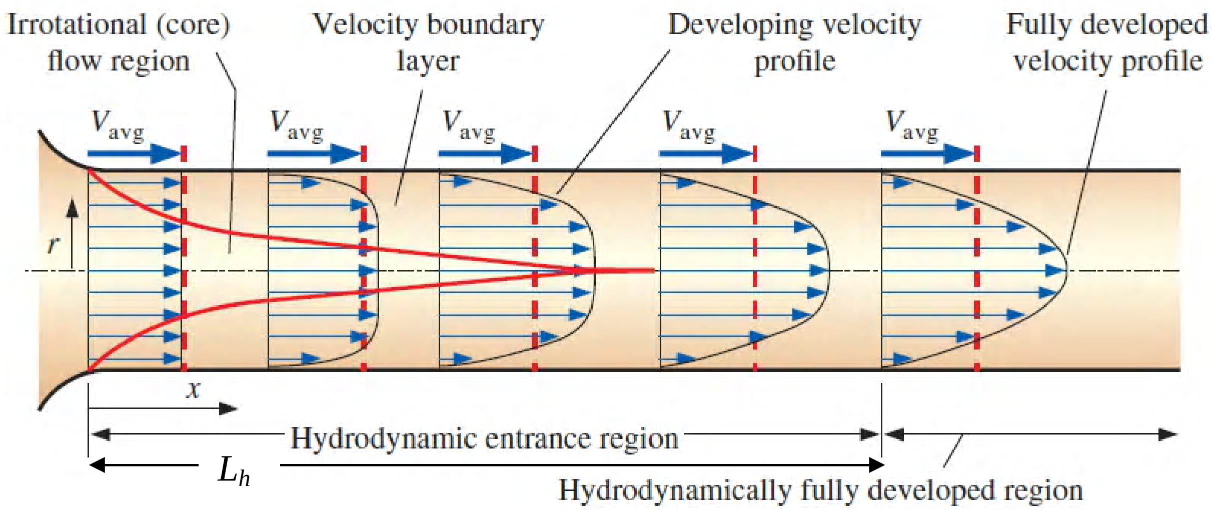
Today, we will:

- Continue discussing the entrance region and the fully developed region for pipe flows
- Discuss the Darcy friction factor, the Moody Chart, and the Colebrook Equation
- Do some example problems – head losses in pipe flows

d. The entrance region (continued)



Last time, we said that L_h is a function of (ρ, μ, D, V_{avg}). Dimensional analysis yields:

$$\boxed{\frac{L_h}{D} = fn(Re)}$$

(let $V_{avg} = V$ for simplicity)

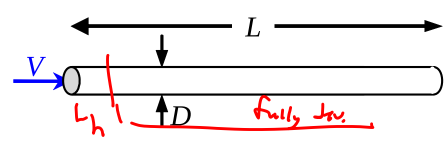
Where $Re = \frac{\rho V_{avg} D}{\mu} = \frac{V_{avg} D}{\nu} = \frac{VD}{\nu}$

Laminar: $\boxed{\frac{L_h}{D} = 0.05\,Re}$

Turbulent: $\boxed{\frac{L_h}{D} \approx 10}$

Example: Hydrodynamic entrance length

Given: Water with $\nu = 1.00 \times 10^{-6}$ m^2/s flows at a steady average speed of 5.70 m/s through a long pipe of diameter 25.4 cm. The pipe is 1.80 km long.



To do: What percent of the pipe length can be considered to be fully developed?

Solution:

$$Re = \frac{VD}{\nu} = \frac{(5.70\ \text{m/s})(0.254\ \text{m})}{1.00 \times 10^{-6}\ \text{m}^2/\text{s}} = \underline{\underline{1.448 \times 10^{6}}} > 4000 \rightarrow \text{turbulent}$$

$$\frac{L_h}{D} = 10 \rightarrow L_h = 10(0.254\ \text{m}) = \underline{\underline{2.54\ \text{m}}} \qquad \% = \frac{L - L_h}{L} = \frac{1800 - 2.54\ \text{m}}{1800\ \text{m}}$$

$$= 99.86\% \approx \boxed{99.9\%}$$

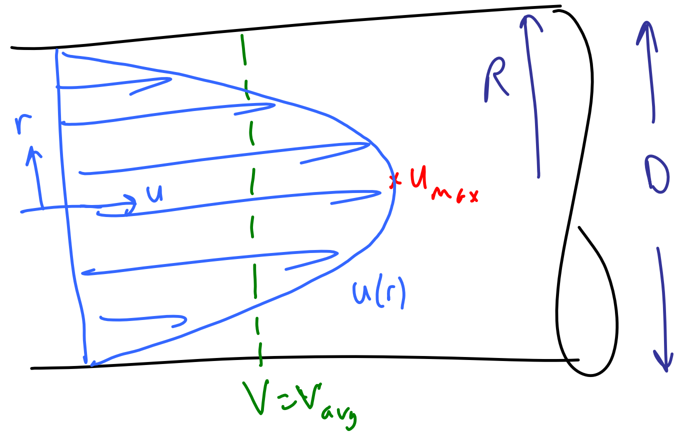
For $\underline{\underline{\text{long}}}$ pipes we typically approx the whole pipe as fully developed

B. Fully Developed Pipe Flow

1. Comparison of laminar and turbulent velocity profiles

Laminar:

- Can solve exactly
- Flow is steady
- Velocity profile is parabolic
- pipe roughness is not important (unless it is huge)



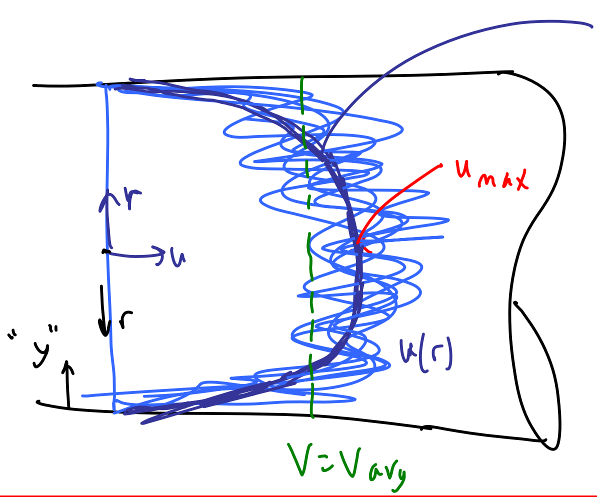
$$\boxed{V_{avg} = \frac{1}{2} u_{max}}$$

eq.

$$\boxed{u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)}$$

Turbulent

- Cannot solve exactly
- flow is not steady (but it is steady in the mean)
- pipe roughness is very important
- Velocity profile is "fuller" (closer to a top-hat or uniform velocity than a parabola)



time averaged (mean) velocity profile

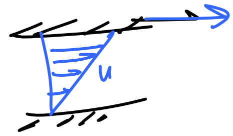
$V_{avg} \approx 85\%$ of u_{max}

No analytical eq. for the profile

See textbook for some empirical Approximate formulas

2. Wall shear stress

recall, for simple shear flow

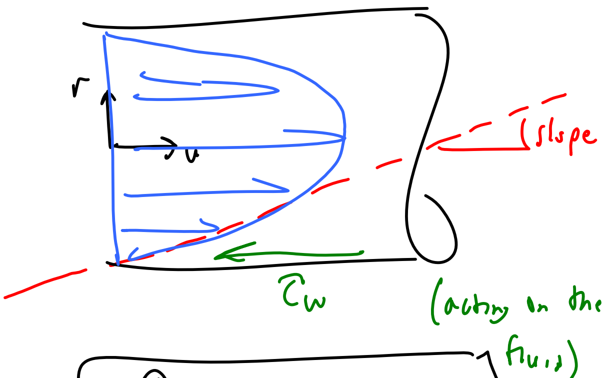


$$\tau = \text{shear stress} = \mu \frac{du}{dy}$$

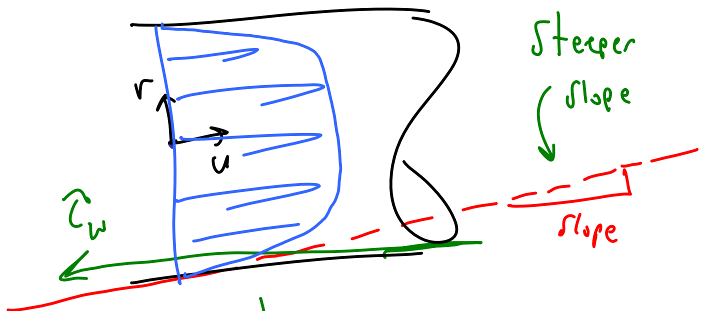
In pipe flow, $\boxed{\tau = \mu \left| \frac{du}{dr} \right|}$

For fully developed flow:

LAM



TURB



$\left| \frac{du}{dr} \right|$ much bigger at wall than for laminar flow

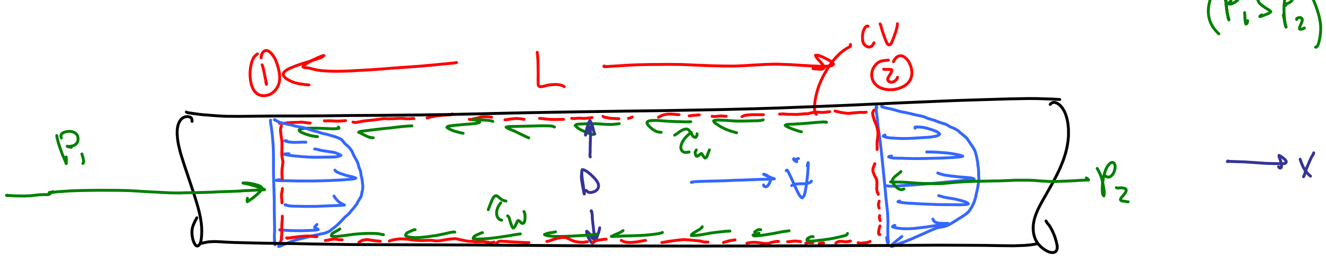
$$\boxed{\tau_{w\,turb} > \tau_{w\,lam}}$$

more friction $\therefore$ higher pressure drop for turb. pipe flow

3. Pressure drop in fully developed pipe flow

C.V. analysis of a fully developed section of pipe, steady, incompressible

$(P_1 > P_2)$



• Cons. of mass

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\rho \dot{\forall}_1 = \rho \dot{\forall}_2$$

$$V_1 \frac{\pi D^2}{4} = V_2 \frac{\pi D^2}{4} \longrightarrow \therefore \boxed{V_1 = V_2}$$

• Con. of momentum (x-direction)

$$\sum F_x = \cancel{\sum F_{x_{grav}}} + \sum F_{x\,press} + \sum F_{x\,visc} + \cancel{\sum F_{x_{other}}} = \sum_{out} \beta \dot{m} u - \sum_{in} \beta \dot{m} u$$

none in x

none for our CV

$$\left(P_1 - P_2\right)\frac{\pi D^2}{4} \qquad -\tau_w \pi L D$$

$$\beta_2 \cancel{\dot{m}} \cancel{V_2} - \beta_1 \cancel{\dot{m}} \cancel{V_1}$$

$V_1 = V_2$

$\beta_1 = \beta_2$ since fully developed

$= 0$

$$\boxed{P_1 - P_2 = 4\tau_w \frac{L}{D}} \qquad (1)$$

Cons. of energy for the same CV

$$\frac{P_1}{\rho g} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} + \cancel{h_{pump,u}} = \frac{P_2}{\rho g} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + \cancel{z_2} + \cancel{h_{turbine,e}} + h_L$$

none

none

$\alpha_1 = \alpha_2$, $V_1 = V_2$

fully developed

horizontal

$$\boxed{P_1 - P_2 = \rho g h_L} \qquad (2)$$

$h_L =$ irreversible loss

here it is due to friction on pipe wall

We call this a "major" loss

Combine (1) & (2)

4. The Darcy Friction Factor, f

equate (1) & (2) $\rightarrow$ $4\tau_w \frac{L}{D} = \rho g h_L$ $\rightarrow$ $\boxed{h_L = \frac{4\tau_w}{\rho g}\frac{L}{D}}$ (3)

(I don't know τ_w)

How to solve for τ_w ?

LAMINAR $\rightarrow$ can solve exactly since we know the eq. for $u(r)$

TURBULENT $\rightarrow$ need experiments (empirical data — curve fits)

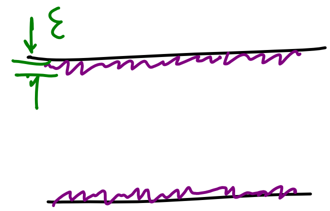
☆ Dimensional analysis is helpful !

$$\tau_w = fnc(\rho, V, \mu, D, \varepsilon)$$

$\downarrow$

practice

pipe roughness (avg height of roughness)



results:

$$\Pi_1 = f = \text{Darcy friction factor} = \boxed{f = \frac{8\tau_w}{\rho V^2}}$$

$$\Pi_2 = \boxed{Re = \frac{\rho V D}{\mu}}$$

$$\Pi_3 = \boxed{\frac{\varepsilon}{D} = \text{roughness factor}}$$

☆ $\boxed{f = fnc(Re, \varepsilon/D)}$ ☆