Today, we will:

- Continue discussing minor losses in pipe flows, and do some example problems

Recall, major and minor head losses:

Major: $\boxed{h_{L,\text{major}} = f\dfrac{L}{D}\dfrac{V^2}{2g}}$ where $\boxed{f = \text{fnc}\left(\text{Re}, \dfrac{\varepsilon}{D}\right)}$ from Moody chart or Colebrook equation.

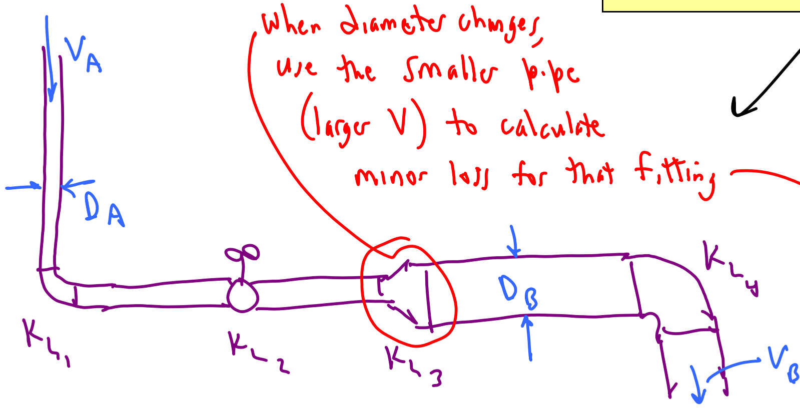
Minor: $\boxed{h_{L,\text{minor}} = K_L\dfrac{V^2}{2g}}$ where K_L = minor loss coefficient, from tables and charts.

In the head form of the energy equation, $\boxed{h_L = \sum h_{L,\text{major}} + \sum h_{L,\text{minor}} = \sum f\dfrac{L}{D}\dfrac{V^2}{2g} + \sum K_L\dfrac{V^2}{2g}}$.

Sum over pipe sections of different diameters

have different V, D

When diameter changes, use the smaller pipe (larger V) to calculate minor loss for that fitting

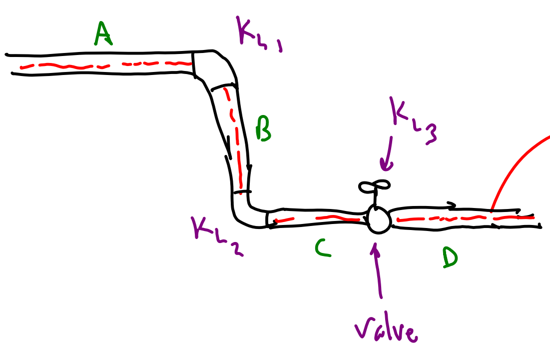


In this example, K_{L_1}, K_{L_2}, & K_{L_3} are with the small pipe. K_{L_4} is with the large pipe

- In most problems, D = constant everywhere ∴ V is const everywhere

$$\boxed{\therefore\ h_L = \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right)}$$

Total length of pipe, adding all sections

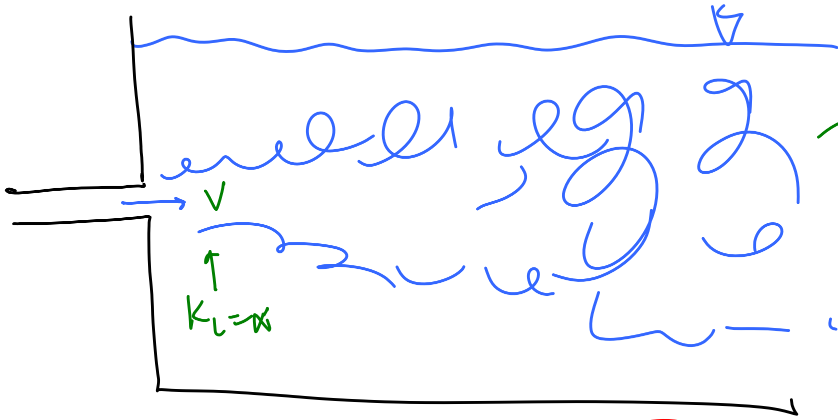


L = total length of all pipe sections

Here, add L_A, L_B, L_C, L_D to get the total L

☆ This is the most common form that we will use in this course

Head loss at outlets

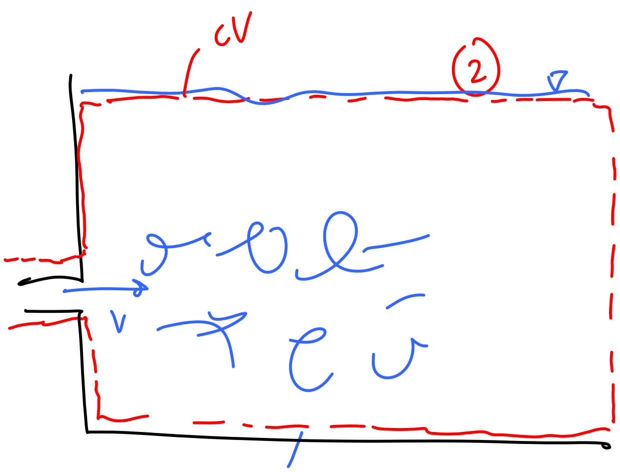


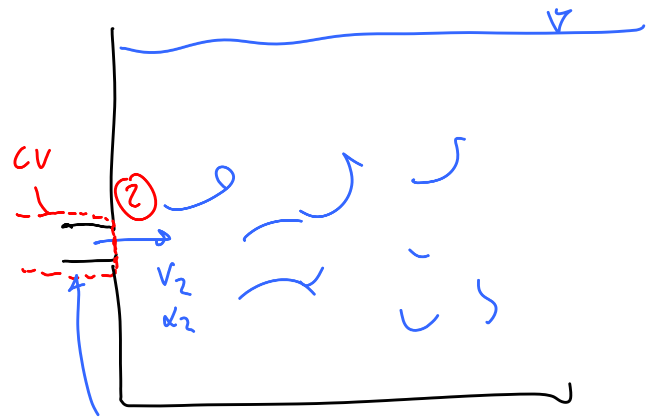
turbulent eddies waste energy! eventually dissipate to heat energy

In energy eq., $\sim + \alpha \frac{V^2}{2g}$ all of this kinetic energy is wasted

$\therefore$ K_L @ an outlet is equal to α

Minor loss @ an outlet is $h_L = K_L \frac{V^2}{2g}$ $K_L = \alpha$





With this CV, include α as one of our K_L's

Energy eq. $+ \cdots \alpha_2 \cancel{\frac{V_2^2}{2g}} + \cdots$ 0 @ 2

But, $K_L = \alpha$ in tank, so get $\alpha \frac{V^2}{2g}$

With this CV Do not include α in the sum of K_L's

However in energy eq.

$+ \cdots \alpha_2 \frac{V_2^2}{2g} + \cdots$ include this term

b. Equivalent length — an alternative to using K_L for minor losses

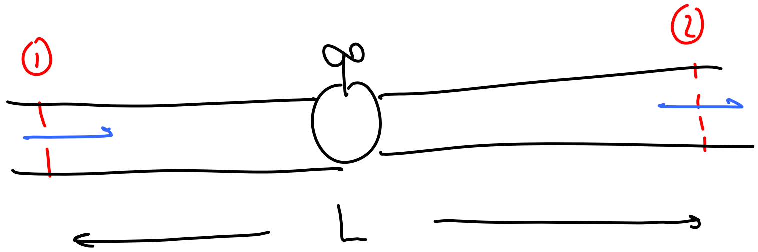
$$\boxed{h_{L_{minor}} = f \frac{L_{equiv}}{D} \frac{V^2}{2g}}$$

→ Treat the minor loss as if it were a longer pipe

equivalent dis

eg. <u>Valve</u>

<u>Real case</u>



<u>Imaginary replacement case</u> [Pretend the pipe is longer by L_{equiv}]

In energy eq., use

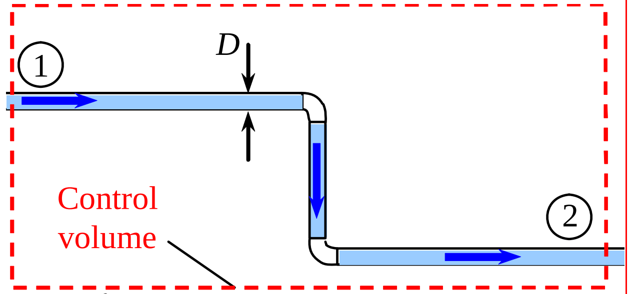
$$\boxed{h_L = f \frac{L_{total}}{D} \frac{V^2}{2g}}$$

$$\boxed{L_{total} = L + L_{equiv}}$$

2. <u>Examples</u>

Example: Major and minor losses

Given: Water ($\rho = 998.$ kg/m^3, $\mu = 1.00 \times 10^{-3}$ kg/m·s) flows at a steady average velocity of 6.45 m/s through a smooth pipe of diameter 2.54 cm. The flow is fully developed through the entire section of pipe. The total pipe length is 10.56 m, and there are two elbows, each with $K_L = 0.90$.



To do:

(a) Calculate the total irreversible head loss in meters through this section of piping due to both major and minor losses.

Solution:

- Draw a CV — see diagram

- $Re = \dfrac{\rho V D}{\mu} = \dfrac{(998\ \text{kg/m}^3)(6.45\ \text{m/s})(0.0254\ \text{m})}{1.00 \times 10^{-3}\ \text{kg/m·s}} = \underline{163{,}502}$ TURBULENT

$(\varepsilon/D = 0)$

- Moody chart or Colebrook eq. $\rightarrow$ $\boxed{f = 0.0163}$

- Minor losses too.

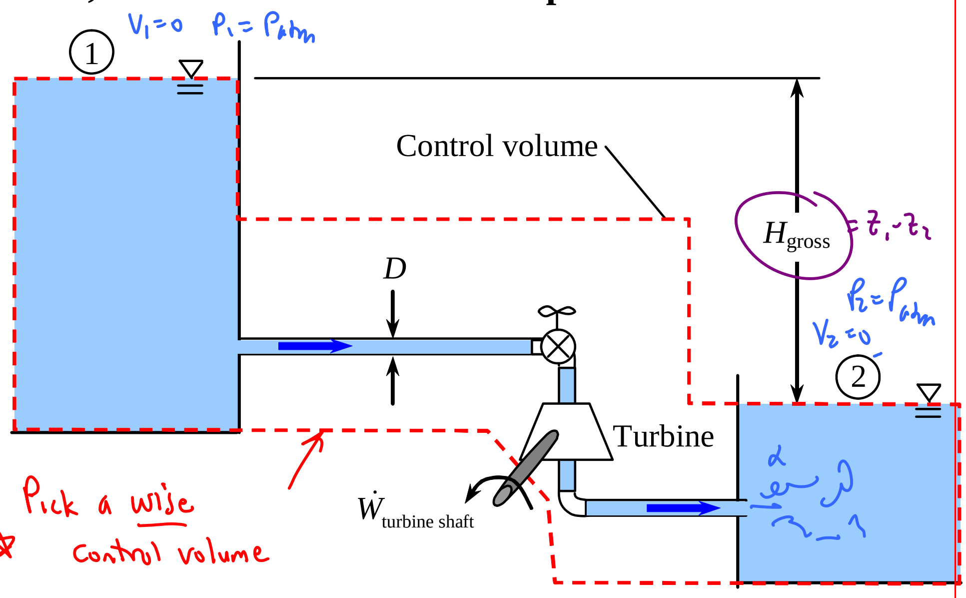
$$\boxed{h_L = \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right)}$$

$$= \frac{(6.45\ \text{m/s})^2}{2(9.807\ \frac{\text{m}}{\text{s}^2})}\left(0.0163\ \frac{10.56\ \text{m}}{0.0254\ \text{m}} + \underbrace{2(0.9)}_{\sum K_L}\right) = \boxed{18.2\ \text{m}}$$

Comment: This is just the irreversible head loss. The flow from ① to ② also includes an elevation or potential energy head loss → need to use the head form of the energy equation to deal with that

Example: Major and minor losses, calculation of turbine shaft power

Given: Water (ρ = 998. kg/m^3, $\mu = 1.00 \times 10^{-3}$ kg/m·s) flows from one large reservoir to another, and through a turbine as sketched. The elevation difference between the two reservoir surfaces is H_{gross} = 120.0 m. The pipe is 5.0 cm I.D. cast iron pipe. The total pipe length is 30.8 m. The entrance is slightly rounded; the exit is sharp. There is one regular flanged 90-degree elbow, and one fully open flanged angle valve. The turbine is 81% efficient. The volume flow rate through the turbine is 0.0045 m^3/s.



To do: Calculate the shaft power produced by the turbine in units of kilowatts.

Solution:

- First we draw a control volume, as shown by the dashed line. We cut through the surface of both reservoirs (inlet 1 and outlet 2), where we know that the velocity is nearly zero and the pressure is atmospheric. We also slice through the turbine shaft. The rest of the control volume simply surrounds the piping system.
- We apply the head form of the energy equation from the inlet (1) to the outlet (2):

$$\boxed{P_1 = P_2 = P_{atm}}$$

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_{pump,u}} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + z_2 + h_{turbine,e} + h_L \rightarrow h_{turbine,e} = \underbrace{(z_1 - z_2)}_{H_{gross}} - h_L$$

$$\boxed{V_1 = V_2 \approx 0}$$

- But by definition of turbine efficiency, $\boxed{h_{turbine,\,e} = \frac{\dot{W}_{turbine\ shaft}}{\eta_{turbine}\dot{m}g}}$ where $\boxed{\dot{m} = \rho\dot{V}}$. Also, since the reference velocity is the same for all the major and minor losses (the pipe diameter is constant throughout), we may use the simplified version of the equation for h_L, i.e., Eq. 8-59: $\boxed{h_L = \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right)}$. Therefore, we solve the energy equation for the desired unknown, namely, turbine shaft power, $\boxed{\dot{W}_{turbine\ shaft} = \eta_{turbine}\rho\dot{V}g\left[H_{gross} - \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right)\right]}$. This is our answer in variable form, but we still need to calculate the values of some of the variables.

The rest of this problem will be solved in class.

- $V = \frac{\dot{V}}{A} = \frac{4\dot{V}}{\pi D^2} = \frac{4(0.0045\ m^3/s)}{\pi(0.050\ m)^2} = 2.29183\ m/s = V$

$\cdot\ Re = \dfrac{\rho V D}{\mu} = \dfrac{(998\ kg/m^3)(2.29183\ m/s)(0.050\ m)}{0.001\ kg/m \cdot s} = \dfrac{1.144 \times 10^5 = Re}{(\text{turbulent})}$

$\cdot\ \varepsilon = \rightarrow$ look up Table 8-2 $\rightarrow \varepsilon = 0.26\ mm \rightarrow \dfrac{\varepsilon}{D} = \dfrac{0.26\ mm}{50\ mm} = \underline{\underline{0.0052}}$

$\cdot$ At this Re & ε/D $\rightarrow$ Moody chart $\rightarrow$ $\underline{\underline{f = 0.03154}}$

$\alpha = 1.05$

$$\cdot\ \sum K_L = \underset{\text{slightly rounded inlet}}{0.12} + \underset{\text{angle valve}}{5} + \underset{\text{flanged elbow}}{0.3} + \underset{\text{outlet into a tank}}{\alpha} = \underline{\underline{6.47}}$$

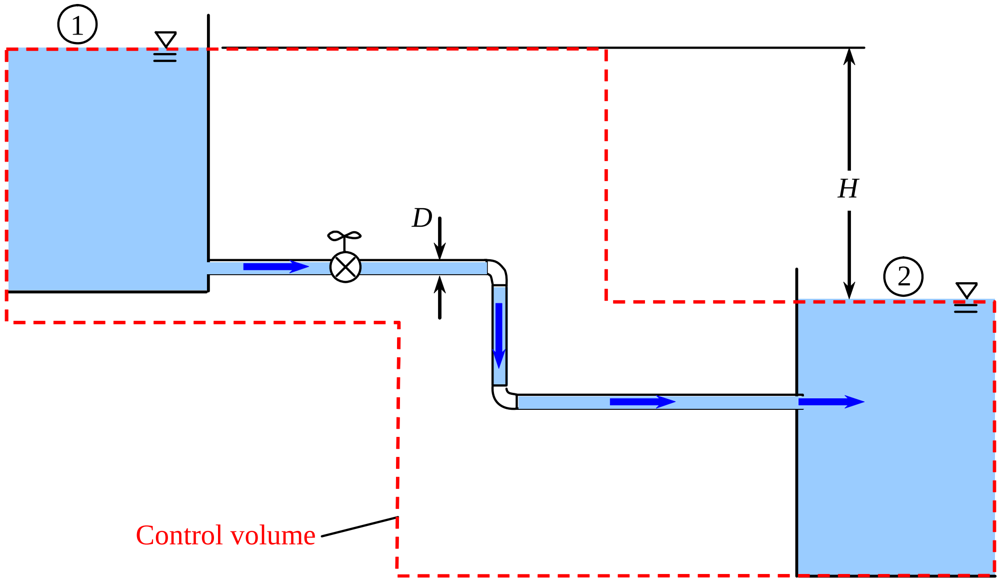
From Table

$$\dot{W}_{\text{turbine shaft}} = \eta_{\text{turbine}}\, \rho \dot{V} g \left[H_{gross} - \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L \right) \right]$$

$$\boxed{\dot{W}_{\text{turbine shaft}} = 4.03\ kW}$$

Example: Major and minor losses, iterating to calculate the flow rate

Given: Water ($\rho = 998.$ kg/m^3, $\mu = 1.00 \times 10^{-3}$ kg/m·s) flows *by gravity alone* from one large tank to another, as sketched. The elevation difference between the two surfaces is $H = 35.0$ m. The pipe is 2.5 cm I.D. with an average roughness of 0.010 cm. The total pipe length is 20.0 m. The entrance and exit are sharp. There are two regular threaded 90-degree elbows, and one fully open threaded globe valve.



To do: Calculate the volume flow rate through this piping system.

Solution:

- First we draw a control volume, as shown by the dashed line. We cut through the surface of both reservoirs (inlet 1 and outlet 2), where we know that the velocity is nearly zero and the pressure is atmospheric. The rest of the control volume simply surrounds the piping system.
- We apply the head form of the energy equation from the inlet (1) to the outlet (2):

$$\boxed{P_1 = P_2 = P_{\text{atm}}}$$

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_{\text{pump,u}}} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + z_2 + \cancel{h_{\text{turbine,e}}} + h_L$$

$$\boxed{V_1 = V_2 \approx 0}$$

  Therefore, the energy equation reduces to $\boxed{h_L = z_1 - z_2 = H}$

- Next, we add up all the irreversible head losses, both major and minor. Since the reference velocity is the same for all the major and minor losses (the pipe diameter is constant throughout), we may use the simplified version of the equation for h_L,

(2)→ $\boxed{h_L = \left(f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g}}$, & $\boxed{\text{Re} = \frac{\rho D V}{\mu}}$ $\boxed{\dot{V} = V\frac{\pi D^2}{4}}$ $\boxed{\frac{\varepsilon}{D} = \frac{0.010 \text{ cm}}{2.5 \text{ cm}} = 0.004}$

- We also need either the Moody chart or one of the empirical equations that can be used in place of the chart (e.g., the Colebrook equation).

The rest of this problem will be solved in class.

$$\sum K_L = \underbrace{0.50}_{\text{inlet}} + \underbrace{2(0.9)}_{\text{2 elbows}} + \underbrace{10}_{\text{globe valve}} + \underbrace{\overset{\alpha}{1.05}}_{\text{outlet}} = 13.35$$

Iteration:

Solve (2) for V → $\boxed{V = \sqrt{\dfrac{2gH}{f\frac{L}{D} + \sum K_L}}}$ (3)

Procedure:

Guess f	Calc. V using (3) (m/s)	Calc Re	Look up f _ Moody chart
Initial guess 0.03	4.287	1.076×10^5	0.029
0.029	4.334	1.081×10^5	0.0294
0.0294	4.315	1.077×10^5	0.02943
0.02943	4.314	1.076×10^5	0.02943 Converged !

Final values → $V = 4.314$ m/s

$$\dot{V} = V \frac{\pi D^2}{4} \rightarrow \boxed{2.12 \times 10^{-3}\ m^3/s}$$

See also the solution using EES

See next pages

EES Solution for Example Problem – Major and Minor Losses in a Piping System

Here is exactly what I typed into the main "Equations Window" of EES:

```
"EES Solution for the class example problem - major and minor losses in a piping system
J. M. Cimbala, February 2005"

"Constants:"

h_L = 35 [m]
rho = 998 [kg/m^3]
mu = 1.00e-3 [kg/(m*s)]
D = 0.025 [m]
L = 20.0 [m]
SigmaK = 13.35

"Equations:"

h_L = (f*L/D + SigmaK)*(V^2)/(2*g#)   "Note that g# is the gravitational constant, pre-defined by EES"
Re = rho*D*V/mu
eps_by_D = 0.004
V_dot = V*PI*(D^2)/4

"Colebrook equation:"   1/sqrt(f) = -2.0*log10(eps_by_D/3.7 + 2.51/(Re*sqrt(f)))

"To solve, click on Calculate and then Solve. Note that it does not converge unless you change the limits and guesses in Options-Variable Info"
```

Note: We could have used the EES function "MoodyChart" instead of the Colebrook equation; i.e.,
f = MoodyChart(Re,eps_by_D)
which gives slightly different answers.

Here is what the "Options-Variable Info" chart looks like:

Variable Information

☑ Show array variables
☐ Show string variables

Variable	Guess	Lower	Upper	Display			Units	Key	Comment
D	1	-infinity	infinity	A	3	N	m		
eps_by_D	1	-infinity	infinity	A	3	N			
f	0.02	1.0000E-03	1.0000E-01	A	3	N			
h_L	1	-infinity	infinity	A	3	N	m		
L	1	-infinity	infinity	A	3	N	m		
mu	1	-infinity	infinity	A	3	N	kg/(m*s)		
Re	10000	4.0000E+03	infinity	A	3	N			
rho	1	0.0000E+00	infinity	A	3	N	kg/m^3		
SigmaK	1	0.0000E+00	infinity	A	3	N			
V	1	0.0000E+00	infinity	A	3	N	m/s		
V_dot	1	0.0000E+00	infinity	A	3	N	m^3/s		

OK   Apply   Print   Update   Cancel

Note: I had to change some of the initial guesses and some of the lower and upper limits in order for EES to converge on a solution. This takes some trial and error.

The Formatted Equations window looks like this (the equations appear in much more readable format):

Formatted Equations

EES Solution for the class example problem - major and minor losses in a piping system
J. M. Cimbala, February 2005

Constants:

$h_L = 35$ [m]

$\rho = 998$ [kg/m^3]

$\mu = 0.001$ [kg/(m*s)]

$D = 0.025$ [m]

$L = 20$ [m]

$SigmaK = 13.35$

Equations:

$$h_L = \left[f \cdot \frac{L}{D} + SigmaK \right] \cdot \frac{V^2}{2 \cdot 9.807 \ [\text{m/s}^2]}$$ Note that g# is the gravitational constant, pre-defined by EES

$$Re = \rho \cdot D \cdot \frac{V}{\mu}$$

$$eps_{by,D} = 0.004$$

$$\dot{V} = V \cdot \pi \cdot \frac{D^2}{4}$$

Colebrook equation: $$\frac{1}{\sqrt{f}} = -2 \cdot \log\left[\frac{eps_{by,D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}} \right]$$

To solve, click on Calculate and then Solve. Note that it does not converge unless you change the limits and guesses in Options-Variable Info

Finally, Calculate and Solve yields the solution:

D=0.025 [m]
eps_by_D=0.004
f=0.02943
h_L=35 [m]
L=20 [m]
mu=0.001 [kg/(m*s)]
Re=107627
rho=998 [kg/m^3]
SigmaK=13.35
V=4.314 [m/s]
V_dot=0.002117 [m^3/s]

This is our final result, i.e., the volume flow rate through the pipe. We can verify that all the variables are correct, and are the same as those calculated by "hand", i.e.,

V_dot = 2.12×10^{-3} m^3/s.