Today, we will: FULLY DEVELOPED PIPE FLOW

- Discuss the Moody chart and the Colebrook equation for friction factor *f*
- Discuss major vs. minor losses in pipe flows, and do some example problems

Consider steady, fully developed, incompressible pipe flow. Last time, we concluded that ~~Darcy friction factor *f*~~ Shear stress τ_w is a function of (ρ, V, μ, D, ε). Dimensional analysis yields:

$$\boxed{f = \text{fnc}\left(\text{Re}, \frac{\varepsilon}{D}\right)} \quad \text{where} \quad \boxed{f = \frac{8\tau_w}{\rho V^2}} \quad \text{and} \quad \boxed{\text{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}}$$

Darcy friction factor

Reynolds number

roughness factor

We also used control volume analysis to show that

$$\boxed{h_L = \frac{4\tau_w}{\rho g}\frac{L}{D}}$$

Combine & solve for h_L

Get $\boxed{h_L = f \frac{L}{D}\frac{V^2}{2g}}$ ☆

we need to find f.

$$\boxed{f = \text{fnc}\left(Re, \frac{\varepsilon}{D}\right)} \text{ ☆}$$

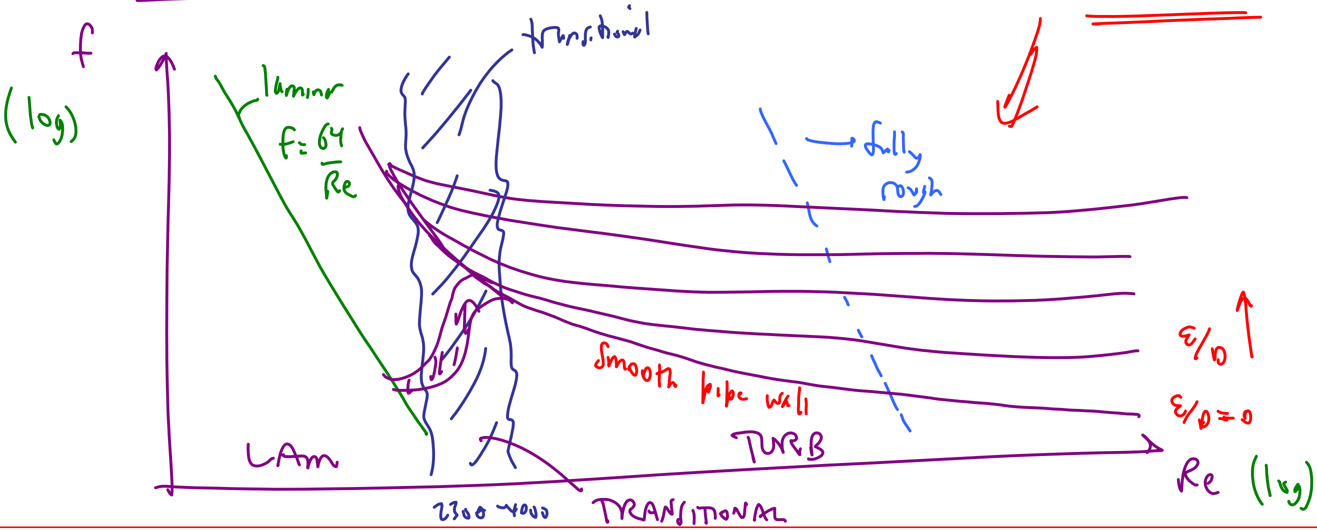
Laminar flow, → exact eq. $\boxed{f = 64/Re}$ (see ch. 9 for proof)

Does not depend on roughness (unless extreme roughness)

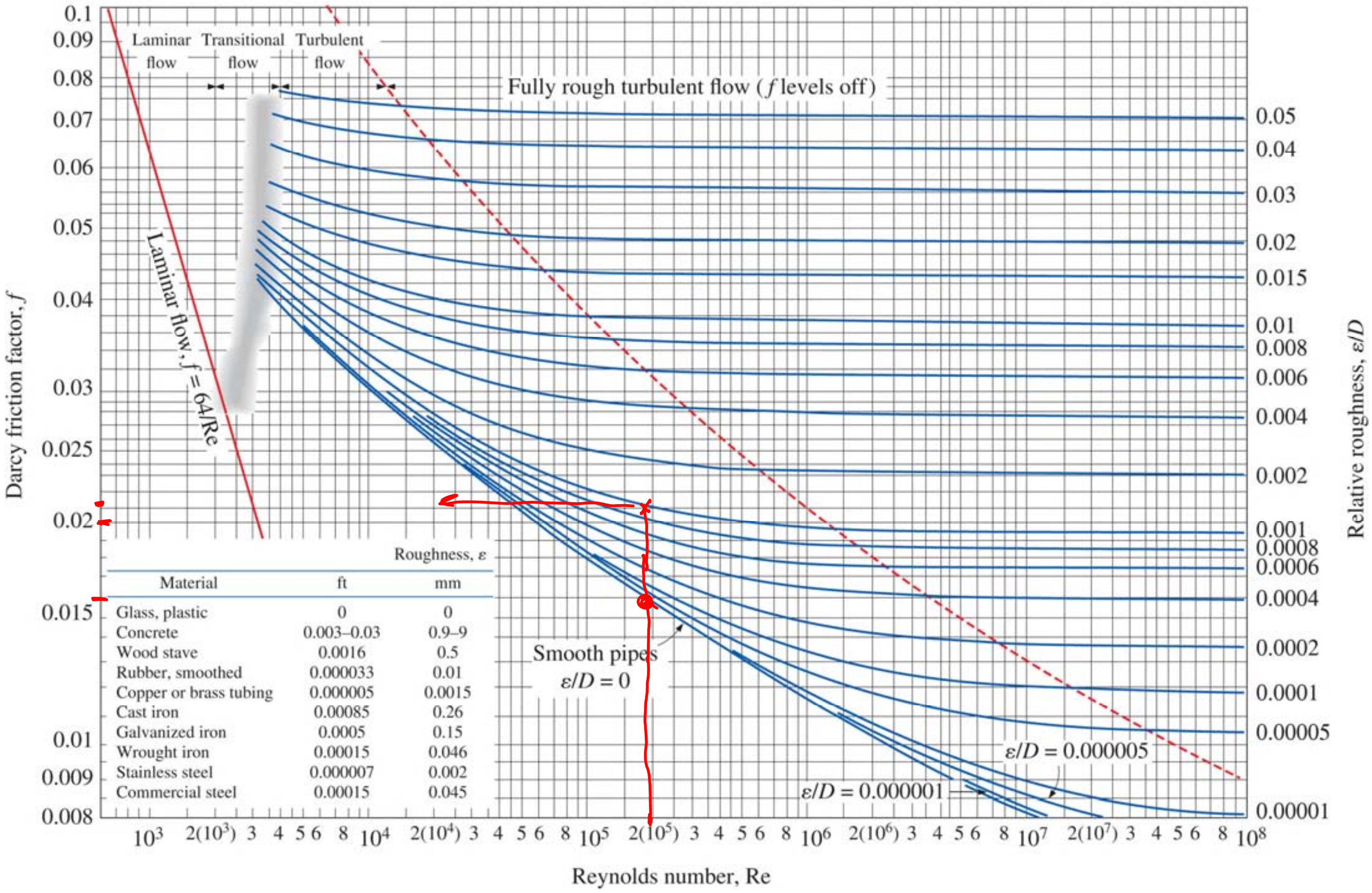
Turbulent flow → no exact eq. → use empirical curve fits & charts

J. The Moody Chart

Construct from the Colebrook Eq.



The Moody Chart (From Appendix, Figure A-12, in the textbook)



6. Empirical Equation for Fully Developed Pipe Flow

There are empirical equations available to use in place of the Moody chart. The most useful one (in fact, the equation with which the turbulent portion of the Moody chart is drawn) is:

The Colebrook equation

$$\boxed{\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\mathrm{Re}\sqrt{f}}\right)}$$

Note: This is $\log_{10}$, not the natural log, ln.

Unfortunately, the Colebrook equation is ***implicit*** in f (since f appears on both sides of the equation), and the equation must be solved by *iteration*. An approximation to the Colebrook equation was created by Haaland, accurate to ±2% compared to the Colebrook equation, and can be used as a quick estimate or as a "first guess" to begin a Colebrook equation iteration:

The Haaland equation

$$\boxed{\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left[\frac{6.9}{\mathrm{Re}}+\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]}$$

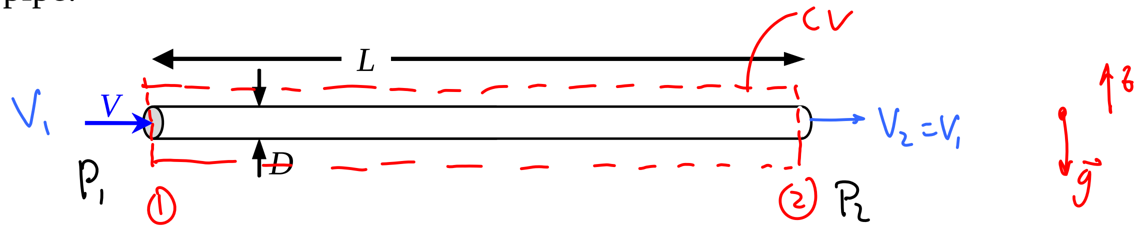
Also $\log_{10}$, not ln.

Finally, there are some other approximations in the textbook, e.g., those of Swamee and Jain. **In this course, we will always use the Colebrook equation or the Moody chart.**

7. Examples

Example: Hydrodynamic entrance length

Given: Water at 10.0 °C flows at a steady volume flow rate of 0.0100 m^3/s through a pipe of diameter 5.00 cm. The pipe is 100 m long, and the flow is fully developed through the entire section of pipe.



To do:

(a) Calculate the pressure drop if roughness height ε = 0.00050 cm.

(b) Calculate the pressure drop if roughness height ε = 0 (hydrodynamically smooth pipe).

Solution:

- Table A-3 → water @ 10°C → $\rho = 999.7\ \mathrm{kg/m^3}$

  $\mu = 1.307 \times 10^{-3}\ \mathrm{kg/m\cdot s}$

- Draw a CV

- Cons. of mass → $\dot{V}_1 = \dot{V}_2$ → $V_1 = V_2 = \text{const.}$

- Cons. of energy in head form

horizontal pipe

$$\frac{P_1}{\rho g} + \cancel{\alpha_1 \frac{V_1^2}{2g}} + \cancel{z_1} + \cancel{h_{pump,u}} = \frac{P_2}{\rho g} + \cancel{\alpha_2 \frac{V_2^2}{2g}} + \cancel{z_2} + \cancel{h_{turbine,e}} + h_L$$

$\alpha_1 = \alpha_2$ (fully dev.)

no pump

$V_1 = V_2$, $\alpha_1 = \alpha_2$

no turbine

$$\boxed{P_1 - P_2 = \Delta P = \rho g h_L} \quad (1)$$

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

need V → $V = \text{avg velocity} = \dfrac{\dot{V}}{A} = \dfrac{4\dot{V}}{\pi D^2}$

need to find f

$$= \frac{4(0.0100\ \mathrm{m^3/s})}{\pi (0.0500\ \mathrm{m})^2} \rightarrow V = 5.092958\ \mathrm{m/s}$$

To find f → $Re = \frac{\rho V D}{\mu} = \frac{(999.7\ kg/m^3)(5.092958\ m/s)(0.050\ m)}{1.307\times10^{-3}\ kg/m\cdot s} = 1.9477\times10^5$

Definitely turbulent

(a) → $\frac{\varepsilon}{D} = \frac{0.0050\ cm}{5.00\ cm} = 0.001 = \varepsilon/D$

- Moody chart → @ $Re = 1.95\times10^5$, $\varepsilon/D = 0.001$ → $f \approx 0.0210$
- Or Colebrook eq. → see example files in Excel, EES

Excel → $f = 0.02107$, EES, colebrook → $f = 0.02107$

EES → Moody Chart $(Re, \varepsilon/D)$ function → $f = 0.002122$

$$\boxed{\Delta P = \rho \frac{V^2}{2} f \frac{L}{D}} = \left(997.2\ \frac{kg}{m^3}\right)\left(\frac{(5.092958\ \frac{m}{s})^2}{2}\right)(0.02107)\left(\frac{100\ m}{0.050\ m}\right)$$

$$\cdot\left(\frac{N\cdot s^2}{kg\cdot m}\right)\left(\frac{kPa\cdot m^2}{1000\ N}\right) \to \boxed{\Delta P = 546.\ kPa}$$

rough pipe

(b) Smooth pipe $(\varepsilon = 0)$

Repeat → $f = 0.0157$ → $\boxed{\Delta P = 408\ kPa}$

Diff ≈ 25%

Roughness makes a big difference!

$\varepsilon/D = 0.001$

$$\boxed{h_L = f \frac{L}{D}\frac{V^2}{2g} \to h_L = \begin{matrix} 55.7\ m \\ 41.6\ m \end{matrix}}$$

$\varepsilon/D = 0.001$

$\varepsilon/D = 0$

C. Minor Losses

1. Terminology → Major losses – losses due to long straight fully developed sections of pipe

Use Moody chart or Colebrook eq.

$$\boxed{h_{L,major} = f \frac{L}{D} \frac{V^2}{2g}}$$

Minor losses → everything else

- fittings
- valves
- elbows
- tees / branches
- inlet region
- outlets

Note: "Minor" losses can actually be bigger than "major" losses

Two ways to account for minor losses

a. Loss coefficient – more popular

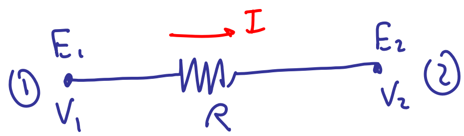
b. Equivalent length

a. Loss Coefficient

$$\boxed{h_{L,minor} = K_L \frac{V^2}{2g}}$$

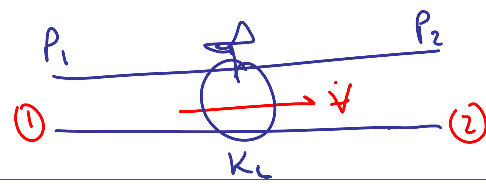
K_L = minor loss coefficient (nondimensional)

Analogy to electrical circuit



Voltage drop due to R

$E_1 > E_2$

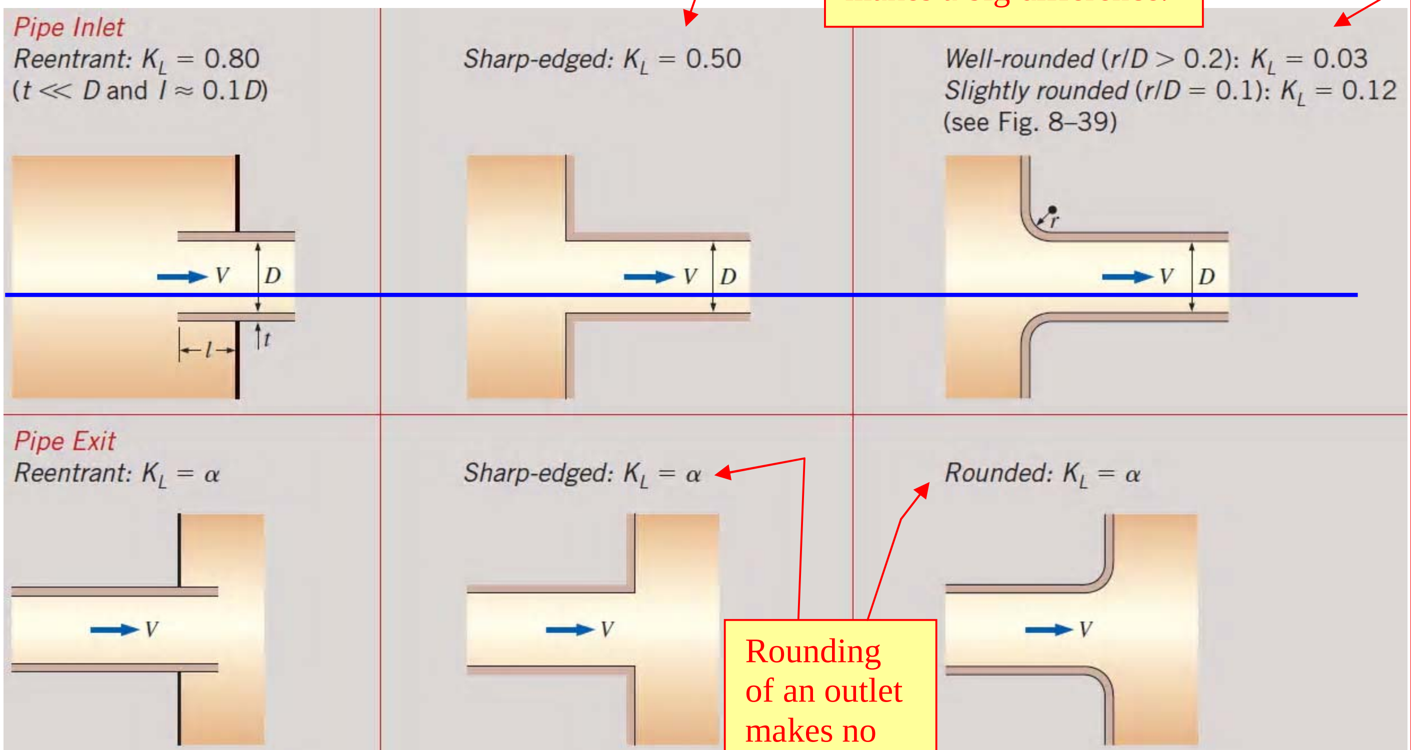


$P_1 > P_2$ due to the minor loss K_L

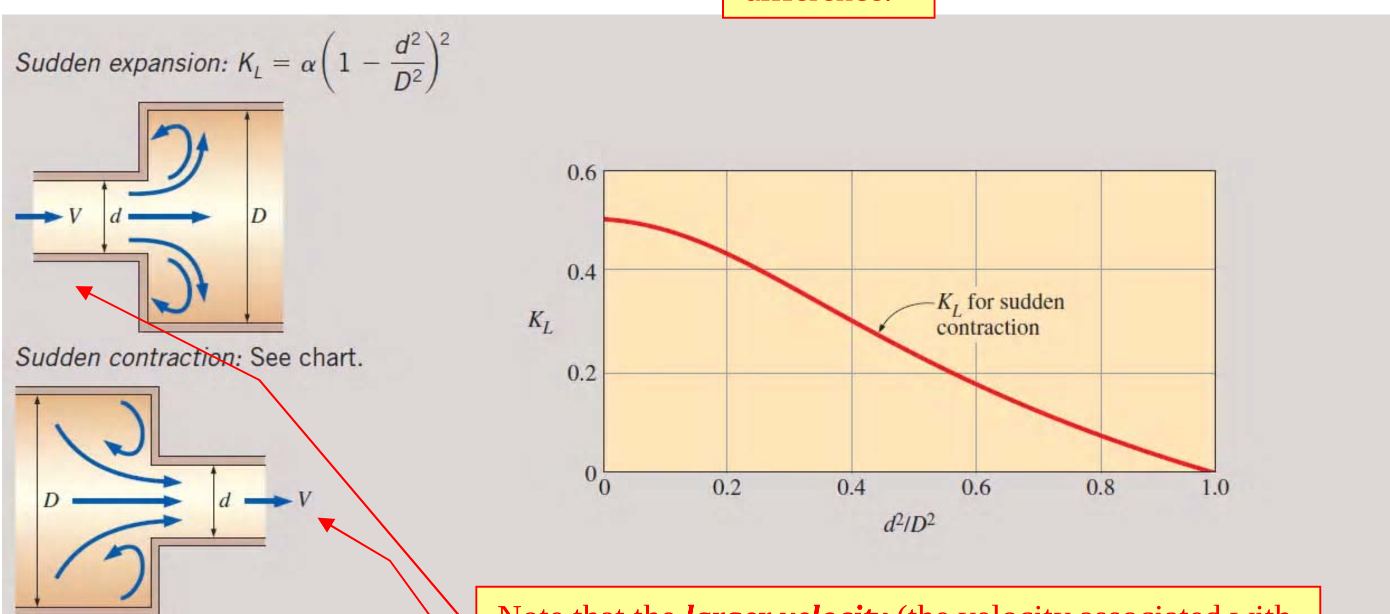
Minor Losses

Here are some sample loss coefficients for various minor loss components. More values are listed in Table 8-4 of the Çengel-Cimbala textbook:

Rounding of an inlet makes a big difference.



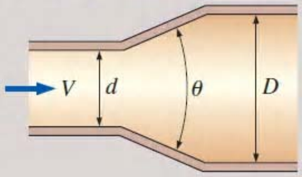
Rounding of an outlet makes no difference.



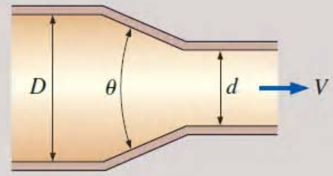
Note that the ***larger velocity*** (the velocity associated with the ***smaller pipe section***) is used by convention in the equation for minor head loss, $h_{L,\,\text{minor}} = K_L \dfrac{V^2}{2g}$.

Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Expansion (for $\theta = 20°$):
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

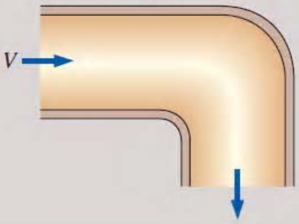


Contraction:
$K_L = 0.02$ for $\theta = 30°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

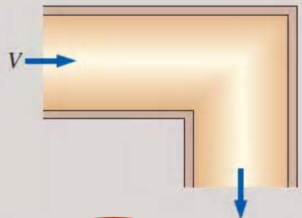


Bends and Branches

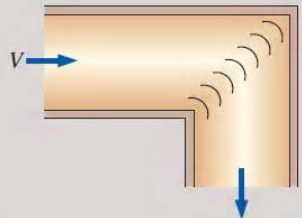
90° *smooth bend:*
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



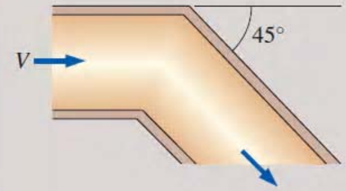
90° *miter bend*
(without vanes): $K_L = 1.1$



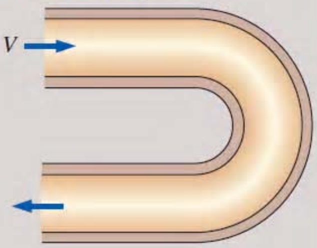
90° *miter bend*
(with vanes): $K_L = 0.2$



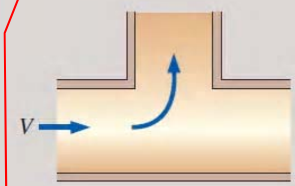
45° *threaded elbow:*
$K_L = 0.4$



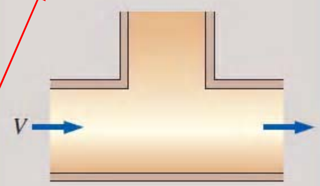
180° *return bend:*
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



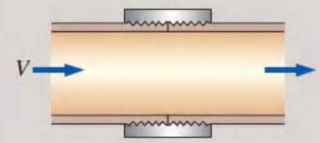
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$



Threaded union:
$K_L = 0.08$



For tees, there are <u>*two*</u> values of K_L, one for *branch flow* and one for *line flow*.

Valves

Globe valve, fully open: $K_L = 10$
Angle valve, fully open: $K_L = 5$
Ball valve, fully open: $K_L = 0.05$
Swing check valve: $K_L = 2$

Gate valve, fully open: $K_L = 0.2$
$\frac{1}{4}$ closed: $K_L = 0.3$
$\frac{1}{2}$ closed: $K_L = 2.1$
$\frac{3}{4}$ closed: $K_L = 17$