Today, we will:

- Discuss applications of the Bernoulli equation
- Derive and discuss the linear momentum equation for a control volume (Chapter 6)
- Discuss the momentum flux correction factor, β
- Discuss all the various forces acting on a control volume, and do some examples

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Recall, the beloved Bernoulli equation:

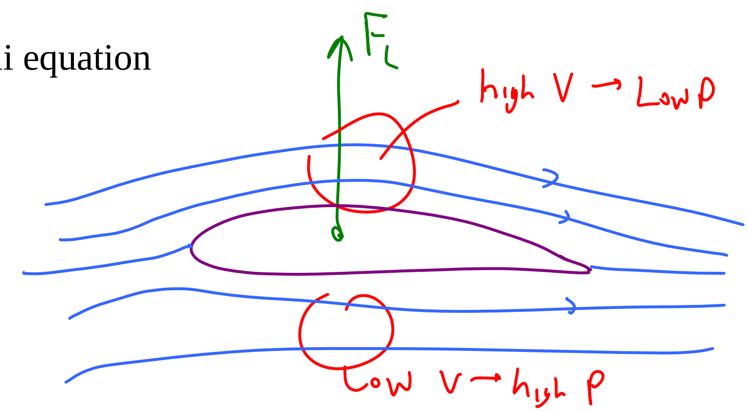
$$\boxed{\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2=\text{ constant along a sreamline}}$$

Note that the Bernoulli equation is valid *only* if *all* of the following limitations are met:

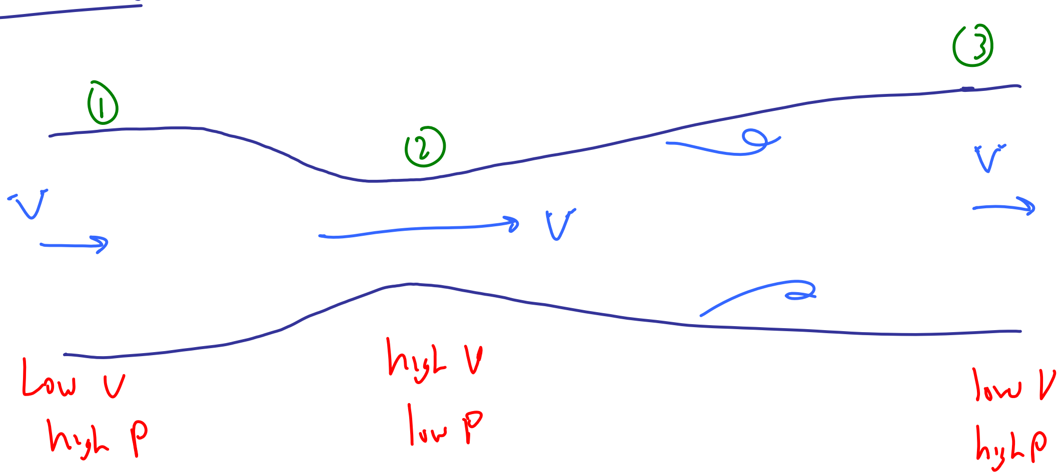
- no pumps or turbines
- steady
- incompressible
- negligible irreversible head losses (this is the limitation that usually restricts its use)

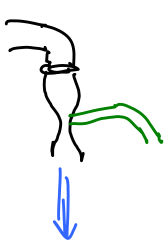
2. Applications of the Bernoulli equation

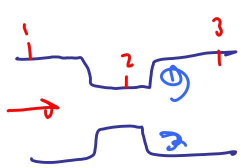
a) e.g. Airplane wing



b) Venturi tube Converging / diverging nozzle







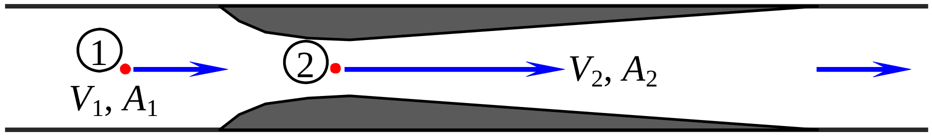
$P_3 > P_2$

$P_3 < P_1$

P @ ③ is not as high as P @ ①

Example: Pressure drop through a Venturi tube

Given: Water flows horizontally in a round pipe with a converging-diverging section (a Venturi tube) as sketched. Cross-sectional area A_1 is four times larger than cross-sectional area A_2 (at the throat). Neglect any irreversible losses in the flow. The average speed at Section 1 is $V_1 = 2.00$ m/s.



To do: If $P_{1,\text{gage}} = 50,000$ Pa, estimate $P_{2,\text{gage}}$ in units of Pa.

Solution:

Bernoulli, ignoring Δz: $P_2 + \frac{1}{2}\rho V_2^2 = P_1 + \frac{1}{2}\rho V_1^2$

$$P_{2_{\text{gage}}} + \frac{1}{2}\rho V_2^2 = P_{1_{\text{gage}}} + \frac{1}{2}\rho V_1^2$$

Cons. of mass $-$ $V_1 A_1 = V_2 A_2 \rightarrow V_2 = V_1\left(\frac{A_1}{A_2}\right)$

$$\boxed{P_{2,\text{gage}} = P_{1,\text{gage}} - \frac{1}{2}\rho V_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}$$

#'s

$$= 50,000 \text{ Pa} - \frac{1}{2}\left(998\,\frac{\text{kg}}{\text{m}^3}\right)\left(2.00\,\frac{\text{m}}{\text{s}}\right)^2\left(4^2 - 1\right)\cdot\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{\text{Pa}\cdot\text{m}^2}{\text{N}}\right)$$

$$= \boxed{20,100 \text{ Pa}}$$

Notice that we used gage pressure everywhere. You can use either absolute or gage pressure, but pick one of them & then use it consistently everywhere

E. The Linear Momentum Equation for a Control Volume (Chapter 6)

1. Equations and Definitions

Recall the RTT at the end of Chapter 4, applied to linear momentum:

$$\boxed{\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \int_{CS} \rho \vec{V}\left(\vec{V}\cdot\vec{n}\right) dA}$$

→ Apply to CVs

approx

Total force (***vector***) acting on the control volume

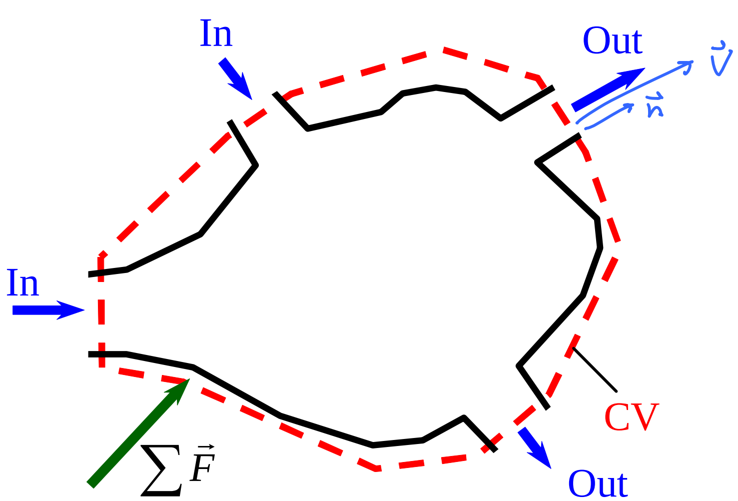
Rate of change of linear momentum inside the control volume

Net rate of linear momentum flow out of the control volume

Use *relative* velocity $\vec{V}_r$ here if we have a moving or deforming control volume

Unlike mass & energy eq's → this is a vector eq (3 components)

Simplification for well-defined inlets and outlets



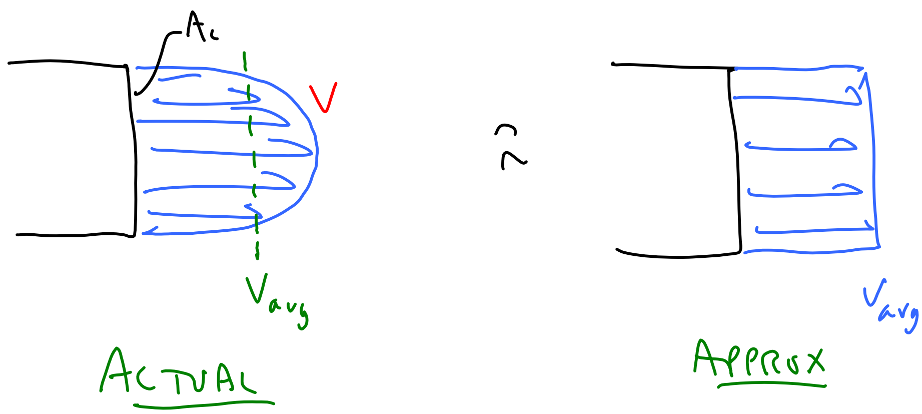
Approximation:

$$\int_{CS} \rho \vec{V}\left(\vec{V}\cdot\vec{n}\right) dA \approx \sum_{out} \dot{m}\vec{V}_{avg} - \sum_{in} \dot{m}\vec{V}_{avg}$$

mass flow rate

$$\boxed{\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \sum_{out} \dot{m}\vec{V} - \sum_{in} \dot{m}\vec{V}}$$

2. The momentum flux correction factor



Because of nonlinearity

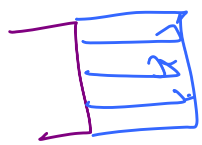
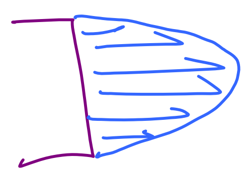
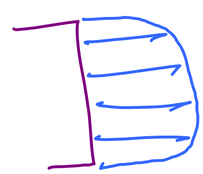
$$\rho\int_{A_c} V^2\, dA > \underbrace{\rho V_{avg} A_c}_{=\dot{m}} V_{avg}$$

Define the momentum flux correction factor, $\boxed{\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{avg}}\right)^2 dA_c}$

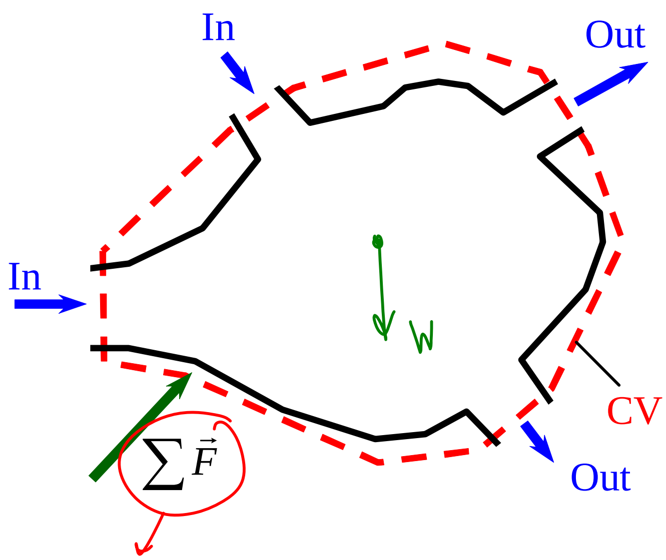
$\boxed{\beta \geq 1}$ *

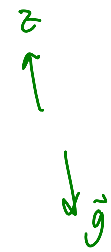
$$\boxed{\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}}$$

e.g.'s

- Uniform flow  V = const, $\boxed{\beta = 1}$
- fully developed laminar flow in a pipe  $\boxed{\beta = \frac{4}{3}}$
- " " turbulent " "  $\boxed{\beta \approx 1.02}$

3. Forces acting on a control volume





Body forces + Surface forces

$$\sum \vec{F} = \sum \vec{F}_{body} + \sum \vec{F}_{surface}$$

gravity → weight

$$= \sum \vec{F}_{gravity} + \sum \vec{F}_{pressure} + \sum \vec{F}_{viscous} + \sum \vec{F}_{other}$$

Plug into our mom eq.

"Workhorse" eq. for momentum analysis

[Most problems we do in this class are also steady, so this unsteady term goes away.]

The approximate, most useful form of the linear momentum equation:

$$\boxed{\sum \vec{F} = \sum \vec{F}_{\text{gravity}} + \sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}}$$

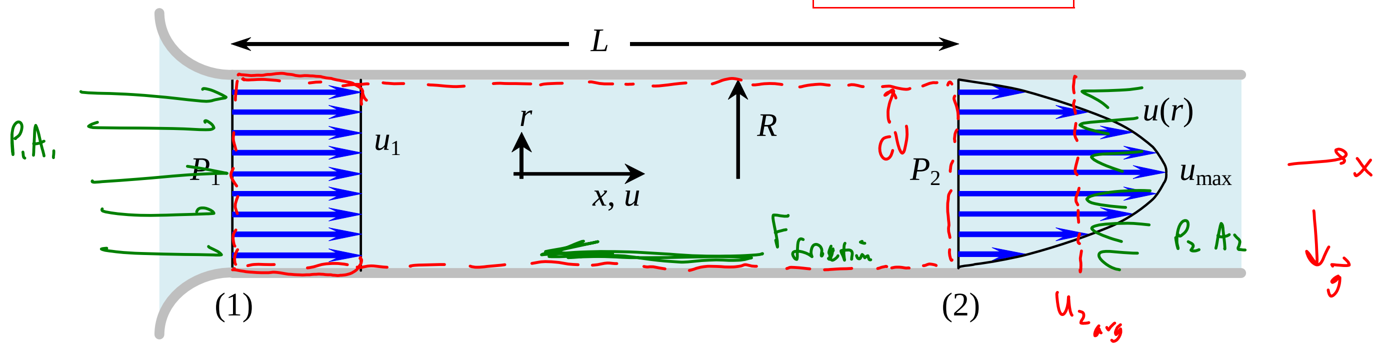
4. Examples

Note: It is a vector eq., so we need to work out all 3 components (2 components for 2-D flow)

Example: Friction force in a pipe

Given: Consider steady, laminar, incompressible, axisymmetric flow of a liquid in a pipe as sketched. At the inlet (1) there is a nice bell mouth, and the velocity is nearly uniform (except for a very thin boundary layer, not shown).

- At (1), $u = u_1$ = constant, $v = 0$, and $w = 0$. P_1 is measured.
- At (2), the flow is fully developed and parabolic: $\boxed{u_2 = u_{\max}\left(1-\frac{r^2}{R^2}\right)}$. P_2 is measured.



To do: Calculate the total friction force acting on the fluid by the pipe wall from 1 to 2.

Solution:

- First step: Pick a CV (shown)

cons of mass → $\dot{m}_{in} = \dot{m}_{out}$, $\dot{m}_1 = \dot{m}_2 = \dot{m}$

$\cancel{\rho} u_1 \cancel{A_1} = \cancel{\rho} u_{2_{avg}} \cancel{A_2}$

$\boxed{u_{2_{avg}} = u_1}$

$\boxed{A = \pi R^2}$

$\boxed{\dot{m} = \rho u_1 A}$

At ①, flow is uniform → $\boxed{\beta_1 = 1}$

At ②, fully dev. lam. pipe flow → $\boxed{\beta_2 = 4/3}$

- Now use the approximate, most useful form of the linear momentum equation,

$$\boxed{\sum \vec{F} = \cancel{\sum \vec{F}_{\text{gravity}}} + \sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \cancel{\sum \vec{F}_{\text{other}}} = \cancel{\frac{d}{dt}\int_{CV} \rho \vec{V} dV} + \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}}$$

in x-dir, no gravity

x-comp: $P_1A_1 - P_2A_2 - F_{friction}$ no "other" forces $=$ steady $\beta_2 \dot{m} u_1 - \beta_1 \dot{m} u_1$

in x-direction

Plug in values: $\boxed{F_{friction} = \pi R^2 \left[P_1 - P_2 - \frac{1}{3}\rho u_1^2\right]}$