**Today, we will**:

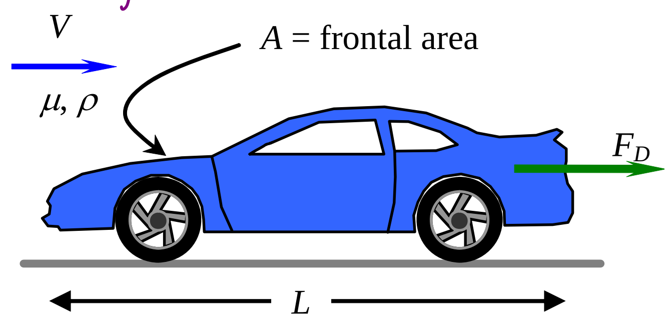
- Finish discussing the example problem from last lecture
- Do some more example problems – dimensional analysis
- Discuss experimental testing and incomplete similarity

## Example: Dimensional analysis – Car drag (continued)

***Given***: The drag force $F_D$ on a car is a function of four variables: air velocity $V$, air density $\rho$, air viscosity $\mu$, and the length $L$ of the car.

***To do***: Express this relationship in terms of nondimensional parameters.

***Solution***: We followed the six steps for the method of repeating variables.




See previous lecture. We completed step 5, and had

$$\Pi_1 = \text{ dependent Pi} = \boxed{C_D = \text{drag coefficient} = \frac{F_D}{\frac{1}{2}\rho V^2 A}}$$

$$\Pi_2 = \text{ independent Pi} = \boxed{\text{Re} = \text{Reynolds number} = \frac{\rho V L}{\mu}}$$

Finally, Step 6 is to write the relationship between the Πs: $\Pi_1 = \text{func}(\Pi_2, \Pi_3, ...)$:

$\boxed{C_D = \text{func}(Re)}$ ☆ → powerful

APPLICATIONS

Designing experiments:

Originally, $\boxed{F_D = \text{func}(V, L, \rho, \mu)}$

1 dependent variable

4 independent variables

reduced to $\boxed{C_D = \text{func}(Re)}$

1 dep. var.

1 indep. var.

We have reduced the number of independent variables by 3 (4−3=1)

(j=3)

reduction

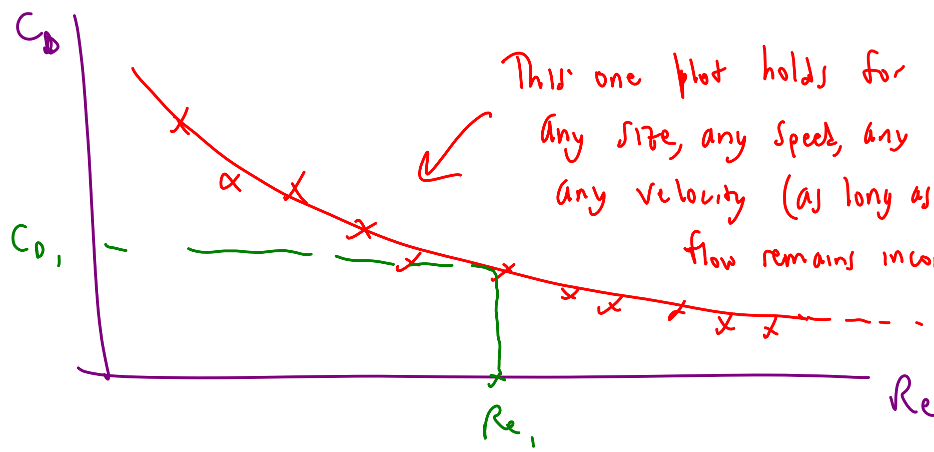
This is what j means

I can use **one** model car $(L \rightarrow$ only one$)$

**one** fluid $(\rho, \mu \rightarrow$ only one$)$

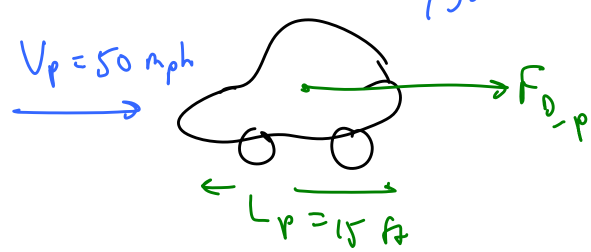
run various velocities $(\text{vary } V)$

$\Big\}$ measure $F_D$




This one plot holds for any size, any speed, any density, any velocity (as long as the flow remains incompressible) ☆

**Given:** **Prototype car** →




**Model car** → $\frac{1}{5}$ **scale** geometrically similar

$L_m = 3$ ft

**To do:** (a) How fast do we need to run the wind tunnel to achieve **dynamic similarity**?

$\Downarrow$

✓ match all nondimensionless params.

Here we need to match Re

$$Re_m = \frac{\rho_m V_m L_m}{\mu_m} = Re_p = \frac{\rho_p V_p L_p}{\mu_p}$$

Solve for $V_m$ → $\boxed{V_m = V_p \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right)\left(\frac{\mu_m}{\mu_p}\right)}$

same air

air @ same T

#s

$$V_m = (50 \text{ mph})(1)(5)(1) = \boxed{V_m = 250 \text{ mph}}$$

(b) Run wind tunnel @ 250 mph → measure $F_D = 25.0$ lbf

Predict $F_{D_p}$

$\boxed{C_D = \text{fn}(Re)}$

$\boxed{\text{If } Re_m = Re_p \text{, then } C_{D_m} = C_{D_p}}$ ☆

$$C_{D_p} = \frac{F_{D_p}}{\frac{1}{2}\rho_p V_p^2 A_p} = \frac{F_{D_m}}{\frac{1}{2}\rho_m V_m^2 A_m} = C_{D_m}$$

Solve for $\boxed{F_{D_p} = F_{D_m}\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{A_p}{A_m}\right)}$

Same air

$\therefore \rho_m = \rho_p$

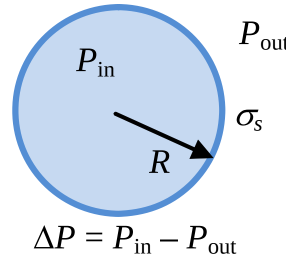
$\mu_m = \mu_p$

$$F_{D_p} = (25.0 \text{ lbf})(1)\left(\frac{50 \text{ mph}}{250 \text{ mph}}\right)^2 (5)^2$$

$$\boxed{F_{D_p} = 25.0 \text{ lbf}}$$

## Example: Dimensional analysis – Soap bubble

**Given**: The difference in pressure $\Delta P$ between the inside and outside of a soap bubble is known to be a function of surface tension $\sigma_s$ and soap bubble radius $R$.




**To do**: Use dimensional analysis to express the relationship between $\Delta P$, $\sigma_s$, and $R$ in dimensionless form.

**Solution**:

Step 1: (list the variables) $\Delta P = \text{fnc}(\sigma_s, R)$ $\boxed{n=3}$

Step 2: (list the dimensions) $\left\{\frac{m}{t^2 L}\right\}$ $\left\{\frac{m}{t^2}\right\}$ $\{L\}$

Step 3: (pick reduction $j$) We see m, L, t → pick $j=3$

$\therefore k = \#\,\Pi\text{'s} = n - j = 3 - 3 = 0\ \Pi\text{'s}$ ??

impossible → set $j = 3-1 = \underline{\underline{2}}$ $\boxed{j=2}$

$\therefore k = n - j = 3 - 2 = \underline{\underline{1\ \Pi}}$ ← expect $1\ \Pi$

Step 4: (pick the repeating variables) pick 2 → $\sigma_s$, $R$

Step 5: (calculate the Πs) $\Pi_1 = \Delta P\, \sigma_s^{a} R^{b}$

calc exp. a — Force $\Pi_1$ to be dimensionless

$$\{m^0 L^0 t^0\} = \left\{\frac{m}{t^2 L}\right\}\left\{\frac{m}{t^2}\right\}^a \{L\}^b$$

m: $0 = 1 + a \rightarrow \boxed{a=-1}$

t: $0 = -2 - 2a \rightarrow \boxed{a=-1}$

L: $0 = -1 + b \rightarrow \boxed{b=1}$

$$\boxed{\Pi_1 = \frac{\Delta P R}{\sigma_s}}$$

Step 6: $\Pi_1 = \text{fnc}(\Pi_2, \Pi_3, \ldots)$

$\Pi_1 = \text{fnc}(\text{nothing}) \rightarrow \boxed{\Pi_1 = \text{constant}}$ $\therefore$ $\boxed{\Delta P = \frac{\text{const}\cdot\sigma_s}{R}}$ ★

• How does $\Delta P$ vary with $R$?   $\Delta P \propto \frac{1}{R}$

if $R \uparrow$ factor of 2 $\rightarrow$ $\Delta P \downarrow$ by a factor of 2

We know that $\boxed{\Delta P = \frac{\text{const} \cdot \sigma_s}{R}}$   without knowing any physics!

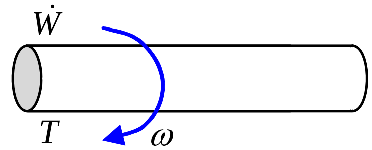
We cannot calc. the constant w/ Dim. Anal.

(we need either an experiment or, analysis)

e.g. we know from previous lectures const = 4

$$\boxed{\Delta P = \frac{4\,\sigma_s}{R}}$$

## Example: Dimensional analysis – shaft power




**Given**: The output power $\dot W$ of a spinning shaft is a function of torque $T$ and angular velocity $\omega$.

**To do**: Use dimensional analysis to express the relationship between $\dot W$, $T$, and $\omega$ in dimensionless form.

[radian is a non dimensional unit]

**Solution**:

Step 1: (list the variables) $\dot W = fnc(T, \omega)$ $\boxed{n=3}$

Step 2: (list the dimensions) $\left\{\frac{mL^2}{t^3}\right\}$ $\left\{\frac{mL^2}{t^2}\right\}$ $\left\{\frac{1}{t}\right\}$

Step 3: (pick reduction $j$) See $m, L, t$ → set $j = 3$ → $k = n - j = 3 - 3 = 0$ $\Pi$'s X

No → set $j = 3-1 = 2$ → expect $k = 3-2 = 1$ $\Pi$ $\boxed{j=2}$

Step 4: (pick the repeating variables) Pick $T$ & $\omega$

Step 5: (calculate the Πs) $\boxed{\Pi_1 = \dot W T^a \omega^b}$ ⇒ force this $\Pi$ to be dimensionless

$$\{m^0 L^0 t^0\} = \left\{\frac{mL^2}{t^3}\right\}\left\{\frac{mL^2}{t^2}\right\}^a\left\{\frac{1}{t}\right\}^b$$

equate exponents:

$m$: $0 = 1 + a$ → $\boxed{a = -1}$

$L$: $0 = 2 + 2a$ → $\boxed{a = -1}$ ✓

$t$: $0 = -3 - 2a - b$ → $b = -3 - 2a$
$= -3 + 2 = -1$

$\boxed{b = -1}$

$$\Pi_1 = \frac{\dot W}{T\omega}$$

Step 6 → $\Pi_1 = fnc(\text{nothing}) = \text{const}$ ∴ $\frac{\dot W}{T\omega} = \text{const}$ → $\boxed{\dot W = \text{const.}\ \omega T}$

(const = 1)

## D. Experimental Testing & Incomplete Similarity

In more complicated cases, we may not be able to match **all** of the $\pi$'s between model & prototype

e.g. Wind tunnel, car drag → we had $C_D = \text{fnc}(Re)$

Suppose scale of model is $\frac{1}{16}$th

Want to simulate 60 mph of the prototype

↓

match Re → $Re_m = Re_p$ → need $V_m = 16(V_p)$

$$\boxed{V_m = 960 \text{ mph}}$$

$C$ = speed of sound = 767 mph

Would be supersonic !!

Compressibility effects would be important

Need to go back & re-do dim anal. with an additional variable, $C$

⇓

get $\boxed{C_D = \text{fnc}(Re, Ma)}$

Mach # $= \frac{V}{C}$

If $Ma \lesssim 0.3$, we can ignore compressibility effects

but if $Ma > 0.3$ we cannot ignore compressibility effects