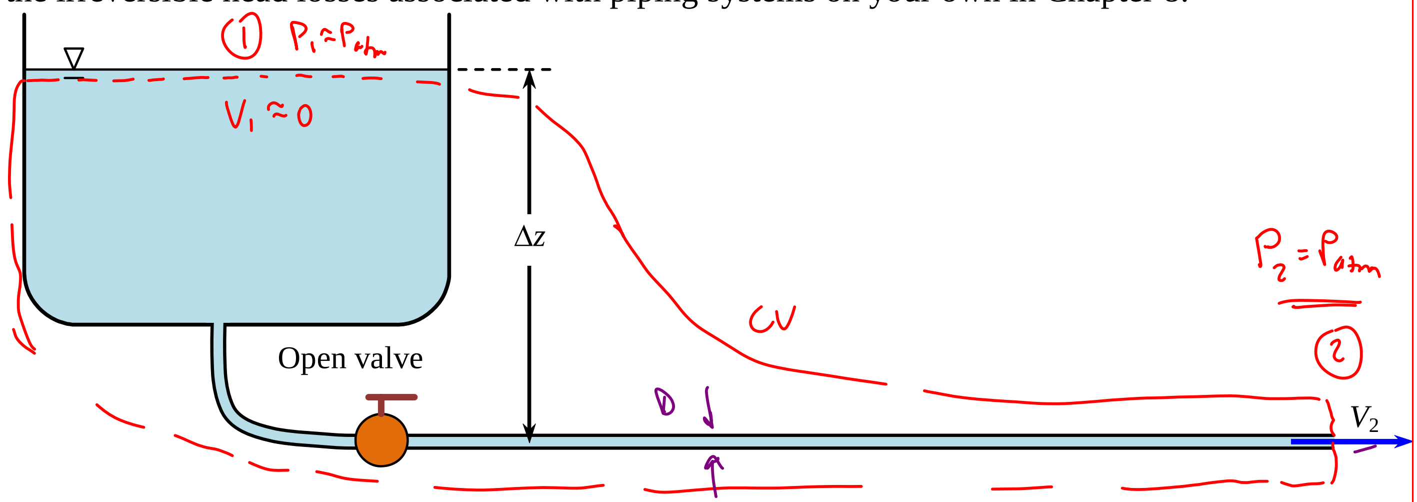
Today, we will:

- Do another example problem – head form of the energy equation
- Discuss grade lines – energy grade line and hydraulic grade line
- Derive and discuss the Bernoulli equation

5. Examples (continued)

Example – Water draining from a tank

Given: Water drains by gravity from a tank exposed to atmospheric pressure. The vertical distance from the pipe outlet to the surface of the water in the tank is Δz = 0.500 m. The irreversible head losses in the piping system (due to friction in the pipe, losses through the valve, elbow, etc.) are estimated as h_L = 0.400 m of equivalent water column height. *Note*: You will learn how to calculate the irreversible head losses associated with piping systems on your own in Chapter 8.



To do: Calculate the average velocity at the outlet, V_2.

Solution:

From previous lecture…use the head form of the conservation of energy equation:

$$\boxed{\left(\cancel{\frac{P_1}{\rho_1 g}}+\alpha_1\cancel{\frac{V_1^2}{2g}}+z_1\right)+\cancel{\sum h_{\text{pump,u}}}=\left(\cancel{\frac{P_2}{\rho_2 g}}+\alpha_2\frac{V_2^2}{2g}+z_2\right)+\cancel{\sum h_{\text{turbine, e}}}+h_L}$$

≈0

Solve for V_2 → $\boxed{V_2 = \left\{\frac{2g}{\alpha_2}\left[\Delta z - h_L\right]\right\}^{1/2}}$ Ans. in variables

what is α_2 ?

3 cases

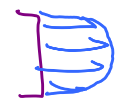
1) $\alpha = 1$ → ignore α: $V_2 = \left\{\frac{2(9.807\ \text{m/s}^2)}{1.00}\left[0.50\ \text{m} - 0.40\ \text{m}\right]\right\}^{1/2} = \boxed{1.40\ \text{m/s}}$

BUT, $\alpha = 1$ real life, $\alpha \neq 1$, $\alpha > 1$

2) $\underline{\alpha = 2.0}$ → Laminar flow → $\boxed{V_2 = \underline{0.990}\ m/s}$ Laminar

Diff is 41% compared to $\alpha = 1$

3) $\underline{\alpha \simeq 1.05}$ – Turbulent flow → $\boxed{V_2 = \underline{1.37}\ m/s}$ Turbulent



Diff is 2.4% compared to $\alpha = 1$

☆ It is wise to always include α in our analyses! ☆

if $Re = \dfrac{\rho V_2 D}{\mu} \gtrsim 2300$ → turbulent

if " " $\lesssim 2300$ → laminar

How we can tell whether the flow is laminar or turbulent

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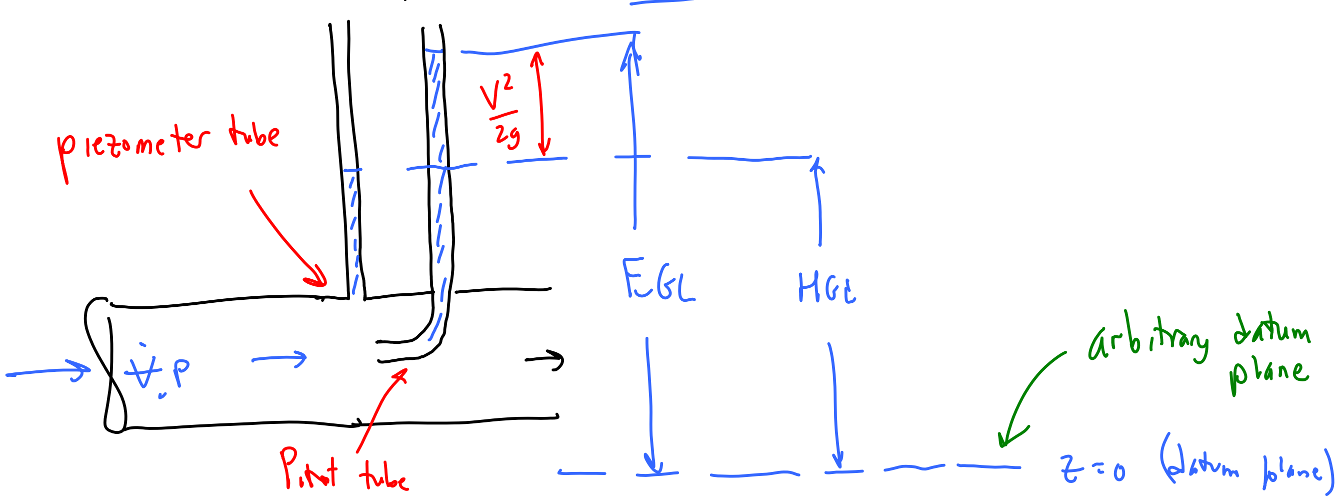
6. Grade Lines

a. Hydraulic Grade Line (HGL) ≡ the height to which a liquid will rise from a pressure tap normal to the flow (a static pressure tap)



b. Energy Grade Line (EGL) ≡ the height to which a liquid will rise from a total pressure probe aligned with the flow. (into the flow)

$$\boxed{HGL = \frac{P}{\rho g} + z} \quad \{L\}$$

$$\boxed{EGL = \frac{P}{\rho g} + \frac{V^2}{2g} + z} \quad \{L\}$$

$$\boxed{EGL - HGL = \frac{V^2}{2g}}$$

Can measure V = avg vel. in pipe

$$\boxed{V = \sqrt{2g\,(EGL - HGL)}}$$

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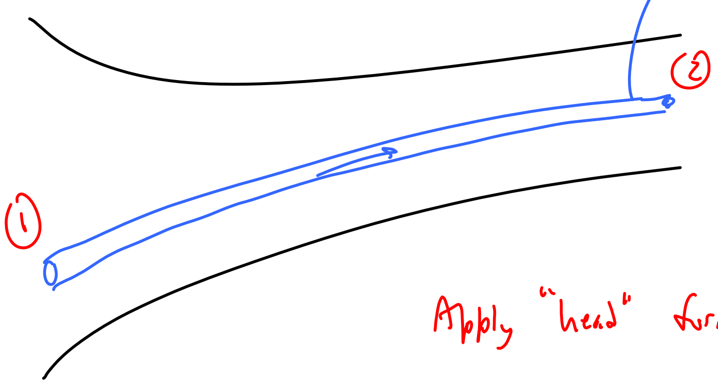
c. Relationship to the Energy Eq.



Streamtube is a "fat" streamline (follows the flow)

Apply "head" form of the energy eq.

=1 for a streamtube ; =1

$$\underbrace{\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1}_{=\,EGL_1} + h_{pump,u} = \underbrace{\frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2}_{=\,EGL_2} + h_{turbine,e} + h_L$$

[Total Bernoulli Head]

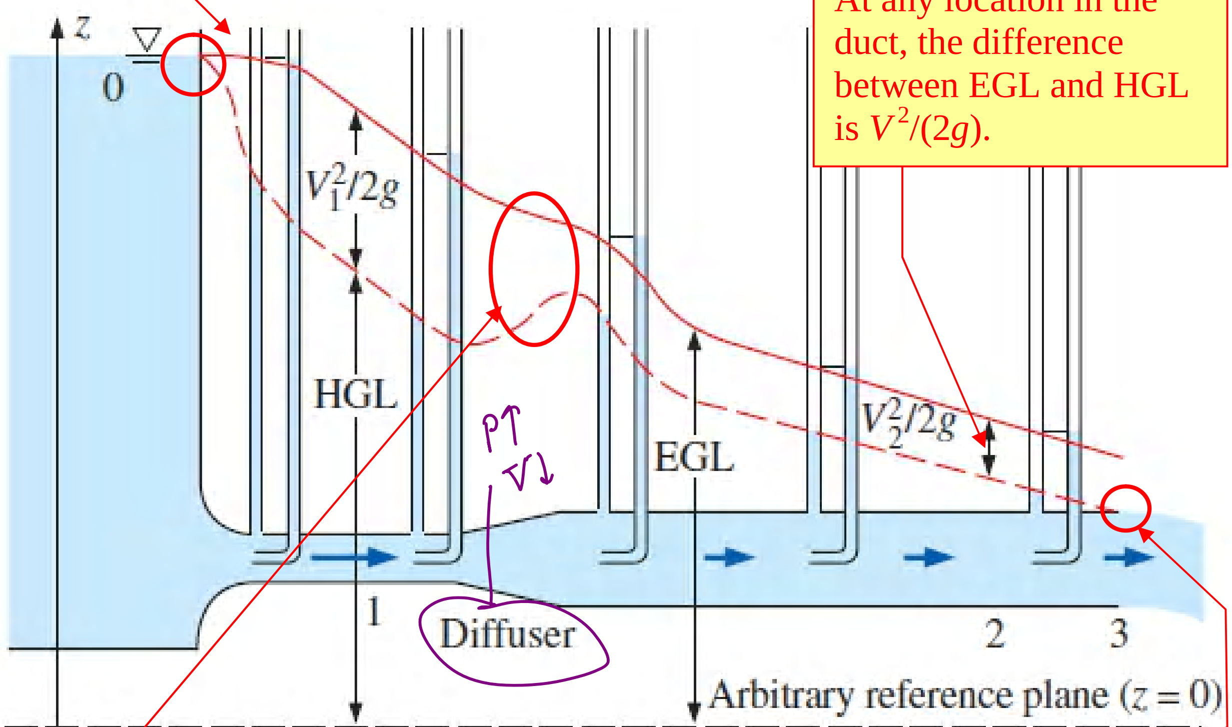
along a streamline or a streamtube

Example of Grade Lines in a Fluid Flow

FIGURE 5–35
The *hydraulic grade line* (HGL) and the *energy grade line* (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser.

At point 0, HGL = EGL inside the tank, since the fluid is at rest ($V = 0$). Neither EGL or HGL can rise above this value unless work is added to the flow (e.g., with a pump).

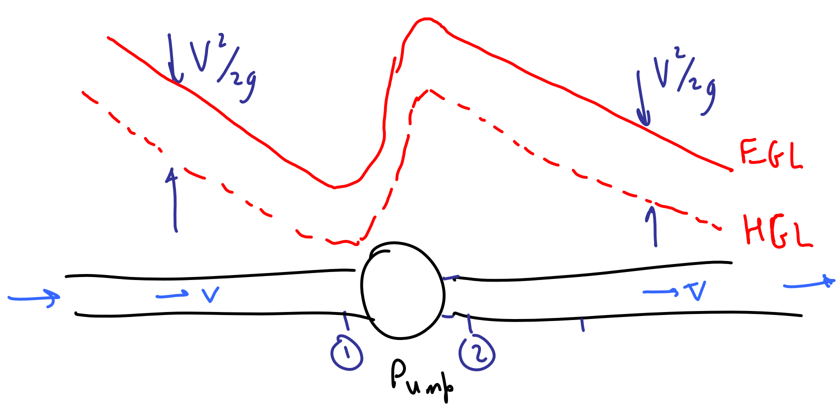
At any location in the duct, the difference between EGL and HGL is $V^2/(2g)$.



EGL continually falls due to irreversible losses, but HGL can rise or fall. Overall, however, HGL also must fall. In fact, HGL can *never* rise above EGL.

Since the jet exits at atmospheric pressure at the outlet of the pipe, $P_3 = P_{atm}$, and HGL is equal to the height of the free surface of the liquid.

- If no pump or turbine, $\boxed{EGL_1 = EGL_2 + h_L}$ → EGL must decrease downstream ✱
- The only way EGL can rise is if we have a pump in the system

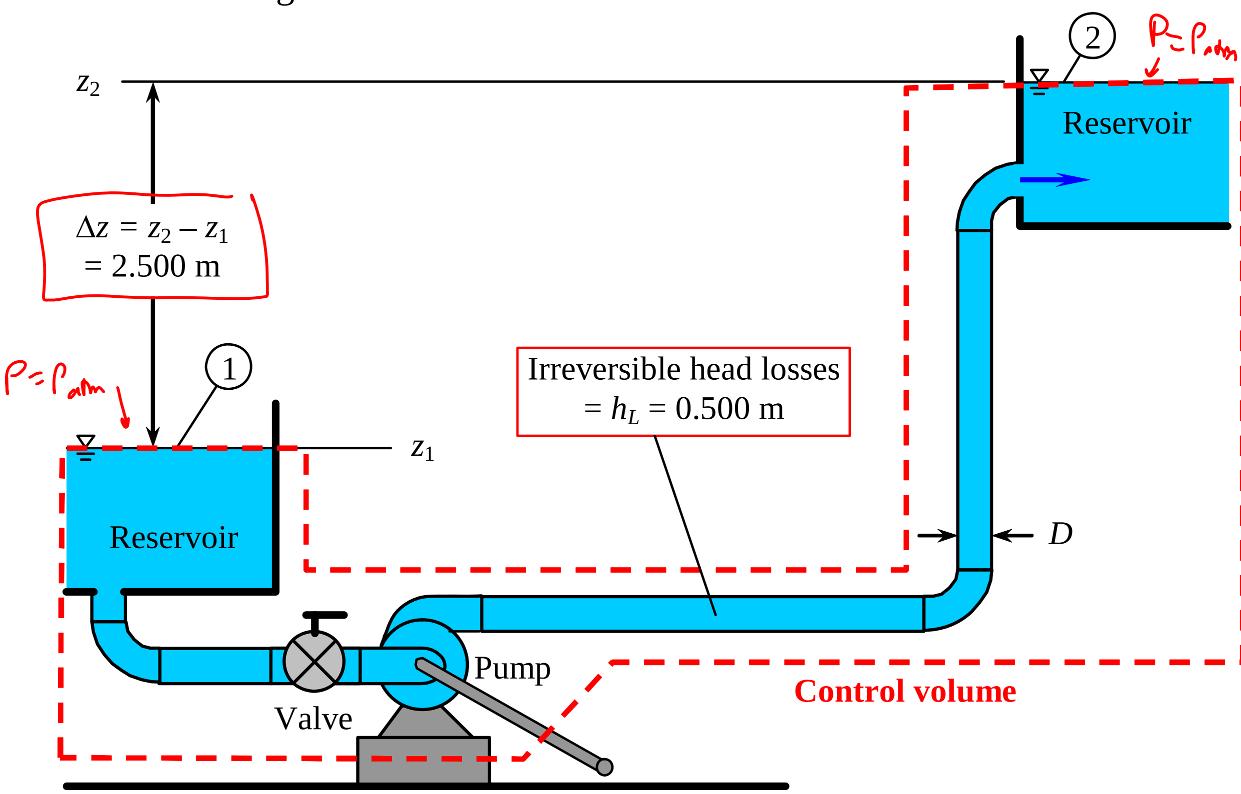


$EGL_2 = EGL_1 + h_{pump,u}$

(the pump increases the head of the water)

Example – Pumping water from one reservoir to another

Given: Water is pumped from one reservoir to another. Both reservoirs are exposed to atmospheric pressure. Δz = 2.50 m. The irreversible head losses in the piping system (due to friction in the pipe, losses through the valve, elbows, etc.) are estimated as h_L = 0.50 m of equivalent water column height.



To do: Calculate $h_{\text{pump,u}}$, the useful pump head supplied to the water in meters of water.

Solution: First we pick a control volume wisely. The CV shown above is a *wise* CV.
Now apply our workhorse equation, the head form of the conservation of energy equation:

$$\boxed{\left(\cancel{\frac{P_1}{\rho_1 g}} + \alpha_1 \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\approx 0} + z_1\right) + \cancel{\sum} h_{\text{pump,u}} = \left(\cancel{\frac{P_2}{\rho_2 g}} + \alpha_2 \underbrace{\cancel{\frac{V_2^2}{2g}}}_{\approx 0} + z_2\right) + \cancel{\sum h_{\text{turbine, e}}} + h_L}$$

$P = P_{atm}$

$$\boxed{h_{\text{pump,u}} = z_2 - z_1 + h_L} = 2.50 + 0.50 = \underline{\underline{3.00 \text{ m}}}$$

The pump does 2 things here: • Increases the elevation of the water
• overcomes irreversible head losses

D. The Bernoulli Equation

1. Derivation

Begin with the head form of the conservation of energy equation, but apply it ***along a streamline***:

$\alpha_1 = 1$
$\alpha_2 = 1$

$$\left(\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1\right) + \cancel{h_{\text{pump},u}} = \left(\frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2\right) + \cancel{h_{\text{turbine},e}} + \cancel{h_L}$$

No irreversible head losses

Approximations & Assumptions:

- No pump
- No turbine
- Negligible irreversible head losses ($h_L = 0$)
- Incompressible, $\rho_1 = \rho_2 = \rho$

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \text{along a streamline}}$$

BERNOULLI EQ (in head form)

"The beloved Bernoulli eq."

A degenerate form of the energy eq.

Everyone loves to use the Bernoulli eq., but it applies only with the above restrictions

Further simplification – if Δz is small or zero (i.e. for gases)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} \approx \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \Rightarrow \boxed{P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2}$$

As $V\uparrow$, $P\downarrow$

CAUTION – DO NOT USE BERNOULLI WHERE IT DOES NOT APPLY !!