Today, we will:

- Discuss pump performance curves
- Discuss how to match a pump and a piping system, and do some example problems

2. Pump Performance

a. Pump performance curves

Note: In pump literature and terminology, H is the **net head of the pump**.
(H is the same as what we called $h_{\text{pump,u}}$ in the energy equation, i.e., the useful head delivered by the pump to the fluid.) So, $\boxed{h_{\text{pump,u}} = H}$

Also recall, **pump efficiency** is defined as

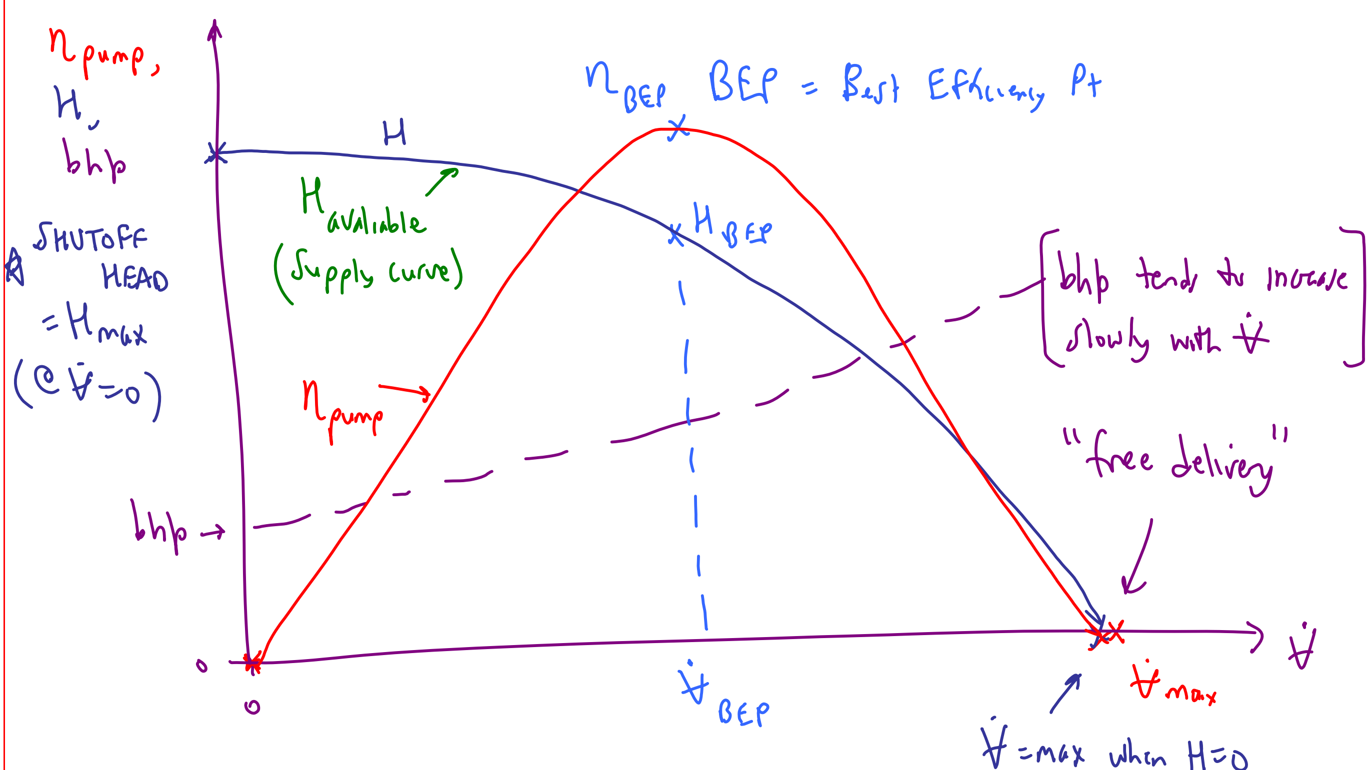
$$\boxed{\eta_{\text{pump}} = \frac{\text{useful power delivered to the fluid}}{\text{shaft power required to run the pump}} = \frac{\text{water horsepower}}{\text{brake horsepower}} = \frac{\dot{m} g h_{\text{pump,u}}}{bhp} = \frac{\rho \dot{V} g H}{bhp}}$$

$$\eta_{\text{pump}} = \frac{\rho \dot{V} g H}{bhp} \rightarrow$$

2 ways for η_{pump} to $= 0$

- $\dot{V} = 0$ (no flow)
- $H = 0$ (no head)

<u>Pump performance curve</u> η vs $\dot{V}$



b. Matching a pump to a pumping system

General Example: Matching a pump to a system.

Given: Consider a typical piping system with a pump that pumps water from a lower reservoir to a higher reservoir. All diameters, heights, minor loss coefficients, etc. are known.

To do: Predict the volume flow rate.

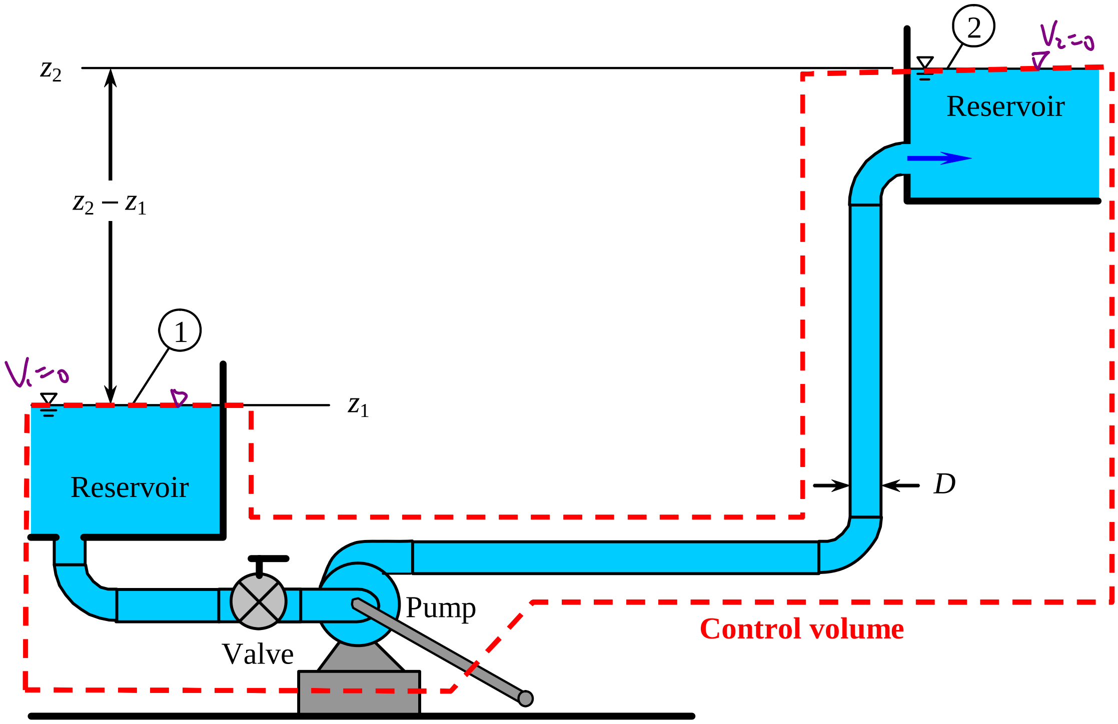
Solution: As always we start by drawing a wise control volume. See CV as drawn.
Then we apply the head form of the conservation of energy equation for a CV from 1 to 2:

No turbine here

$$\frac{P_1}{\rho_1 g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho_2 g}+\alpha_2\frac{V_2^2}{2g}+z_2+\cancel{h_{\text{turbine,e}}}+h_L$$

Solve for H = net pump head delivered or required = $h_{\text{pump,u}}$ = useful pump head:

$$\boxed{H=H_{\text{required}}=h_{\text{pump,u}}=\underbrace{\frac{P_2}{\rho_2 g}-\frac{P_1}{\rho_1 g}}_{\text{I}}+\underbrace{\frac{\alpha_2 V_2^2-\alpha_1 V_1^2}{2g}}_{\text{II}}+\underbrace{z_2-z_1}_{\text{III}}+\underbrace{h_L}_{\text{IV}}} \qquad (1)$$



In general, from Eq. (1), we see that the pump must do four things:

I Change the pressure in the flow from inlet to outlet — Can be ⊕ or ⊖ — Here, $=0$

II Change the kinetic energy in the flow from inlet to outlet — " " ⊕ or ⊖ — Here, $=0$

III Change the elevation in the flow from inlet to outlet — " " ⊕ or ⊖ — Here, ⊕ (z_2-z_1)

IV Overcome irreversible head losses — Always ⊕ !

Here, solve (1) for $\boxed{H_{\text{required}} = z_2 - z_1 + h_L}$ — h_L depends on $\dot{V}$

H_{required}: system; z_2-z_1: const

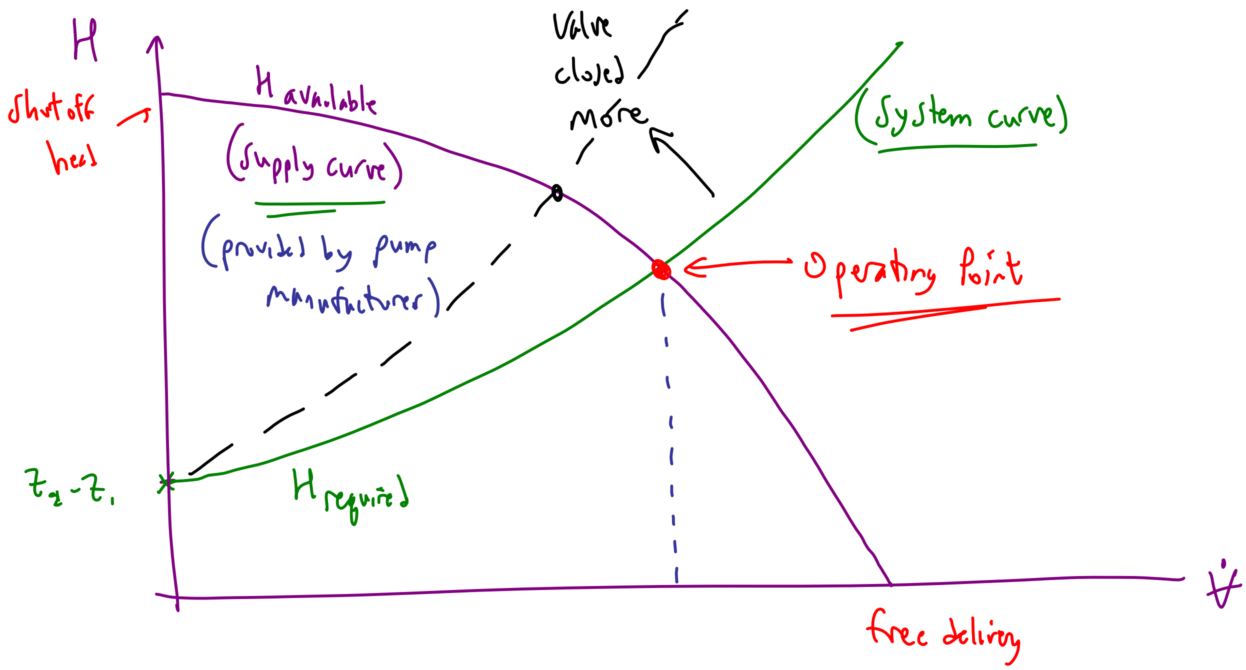
SYSTEM CURVE

$\therefore H_{\text{required}}$ depends on $\dot{V}$

As $\dot{V}\uparrow$, $h_L\uparrow$

b. Matching pump to system

$\left[H_{\text{required}} = z_2 - z_1 \text{ when } \dot{V} = 0\right]$



The flow will automatically adjust itself to this operating point

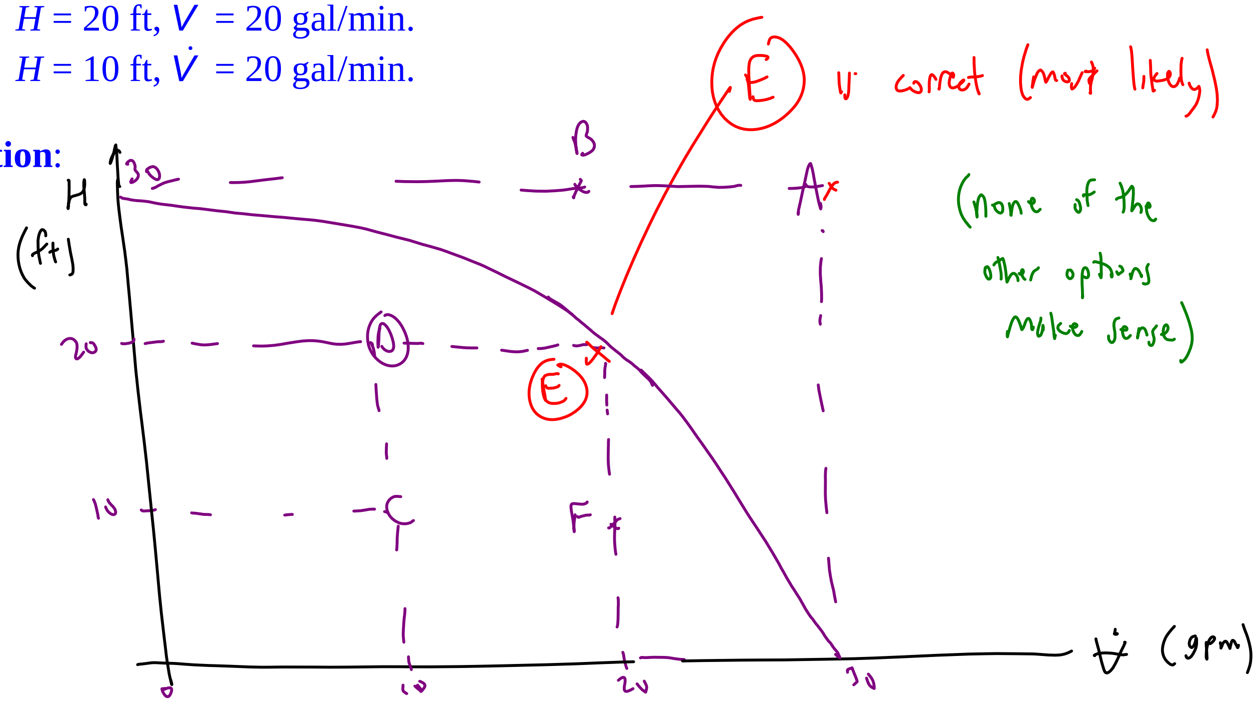
Example: Matching a Pump to a Piping System

Given: James buys a water pump. The specifications say "maximum head = 30 feet of water" and "maximum flow rate = 30 gallons per minute".

To do: Which of the following choices is most likely to be the actual pump performance?

(a) H = 30 ft, $\dot{V}$ = 30 gal/min.
(b) H = 30 ft, $\dot{V}$ = 20 gal/min.
(c) H = 10 ft, $\dot{V}$ = 10 gal/min.
(d) H = 20 ft, $\dot{V}$ = 10 gal/min.
(e) H = 20 ft, $\dot{V}$ = 20 gal/min.
(f) H = 10 ft, $\dot{V}$ = 20 gal/min.

Solution:



NOTE: WHEN THE PUMP MANUFACTURER CLAIMS 30 gpm And 30 ft, Caution → You don't get both of these at the same time!

(max head = $H_{shutoff}$ occurs @ $\dot{V} = 0$)

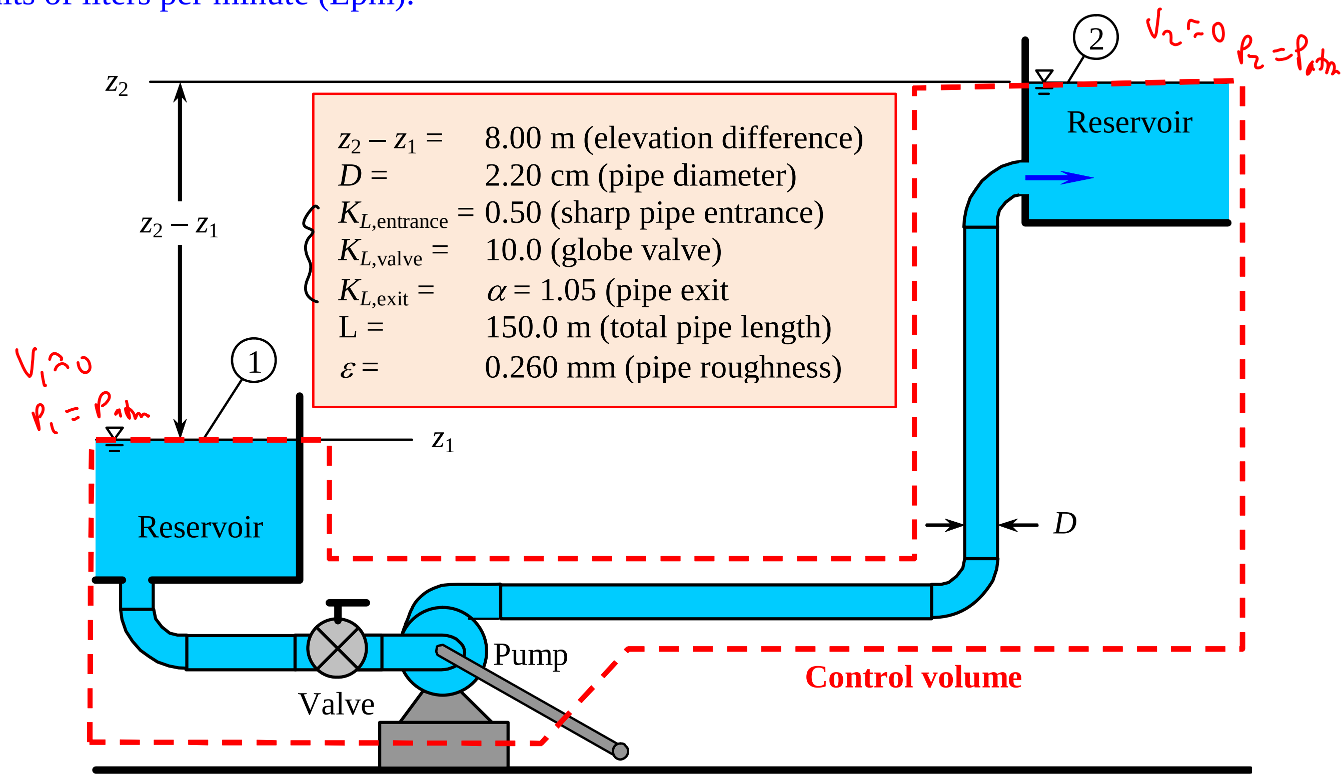
(max $\dot{V} = \dot{V}_{free\ delivery}$ occurs @ $H = 0$)

Example: Matching a Pump to a Piping System

Given: Water (ρ = 1000 kg/m^3, μ = 1.00 × 10^{-3} kg/m·s) is pumped from one large reservoir to another large reservoir that is at a higher elevation. The free surfaces of both reservoirs are exposed to atmospheric pressure, as sketched. The dimensions and minor loss coefficients are provided in the figure. The pipe is 2.20 cm I.D. cast iron pipe. The total pipe length is 150.0 m. The entrance and exit are sharp. There are three regular threaded 90-degree elbows, and one fully open threaded globe valve. The pump's performance (supply curve) is approximated by the expression

$$H_{\text{available}} = h_{\text{pump,u, supply}} = H_0 - a\dot{V}^2$$ ⟵ Manufacturer's specs

where shutoff head H_0 = 20.0 m of water column, coefficient a = 0.0720 m/Lpm^2 = 2.592×10^8 s^2/m^2, available pump head $H_{\text{available}}$ is in units of meters of water column, and volume flow rate $\dot{V}$ is in units of liters per minute (Lpm).



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To do: Calculate the volume flow rate through this piping system.

Solution:

- First we draw a control volume, as shown by the dashed line. We cut through the pump shaft and through the surface of both reservoirs (inlet 1 and outlet 2), where we know that the velocity is nearly zero and the pressure is atmospheric. The rest of the control volume simply surrounds the piping system.
- We apply the head form of the energy equation from the inlet (1) to the outlet (2):

$\boxed{P_1 = P_2 = P_{\text{atm}}}$ $\boxed{V_1 = V_2 \approx 0}$

$$\cancel{\frac{P_1}{\rho_1 g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + h_{\text{pump,u}} = \cancel{\frac{P_2}{\rho_2 g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + z_2 + \cancel{h_{\text{turbine,e}}} + h_L$$

$\boxed{\text{We call this } h_{\text{pump, u, system}} = H_{\text{required}} \text{ since it is the required pump head for the given piping system.}}$

The rest of this problem will be solved in class.

$$H_{required} = h_{pump,u\_system} = h_L + (z_2 - z_1)$$

$$= \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right) + (z_2 - z_1)$$

$$\dot{V} = V\frac{\pi D^2}{4} \longrightarrow V = \frac{4\dot{V}}{\pi D^2}$$

$$\boxed{H_{required} = \frac{8\dot{V}^2}{\pi^2 g D^4}\left(f\frac{L}{D} + \sum K_L\right) + (z_2 - z_1)}$$

System curve (required pump head)

Equate this to the <u>supply curve</u> (manufacturer's spec for pump) — available head

$$\boxed{\underbrace{H_0 - a\dot{V}^2}_{\text{Supply}} = \underbrace{\frac{8\dot{V}^2}{\pi^2 g D^4}\left(f\frac{L}{D} + \sum K_L\right) + (z_2 - z_1)}_{\text{System}}}$$

Solve several simultaneous equations since $f = f(Re, \varepsilon/D)$

$$Re = \frac{\rho V D}{\mu}$$

V = fnc of $\dot{V}$

$\sum K_L = 14.25$ (add up all the K_L's)

Solve this however we can

$\boxed{\text{I solved in } \underline{\underline{\text{EES}}}}$

See website for the file

★ $\boxed{\dot{V} = 1.796 \times 10^{-4} \ m^3/s = 10.8 \ \text{LPM}}$

EES Solution for Example Problem – Matching a Pump to a Piping System

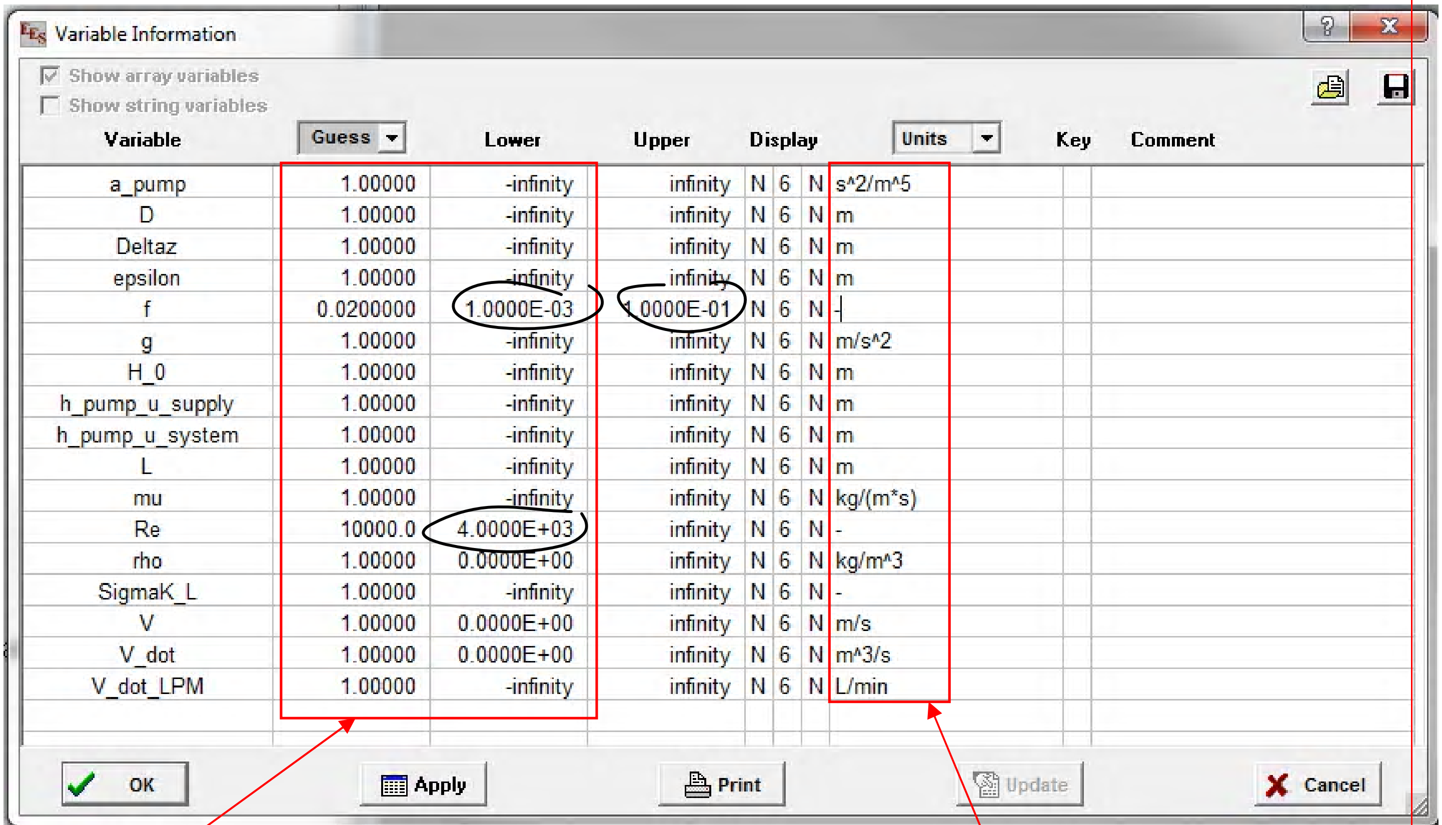
Here is exactly what I typed into the main “Equations Window” of EES:

Equations Window

```
"EES Solution for the class example problem - matching a pump to a piping system, J. M. Cimbala"

"Constants:"
Deltaz = 8.0 [m]
rho = 1000 [kg/m^3]
mu = 1.00e-3 [kg/(m*s)]
D = 0.022 [m]
L = 150 [m]
SigmaK_L = 0.5 + 10 + 3*0.9 + 1.05
epsilon = 0.00026 [m]
g = g# "(gravitational constant, predifined by EES)"

"Pump performance curve:"
H_0 = 20.0 [m]
a_pump = 2.592e8 [s^2/m^5]

"Equations:"
Re = (rho*D*V)/mu
V_dot = V*PI*(D^2)/4
h_pump_u_supply = H_0 - a_pump*V_dot^2
h_pump_u_system = 8*V_dot^2/(PI^2*g*D^4)*(f*L/D + SigmaK_L) + Deltaz
h_pump_u_supply = h_pump_u_system
V_dot_LPM = V_dot*Convert(m^3/s,L/min)
1/sqrt(f) = -2.0*log10((epsilon/D)/3.7 + 2.51/(Re*sqrt(f)))        "Colebrook equation"

"To solve, click on Calculate and then Solve. Note that it does not converge unless you change the limits
and guesses in Options-Variable Info"
```

X | Line: 1 Char: 4 | Wrap: On | Insert | Caps Lock: Off | SI K kPa kJ molar deg | Warnings: On | Unit Chk: On | Complex: Off

Here is what the "Options-Variable Info" chart looks like:



Variable	Guess	Lower	Upper	Display			Units	Key	Comment
a_pump	1.00000	-infinity	infinity	N	6	N	s^2/m^5		
D	1.00000	-infinity	infinity	N	6	N	m		
Deltaz	1.00000	-infinity	infinity	N	6	N	m		
epsilon	1.00000	-infinity	infinity	N	6	N	m		
f	0.0200000	1.0000E-03	1.0000E-01	N	6	N	-		
g	1.00000	-infinity	infinity	N	6	N	m/s^2		
H_0	1.00000	-infinity	infinity	N	6	N	m		
h_pump_u_supply	1.00000	-infinity	infinity	N	6	N	m		
h_pump_u_system	1.00000	-infinity	infinity	N	6	N	m		
L	1.00000	-infinity	infinity	N	6	N	m		
mu	1.00000	-infinity	infinity	N	6	N	kg/(m*s)		
Re	10000.0	4.0000E+03	infinity	N	6	N	-		
rho	1.00000	0.0000E+00	infinity	N	6	N	kg/m^3		
SigmaK_L	1.00000	-infinity	infinity	N	6	N	-		
V	1.00000	0.0000E+00	infinity	N	6	N	m/s		
V_dot	1.00000	0.0000E+00	infinity	N	6	N	m^3/s		
V_dot_LPM	1.00000	-infinity	infinity	N	6	N	L/min		

Note: I had to change some of the initial guesses and some of the lower and upper limits in order for EES to converge on a solution. This takes some trial and error.

I also had to manually insert some of the units. It seems that EES is not smart enough to calculate the units on its own.

Here is what the Windows-Formatted Equations window looks like (much “cleaner” looking equations – and easier to spot typo errors):

EES Solution for the class example problem - matching a pump to a piping system, J. M. Cimbala

Constants:

$\delta z = 8$ [m]

$\rho = 1000$ [kg/m^3]

$\mu = 0.001$ [kg/(m*s)]

$D = 0.022$ [m]

$L = 150$ [m]

$SigmaK_L = 0.5 + 10 + 3 \cdot 0.9 + 1.05$

$\varepsilon = 0.00026$ [m]

$g = 9.807$ [m/s^2] (gravitational constant, predifined by EES)

Pump performance curve:

$H_0 = 20$ [m]

$a_{pump} = 2.592 \times 10^8$ [s^2/m^5]

Equations:

$$Re = \frac{\rho \cdot D \cdot V}{\mu}$$

$$\dot{V} = V \cdot \pi \cdot \frac{D^2}{4}$$

$$h_{pump,u,supply} = H_0 - a_{pump} \cdot \dot{V}^2$$

$$h_{pump,u,system} = 8 \cdot \frac{\dot{V}^2}{\pi^2 \cdot g \cdot D^4} \cdot \left[f \cdot \frac{L}{D} + SigmaK_L \right] + \delta z$$

$$h_{pump,u,supply} = h_{pump,u,system}$$

$$\dot{V}_{LPM} = \dot{V} \cdot \left| 60000 \cdot \frac{\text{L/min}}{\text{m}^3\text{/s}} \right|$$

$$\frac{1}{\sqrt{f}} = -2 \cdot \mathbf{log} \left[\frac{\varepsilon}{D \cdot 3.7} + \frac{2.51}{Re \cdot \sqrt{f}} \right]$$ Colebrook equation

To solve, click on Calculate and then Solve. Note that it does not converge unless you change the limits and guesses in Options-Variable Info

Finally, Calculate and Solve (or click on the calculator icon) to yield the solution, as shown in the Solution window:

Solution

Main

Unit Settings: SI K kPa kJ molar deg

a_{pump} = 2.59200E8 [s²/m⁵]	D = 0.0220000 [m]
δz = 8.00000 [m]	ε = 0.000260000 [m]
f = 0.0447730 [-]	g = 9.80700 [m/s²]
H_0 = 20.0000 [m]	$\boxed{h_{pump,u,supply} = 11.6373 \text{ [m]}}$
$\boxed{h_{pump,u,system} = 11.6373 \text{ [m]}}$	L = 150.000 [m]
μ = 0.00100000 [kg/(m*s)]	Re = 10395.5 [-]
ρ = 1000.00 [kg/m³]	$SigmaK_L$ = 14.2500 [-]
V = 0.472521 [m/s]	$\dot{V}$ = 0.000179621 [m³/s]
$\boxed{\dot{V}_{LPM} = 10.7773 \text{ [L/min]}}$	

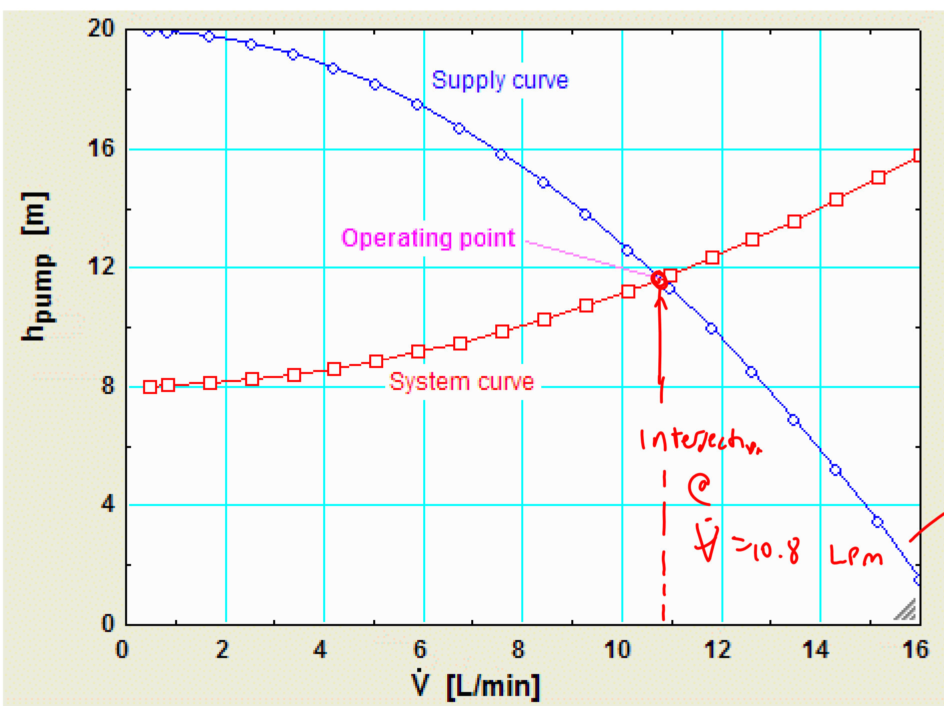
Notice that the system head matches the supply head at the converged value of volume flow rate. This serves as a check that the solution indeed converged correctly.

This is our final answer for volume flow rate in units of liters per minute.

No unit problems were detected.

Calculation time = .0 sec.

Note: It is also possible to plot with EES. Here is a plot of supply head and system head as functions of volume flow rate. As you can see, they intersect at the operating point, which is around 10.8 Lpm.



Same as our calculation!