}_{net\ in}}{\dot{m} g}}$$

If ideal → no friction
no irreversible losses
of any kind

IDEAL →

then $\boxed{u_2 - u_1 \text{ would } = \frac{\dot{Q}_{net\ in}}{\dot{m}}}$ (reversible)

Internal energy would rise only because of heat transfer

REAL LIFE → Friction & other irreversibilities exist

∴ Internal energy rise is greater than the net heat transfer in

$$\boxed{u_2 - u_1 > \frac{\dot{Q}_{net\ in}}{\dot{m}}} \quad \therefore \boxed{h_L > 0}$$

$$\dot{W}_{shaft\ net\ in} = \cancel{\sum} \dot{W}_{pump} - \cancel{\sum} \dot{W}_{turbine}$$

[typically only one pump or turbine, so usually remove the $\sum$'s]

Head form of en. eq:

$$\left(\frac{P}{\rho g} + \alpha \frac{V^2}{2g} + z\right)_1 + \sum h_{pump,u} = \left(\frac{P}{\rho g} + \alpha \frac{V^2}{2g} + z\right)_2 + \sum h_{turbine,e} + h_L$$

Finally, here is the head form of the energy equation in its most useful form:

$$\boxed{\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^{\,2}}{2g} + z_1 + \sum h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^{\,2}}{2g} + z_2 + \sum h_{\text{turbine},e} + h_L} \qquad (3)$$

The head form of the energy equation for *one* pump and *one* turbine:

$$\boxed{\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump},u}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine},e}+h_L}$$

This is our "workhorse" eq. We will use it a lot

$h_{\text{pump},u}$ = useful head supplied by the pump

PUMPS

"pump head" → $$\boxed{h_{\text{pump},u}=\eta_{\text{pump}}\frac{\dot{W}_{\text{pump}}}{\dot{m}g}}$$ ← Do not include the pump inefficiency in the h_L term

$$\boxed{0<\eta_{\text{pump}}<1}$$

$$\boxed{\eta_{\text{pump}}=\text{pump efficiency}=\frac{\text{useful power to fluid}}{\text{actual shaft power}}}$$

TURBINES

let $$\boxed{\eta_{\text{turbine}}=\text{turbine efficiency}=\frac{\text{actual shaft power out}}{\text{power extracted by turbine}}}$$

$$\boxed{\eta_{\text{pump}}=\frac{\text{water horsepower}}{\text{brake horsepower}}<1}$$

useful

shaft power

Let "bhp" = brake horsepower = actual shaft power

$$\boxed{\eta_{\text{turbine}}=\frac{\text{brake horsepower}}{\text{water horsepower}}<1}$$

h_L does not include the pump & turbine losses *

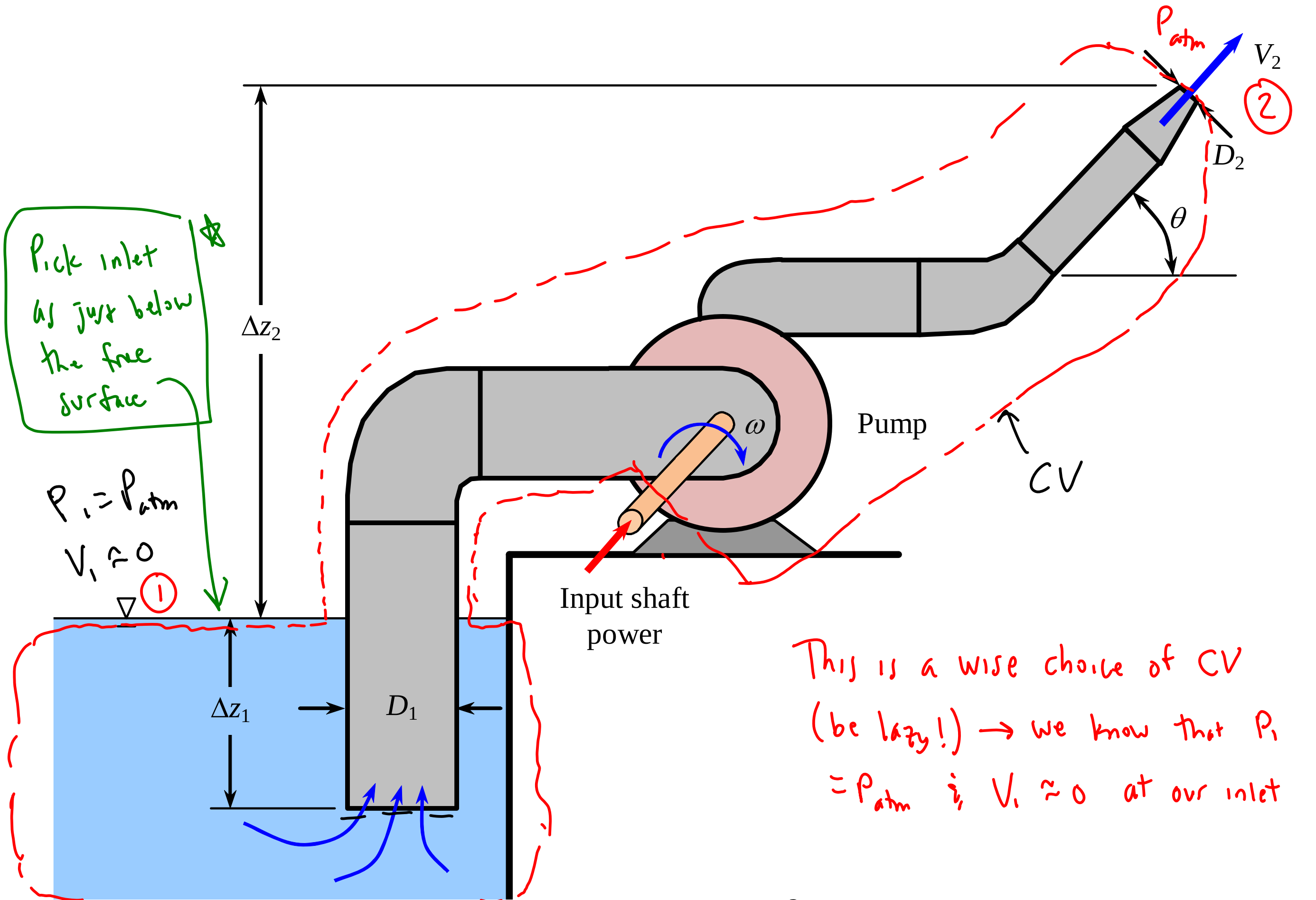
$h_{\text{turbine},e}$ = extracted head, due to the turbine

$$\boxed{h_{\text{turbine},e}=\frac{1}{\eta_{\text{turbine}}}\frac{\dot{W}_{\text{turbine}}}{\dot{m}g}}$$

$$\boxed{0<\eta_{\text{turbine}}<1}$$

Example – Fire-fighting water pump

Given: A self-priming pump is used to draw water from a lake and shoot it through a nozzle, as sketched. The diameter of the pump inlet is D_1 = 12.0 cm. The diameter of the nozzle outlet is D_2 = 2.54 cm, and the average velocity at the nozzle outlet is V_2 = 65.8 m/s. The pump efficiency is 80%. The vertical distances are Δz_1 = 1.00 m and Δz_2 = 2.00 m. The irreversible head losses in the piping system (not counting inefficiencies associated with the pump itself) are estimated as h_L = 4.50 m of equivalent water column height. *Note*: Later on, in Chapter 8, you will learn how to calculate the irreversible head losses associated with piping systems on your own. For now, they are given.



(**a**) **To do**: Calculate the volume flow rate of the water in m³/hr and gallons per minute (gpm).

Solution: At the outlet, $\dot{V} = V_{2,\text{ avg}} A_2 = V_2 \dfrac{\pi D_2^{\,2}}{4} = \left(65.8 \dfrac{\text{m}}{\text{s}}\right) \dfrac{\pi \left(0.0254 \text{ m}\right)^2}{4} = 0.033341 \dfrac{\text{m}^3}{\text{s}}$, where we have dropped the subscript "avg" for convenience. We convert to the required units as follows: $\dot{V} = 0.033341 \dfrac{\text{m}^3}{\text{s}} \left(\dfrac{3600 \text{ s}}{\text{hr}}\right) = \mathbf{120.\ \dfrac{m^3}{hr}}$ and $\dot{V} = 0.033341 \dfrac{\text{m}^3}{\text{s}} \left(\dfrac{15{,}850 \text{ gpm}}{\text{m}^3/\text{hr}}\right) = \mathbf{528.\ gpm}$, where both answers are given to three significant digits of precision.

(**b**) **To do**: Calculate the power delivered by the pump to the water, i.e. calculate the ***water horsepower*** $\dot{W}_{\text{water horsepower}}$ in units of kW.

(**c**) **To do**: Calculate the required shaft power to the pump, i.e. calculate the ***brake horsepower*** bhp in units of kW.

Solutions for parts (b) and (c) to be completed in class.

First, pick a control volume (always the first step!). ✓

Now apply the head form of the energy equation:

(b)

$$\boxed{\cancel{\frac{P_1}{\rho g}} + \alpha_1 \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\approx 0} + z_1 + h_{\text{pump},u} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \cancel{h_{\text{turbine},e}} + h_L}$$

$P_1 = P_2 = P_{atm}$

$$\boxed{h_{\text{pump},u} = \underbrace{\alpha_2 \frac{V_2^2}{2g}}_{A} + \underbrace{(z_2 - z_1)}_{B} + \underbrace{h_L}_{C}} \quad ☆$$

$= \Delta z_2$ in figure

The pump does 3 things: ☆

(A) Increases the kinetic energy of the water

(B) " " potential " " " "

(C) overcomes irreversible head losses

Solve for $\dot{W}_{\text{water horsepower}}$:

$$\boxed{\dot{W}_{\text{water horsepower}} = \dot{m} g h_{\text{pump},u} = \rho \dot{V} g \left[\alpha_2 \frac{V_2^2}{2g} + \Delta z_2 + h_L\right]}$$

Answer in variables

• Plug in numbers:

(@ T = 20°C)

Assume fully developed turbulent pipe flow at the outlet

$$\dot{W}_{\text{water horsepower}} = \left(998 \frac{\text{kg}}{\text{m}^3}\right)\left(0.033341 \frac{\text{m}^3}{\text{s}}\right)\left(9.807 \frac{\text{m}}{\text{s}^2}\right)\left[1.05 \frac{(65.8 \text{ m/s})^2}{2(9.807 \text{ m/s}^2)} + 2.00 \text{ m} + 4.50 \text{ m}\right]$$

$$\cdot \left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{\text{W}}{\text{N}\cdot\text{m/s}}\right)\left(\frac{\text{kW}}{1000 \text{ W}}\right) = 77.756 \text{ kW} \approx \boxed{77.8 \text{ kW}}$$

(c) $\text{bhp} = \dfrac{\dot{W}_{\text{water horsepower}}}{\eta_{\text{pump}}} = \dfrac{77.756 \text{ kW}}{0.80} = \boxed{97.2 \text{ kW}}$

☆ Notice: bhp > $\dot{W}_{\text{water horsepower}}$ due to pump inefficiencies