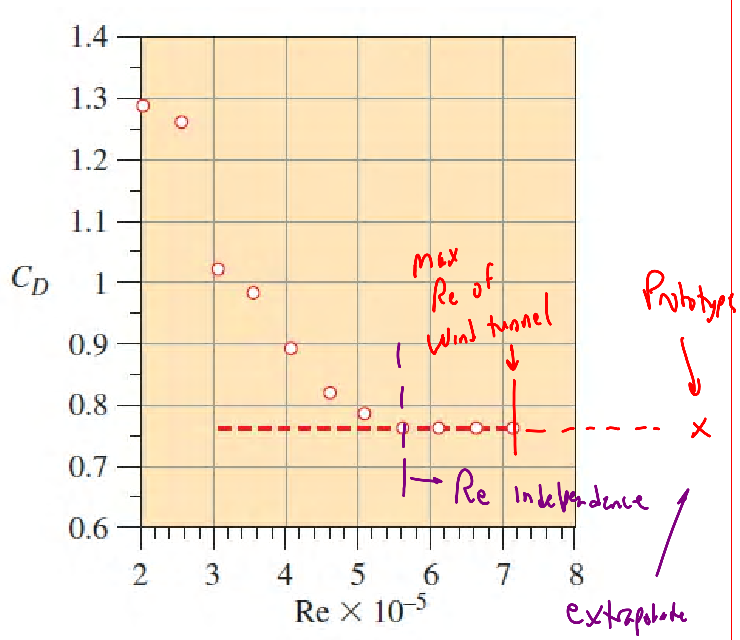
Today, we will:

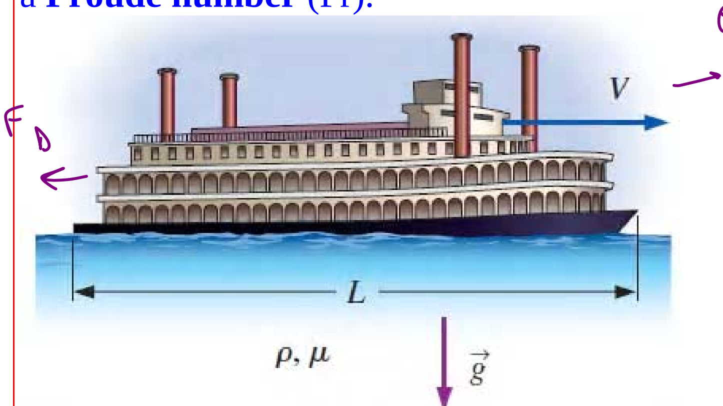
- Finish Chapter 7 – Dimensional Analysis (incomplete similarity)
- Begin Chapter 8 – Internal Flow (flow in pipes)

Incomplete Similarity: Sometimes, we cannot exactly match all the dimensionless parameters. For example, we might not be able to run a wind tunnel at high enough speed to match the Reynolds number. Even if we *could* run the wind tunnel fast enough, we would run into compressibility effects. Fortunately, drag coefficient often becomes *independent* of Reynolds number, beyond a certain threshold. $C_D = \text{fn}(Re)$

Example (see textbook, Example 7-10):
Model truck wind tunnel experiments.



Another example of incomplete similarity is flows with ***free surfaces*** (e.g., rivers, boats, etc.). It turns out that dimensional analysis usually yields both a **Reynolds number** (Re) and a **Froude number** (Fr).



$$\text{Re} = \frac{\rho V L}{\mu} = \frac{VL}{\nu} \qquad \text{Fr} = \frac{V}{\sqrt{gL}}$$

e.g. $C_D = (Re, Fr)$

for complete similarity, must match both Re & Fr

To match both,

Re: $\dfrac{V_m L_m}{\nu_m} = \dfrac{V_p L_p}{\nu_p}$ $\Downarrow$ $\boxed{\dfrac{V_p}{V_m} = \dfrac{L_m}{L_p}\dfrac{\nu_p}{\nu_m}}$

Fr: $\dfrac{V_m}{\sqrt{g L_m}} = \dfrac{V_p}{\sqrt{g L_p}}$ $\Downarrow$ $\boxed{\dfrac{V_p}{V_m} = \sqrt{\dfrac{L_p}{L_m}}}$

$$\boxed{\frac{V_p}{V_m} = \frac{L_m}{L_p}\frac{\nu_p}{\nu_m}} = \boxed{\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}}}$$

To match both Re & Fr we need a liquid with kinematic viscosity

$$\nu_m = \nu_p \left(\frac{L_p}{L_m}\right)^a \leftarrow \text{what is exponent } a\,?$$

$$\left(\frac{L_p}{L_m}\right)^{\frac{1}{2}} = \frac{L_m}{L_p}\frac{\nu_p}{\nu_m}$$

$$= \left(\frac{L_p}{L_m}\right)^{-1}\frac{\nu_p}{\nu_m} \;\text{—}\; \frac{\nu_p}{\nu_m} = \left(\frac{L_p}{L_m}\right)^{3/2} \rightarrow \boxed{\nu_m = \nu_p\left(\frac{L_p}{L_m}\right)^{-3/2}} \quad \boxed{a = -1.5}$$

eg water for prototype

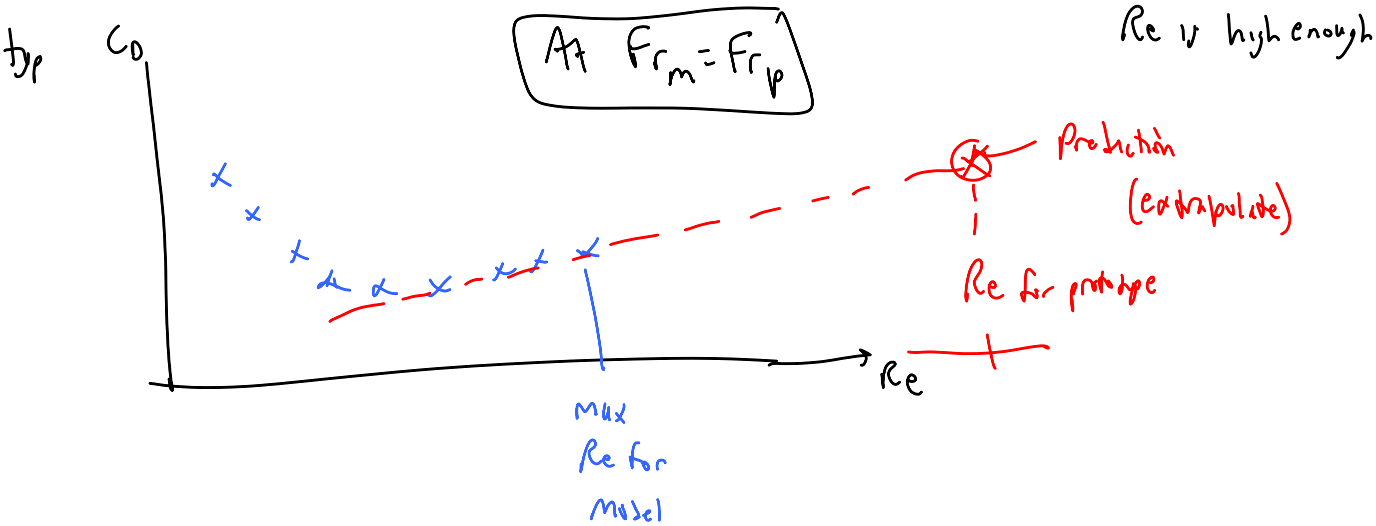
$$\nu_p = 1.0 \times 10^{-6}\ m^2/s$$

Fr $L_m = \frac{1}{100} L_p$ $\rightarrow$ Calc $\nu_m = \nu_p (100)^{-3/2} = 1.0 \times 10^{-9}\ m^2/s$

I need a liquid with $\nu_m = 1.0 \times 10^{-9}\ m^2/s$

Problem – no such liquid exists!

Fr is more important than Re for these kinds of flows, as long as Re is high enough



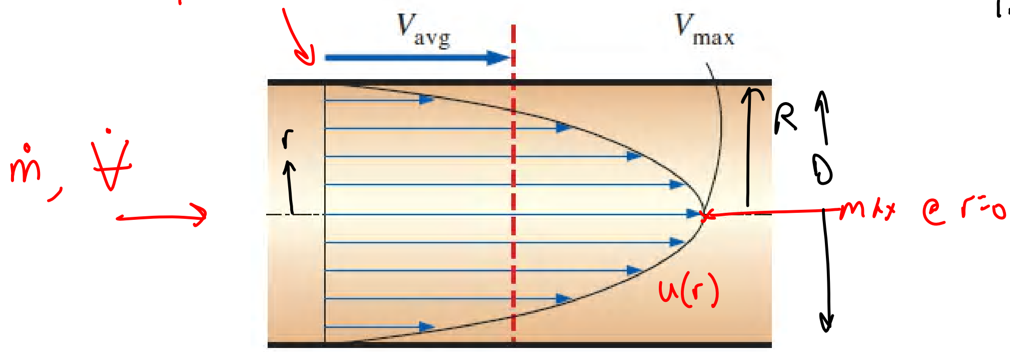
We do the experiment matching Froude #, the extrapolate to the prototype Reynolds number $\rightarrow$ INCOMPLETE SIMILARITY

VI. Internal Flow in Pipes (Chapter 8)

A. Introduction

1. Average velocity in a pipe

no slip condition → Causes friction on the wall → pressure drop in the pipe



We use V_{avg} for our analyses, where we write V as V_{avg}

$$\dot m = \rho V_{avg.} A = \rho V A \qquad A = \pi R^2 = \frac{\pi D^2}{4}$$

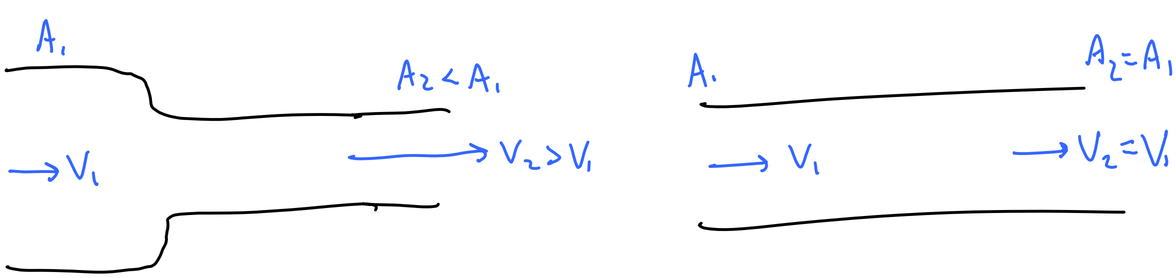
cross sectional area

Along the pipe, $\dot m$ = const for steady flow (cons. of mass)

$$\dot m_1 = \rho_1 V_1 A_1 = \dot m_2 = \rho_2 V_2 A_2$$

If incomp. → $V_1 A_1 = V_2 A_2$ As $A\uparrow, V\downarrow$

or $A\downarrow, V\uparrow$



2. Laminar versus turbulent flow – a comparison

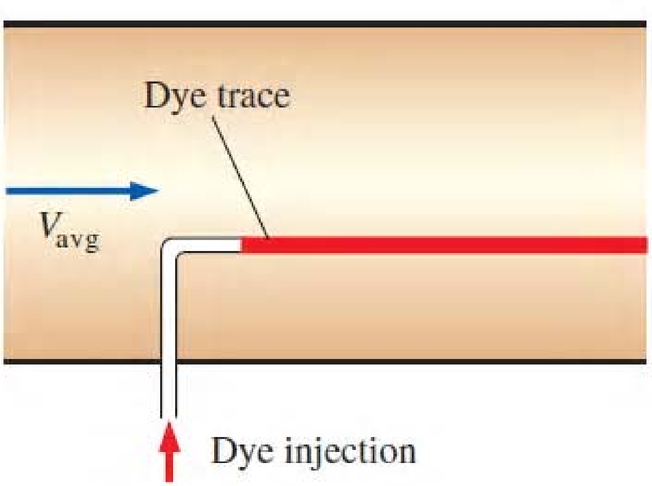
Laminar Flow

Can be steady or unsteady.

(Steady means that the flow field at any instant in time is the same as at any other instant in time.)

Can be one-, two-, or three-dimensional.

Has regular, *predictable* behavior



Analytical solutions are possible (see Chapter 9).

Occurs at *low* Reynolds numbers.

Turbulent Flow

Is always *unsteady*. – can be steady in the mean

Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow.

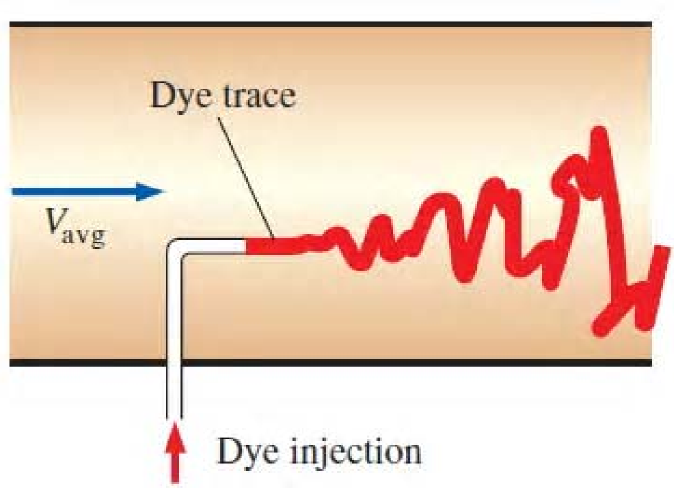
Note: However, a turbulent flow can be steady *in the mean*. We call this a *stationary turbulent flow*.

Is always *three-dimensional*.

Why? Again because of the random swirling eddies, which are in all directions.

Note: However, a turbulent flow can be 1-D or 2-D *in the mean*.

Has irregular or *chaotic* behavior (cannot predict exactly – there is some randomness associated with any turbulent flow.

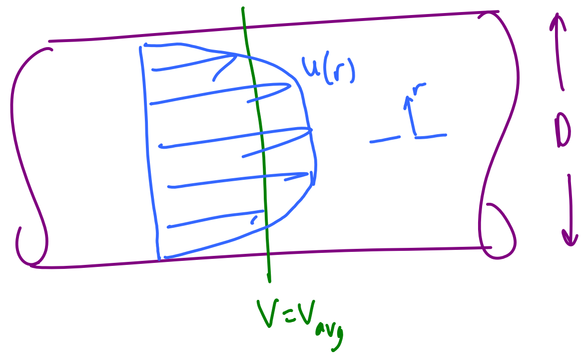


No analytical solutions exist! (It is too complicated, again because of the 3-D, unsteady, chaotic swirling eddies.)

Occurs at *high* Reynolds numbers.

b. Critical Re for pipe flow

$[\nu = \mu/\rho]$



$$\boxed{Re = \frac{\rho V_{avg} D}{\mu} = \frac{V_{avg} D}{\nu}}$$

Reynolds # for pipe flow

For a round pipe:

define $Re_{critical} \simeq 2300$

$Re \lesssim 2300 \rightarrow$ laminar flow (definitely laminar)

$2300 \lesssim Re \lesssim 4000 \rightarrow$ transitional (in between lam. & turb. or oscillatory between them) — unsure

(fuzzy zone)

$Re \gtrsim 4000 \rightarrow$ definitely turbulent

Actual values depend on pipe roughness, vibrations, upstream turbulence, entrance region etc., what is upstream (e.g. elbows, etc.)

c. Hydraulic Diameter → for non-round pipes
D_h

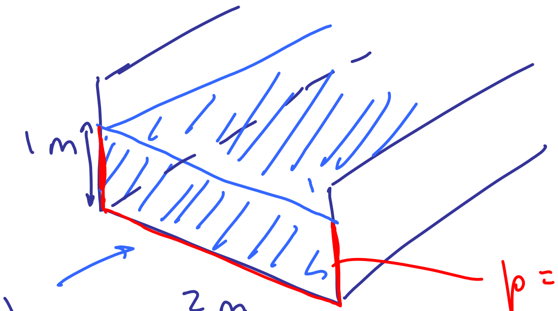
Define $\boxed{D_h = \frac{4 A_c}{p}}$

A_c: cross-sectional area

p: wetted perimeter → part of wall in contact with the fluid

Our eqs & correlations are for round pipes

So we use D_h in place of D as an equivalent round pipe diameter e.g. open channel

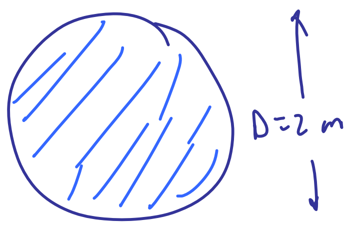


$p = 1 + 2 + 1\ m = 4\ m$

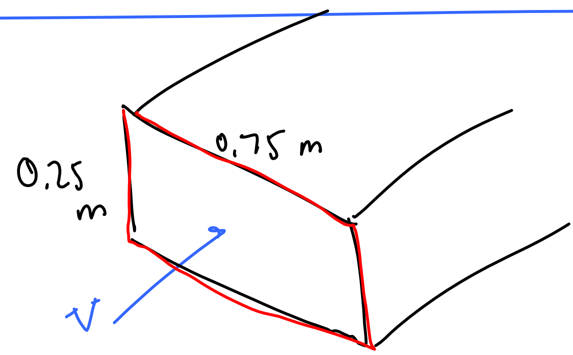
$$D_h = \frac{4A_c}{p} = \frac{4(1m \times 2m)}{4\ m} = \boxed{2\ m = D_h}$$

pretend I have a round pipe with Diameter $D = D_h = 2m$

"equivalent" to



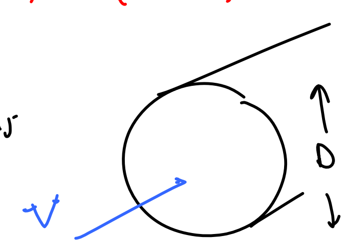
An A/C duct, air,



$$\text{Calc. } D_h = \frac{4A_c}{p} = \frac{4\ (0.25\ m)(0.75\ m)}{2(0.25\ m) + 2(0.75\ m)} = \underline{0.375\ m} = D_h$$

In m

equiv. round pipe is



$D = D_h = 0.375\,m$

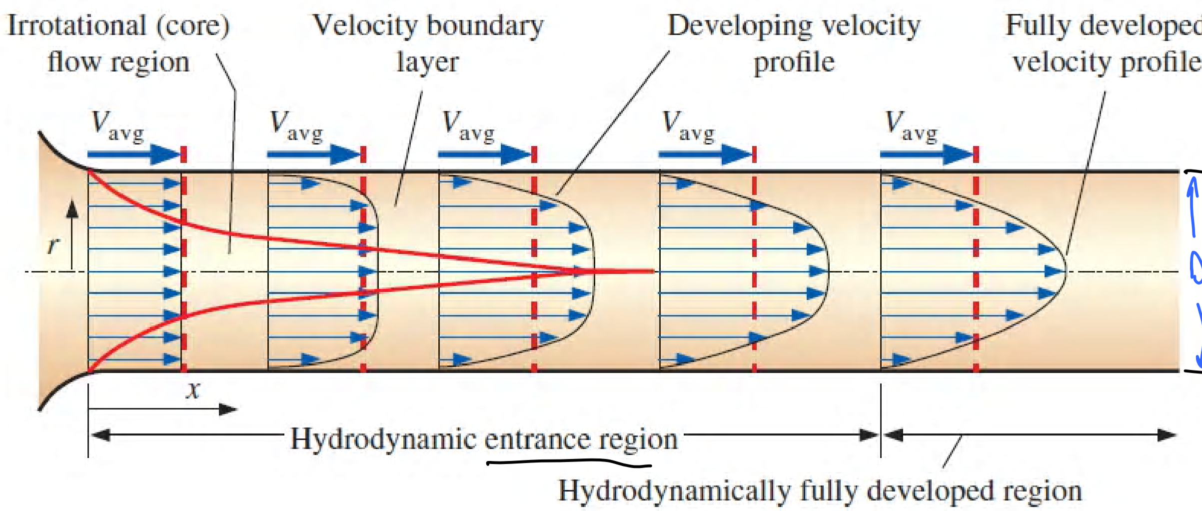
we $Re = \frac{V D_h}{\nu}$

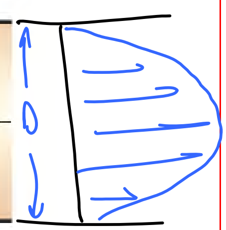
From here on, we pretend we have this pipe & do all our analysis based on the round pipe

d. The entrance region

The velocity profile "develops" for some distance called the Entrance Length L_h

Beyond the entrance length, the flow is fully developed – velocity profile does not change anymore





L_h | fully developed

Pressure continues to drop downstream, but velocity profile stays the same shape

$$\boxed{L_h = fnc(\rho, \mu, D, V_{avg})}$$

– good practice problem for Dimensional Analysis!