Today, we will:

- Discuss dimensionless parameters in pump performance – **the affinity laws**, and do some example problems [*Note*: This material is from Chapter 14]
- Discuss turbines – the various types of turbines

c. Dimensionless parameters in pump performance [Section 14-3]

(1) Nondimensional groups

pump diameter (impeller blades)

Dimensional analysis is extremely useful in pump design and analysis!
Start with the functional relationship, and perform the method of repeating variables:

$$gH = f\left(\dot{V}, D, \varepsilon, \omega, \rho, \mu\right)$$

rotational speed (rad/s)

gH occurs together – so we treat gH as one variable

wall roughness

Try it yourself – great review of dimensional analysis and the method of repeating variables:

$$\boxed{\frac{gH}{\omega^2 D^2} = \text{function of}\left(\frac{\dot{V}}{\omega D^3}, \frac{\rho\omega D^2}{\mu}, \frac{\varepsilon}{D}\right)}$$

A similar analysis with input brake horsepower (*bhp*) as a function of the same variables results in

$$\boxed{\frac{bhp}{\rho\omega^3 D^5} = \text{function of}\left(\frac{\dot{V}}{\omega D^3}, \frac{\rho\omega D^2}{\mu}, \frac{\varepsilon}{D}\right)}$$

Let's name these Π's:

ωD = velocity

$$\boxed{C_H = \frac{gH}{\omega^2 D^2} = \text{head coefficient}}$$

$$\boxed{\mathrm{Re} = \frac{\rho\omega D^2}{\mu} = \text{Reynolds number}}$$

$$\boxed{\frac{\varepsilon}{D} = \text{roughness ratio}}$$

"capacity" means volume flow rate

$$\boxed{C_Q = \frac{\dot{V}}{\omega D^3} = \text{capacity coefficient}}$$

$$\boxed{C_P = \frac{bhp}{\rho\omega^3 D^5} = \text{power coefficient}}$$

many fluids books use $Q = \dot{V}$

So, we write

not real important as long as Re is "big enough"

$$\boxed{C_H = \text{function}\left(C_Q, \text{Re}, \varepsilon/D\right)} \qquad \text{and} \qquad \boxed{C_P = \text{function}\left(C_Q, \text{Re}, \varepsilon/D\right)}$$

But for many pumps, effects of Re and roughness are small at high Re, and thus,

$$\boxed{C_H \approx \text{function}\left(C_Q\right)} \qquad \text{and} \qquad \boxed{C_P \approx \text{function}\left(C_Q\right)}$$

Finally, pump efficiency is already dimensionless, and we write η_{pump} as

$$\boxed{\eta_{\text{pump}} = \frac{\rho\left(\dot{V}\right)(gH)}{bhp} = \frac{\cancel{\rho}\left(\cancel{\omega}\cancel{D^3}C_Q\right)\left(\cancel{\omega^2}\cancel{D^2}C_H\right)}{\cancel{\rho}\cancel{\omega^3}\cancel{D^5}C_P} = \frac{C_Q C_H}{C_P} = \text{function}\left(C_Q\right)}$$

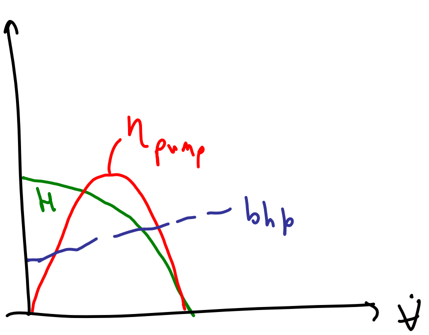
$$\boxed{\eta_{\text{pump}} = f(C_Q)}$$

(2) The Affinity Laws

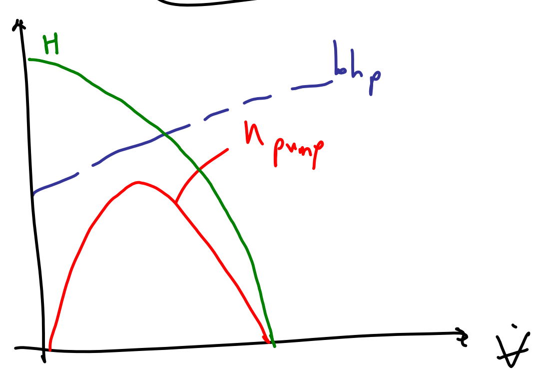
"Affinity" means "inherent likeness"

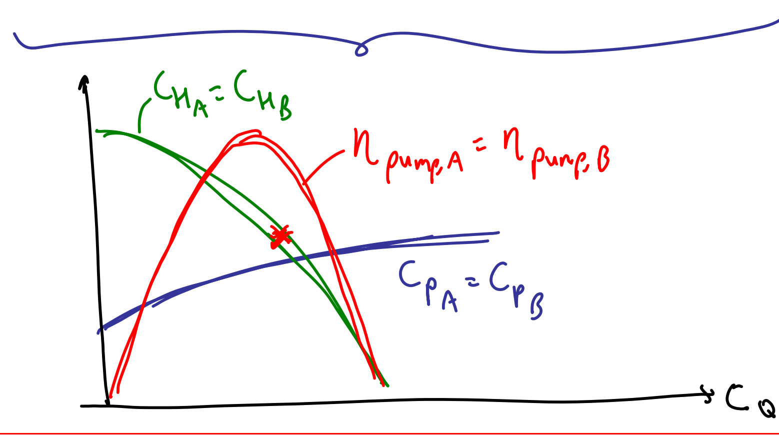
If 2 pumps are geometrically similar & dynamically similar then both pump performance curves fall on top of each other in nondimensional plots

(little) Pump A

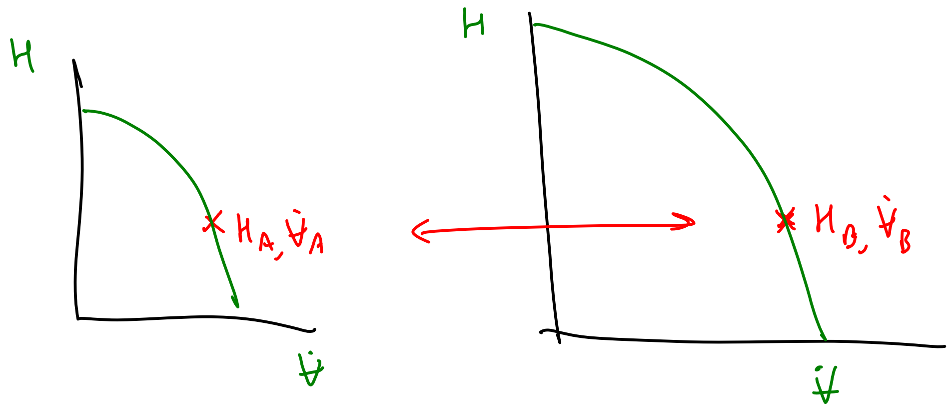


(big) Pump B





We can scale up pumps to design pumps using the Affinity Laws



This occurs at "homologous points"

At a homologous point,

$$\boxed{C_{Q_B} = C_{Q_A}, \quad C_{H_B} = C_{H_A}, \quad C_{P_B} = C_{P_A}, \quad \eta_{pump_B} = \eta_{pump_A}}$$

$$C_{Q_A} = \frac{\dot{V}_A}{\omega_A D_A^3} = C_{Q_B} = \frac{\dot{V}_B}{\omega_B D_B^3} \Rightarrow \boxed{\frac{\dot{V}_B}{\dot{V}_A} = \frac{\omega_B}{\omega_A}\left(\frac{D_B}{D_A}\right)^3}$$

Do this for the other π's also

"The Affinity Laws"

The **Pump Affinity Laws** (Eq. 14.38 in the textbook):

V

$$\frac{\dot{V}_B}{\dot{V}_A} = \frac{\omega_B}{\omega_A}\left(\frac{D_B}{D_A}\right)^3$$

$$\frac{H_B}{H_A} = \left(\frac{\omega_B}{\omega_A}\right)^2\left(\frac{D_B}{D_A}\right)^2$$

$$\frac{bhp_B}{bhp_A} = \frac{\rho_B}{\rho_A}\left(\frac{\omega_B}{\omega_A}\right)^3\left(\frac{D_B}{D_A}\right)^5$$

For one pump operating at various ω values

Jingle – "Very Hard Problems are as easy as 1, 2, 3"!

$$\frac{\dot{V}_B}{\dot{V}_A} = \left(\frac{\omega_B}{\omega_A}\right)^1 \qquad \frac{H_B}{H_A} = \left(\frac{\omega_B}{\omega_A}\right)^2 \qquad \frac{bhp_B}{bhp_A} = \left(\frac{\omega_B}{\omega_A}\right)^3$$

Example: Scaling up a pump using the affinity laws

Given: An existing pump (A): Fluid is water at 20°C, D_A = 6.50 cm, and $\dot{n}_A = 1500$ rpm. At BEP, $\dot{V}_A = 455\ \text{cm}^3/\text{s}$ at $H_A = 1.44$ m. We are designing a new larger pump (B) that is geometrically similar with D_B = 8.20 cm. It still uses water at 20°C, but rotates at a higher rpm, $\dot{n}_B = 1750$ rpm.

To do: (a) Predict H_B and $\dot{V}_B$ for operation of pump B at its BEP.

(b) Estimate the % increase in required brake horsepower from pump A to pump B.

Solution:

(a) At homologous points, the two turbines are dynamically similar. Apply the affinity laws:

$$\frac{\dot{V}_B}{\dot{V}_A} = \frac{\omega_B}{\omega_A}\left(\frac{D_B}{D_A}\right)^3 \qquad \frac{H_B}{H_A} = \left(\frac{\omega_B}{\omega_A}\right)^2\left(\frac{D_B}{D_A}\right)^2 \qquad \frac{bhp_B}{bhp_A} = \frac{\rho_B}{\rho_A}\left(\frac{\omega_B}{\omega_A}\right)^3\left(\frac{D_B}{D_A}\right)^5$$

$\dot{V}_B \rightarrow$ $\dot{V}_B = \dot{V}_A\left(\frac{\omega_B}{\omega_A}\right)\left(\frac{D_B}{D_A}\right)^3 = \dot{V}_A\left(\frac{\dot{n}_B}{\dot{n}_A}\right)\left(\frac{D_B}{D_A}\right)^3$

$\left[\dot{n}\left(\frac{2\pi\ \text{rad}}{\text{rot}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) \rightarrow \omega\right]$

$\dot{V}_B = \left(455\ \frac{\text{cm}^3}{\text{s}}\right)\left(\frac{1750}{1500}\right)\left(\frac{8.20\ \text{cm}}{6.50\ \text{cm}}\right)^3 = 1065.76$ $\boxed{\dot{V}_B = 1070\ \text{cm}^3/\text{s}}$

H_B in m $\rightarrow$ $H_B = H_A\left(\frac{\dot{n}_B}{\dot{n}_A}\right)^2\left(\frac{D_B}{D_A}\right)^2$

$= (1.44\ \text{m})\left(\frac{1750}{1500}\right)^2\left(\frac{8.20}{6.50}\right)^2 = 3.1193$ $\boxed{H_B = 3.12\ \text{m}}$

The two pumps are operating at homologous points, namely BEP

(b) $bhp_B = bhp_A\left(\cancel{\frac{\rho_B}{\rho_A}}\right)\left(\frac{\dot{n}_B}{\dot{n}_A}\right)^3\left(\frac{D_B}{D_A}\right)^5$

$\frac{bhp_B}{bhp_A} = (1)\left(\frac{1750}{1500}\right)^3\left(\frac{8.20}{6.50}\right)^5 = 5.0739$

More than 5 times as much power is required to run pump B!

$\boxed{\%\ \text{increase} = 407\%}$

3. Turbines

a. Introduction and Terminology:

- "***Turbine***" is a general term for any device that extracts mechanical energy from a fluid – generally converting it to rotating energy of a turbine wheel.
- For liquids, we usually call them "***hydraulic turbines***" or "***hydroturbines***".
- For gases, we usually call them "***wind turbines***", "***gas turbines***", or "***steam turbines***", depending on the type of gas being used.
- Just as with pumps, there are two basic types of turbine:
    - ***Positive displacement turbines*** – fluid is forced into a closed volume, and then the fluid is pushed out.
    - ***Dynamic turbines*** – no closed volume is involved; instead, rotating blades called ***runner blades*** or ***buckets*** extract energy from the fluid.
- In general, positive-displacement turbines are used for flow measurement, rather than for production of power, whereas dynamic turbines are used for both power generation *and* flow measurement.

Positive-Displacement Turbines:

- The nutating disc flowmeter, commonly used to measure the volume of water supplied to a house, is an example of a positive-displacement turbine.

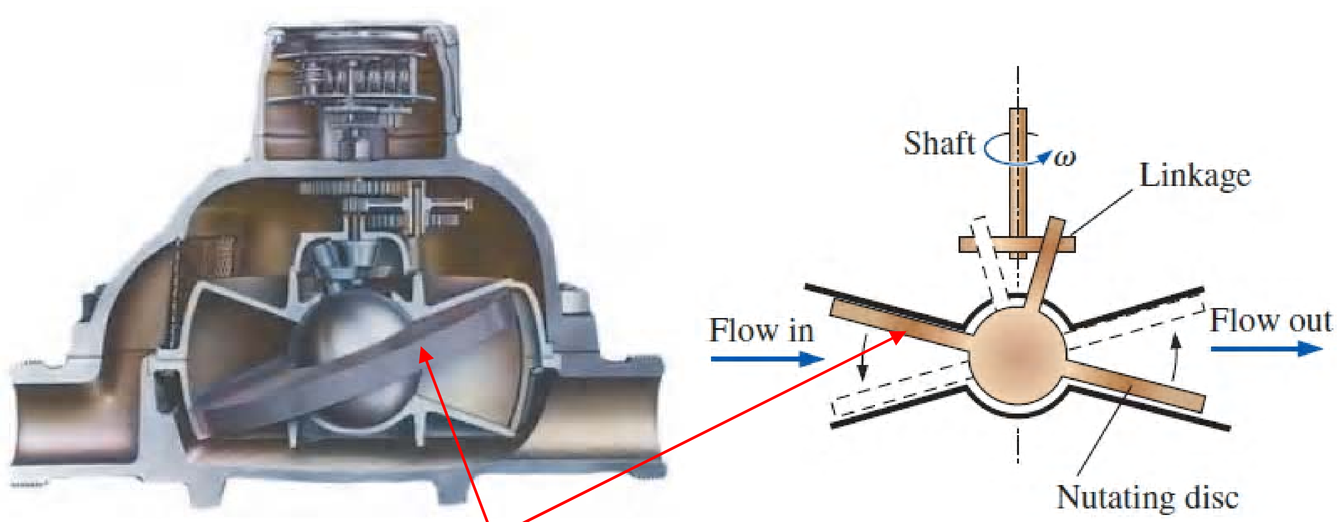


FIGURE 14–80
The *nutating disc fluid flowmeter* is a type of *positive-displacement turbine* used to measure volume flow rate: (*a*) cutaway view and (*b*) diagram showing motion of the nutating disc. This type of flowmeter is commonly used as a water meter in homes.
Photo courtesy of Niagara Meters, Spartanburg, SC.

- Other geometries are also used for positive-displacement turbines; e.g., a flowmeter that uses a double helical three-lobe impeller design, as discussed in Chapter 8:

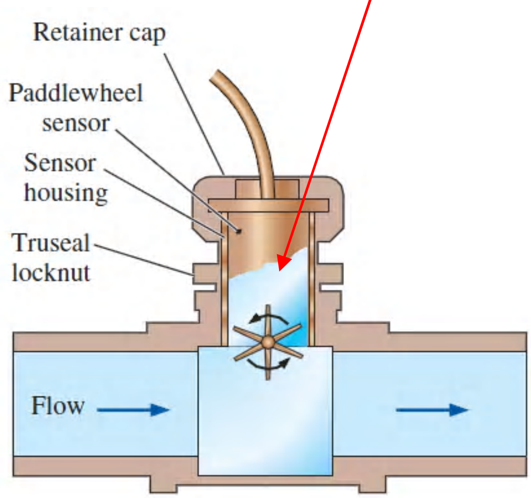
FIGURE 8–63
A positive displacement flowmeter with double helical three-lobe impeller design.
Courtesy Flow Technology, Inc. Source: www.ftimeters.com.

Dynamic Turbines:

- Dynamic turbines do not have closed volumes. Instead, spinning blades called ***runners*** or ***buckets*** transfer kinetic energy and extract momentum from the fluid.
- Dynamic turbines are used for both flow measurement and power production. For example, turbine flowmeters for air and water are discussed in Chapter 8.

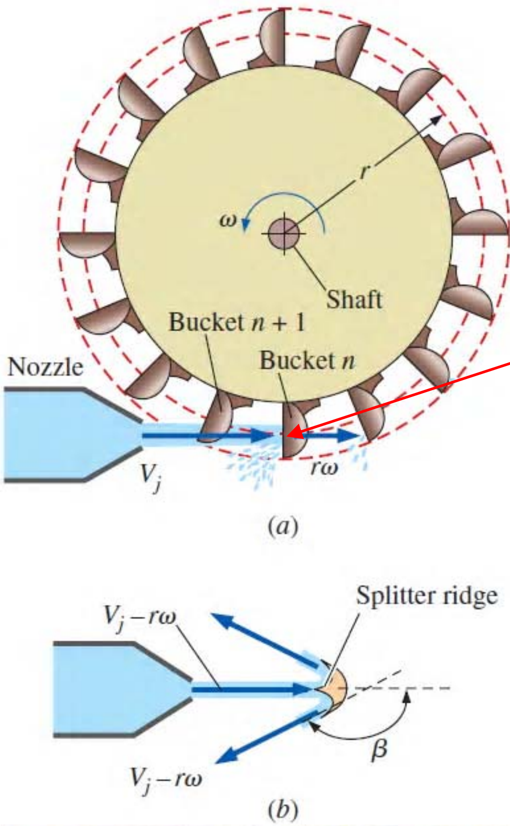
Turbine used for measurement of air speed

Turbine used for measurement of volume flow rate of water flowing in a pipe



There are two main types of dynamic turbines: ***impulse turbines*** and ***reaction turbines***.

- ***Impulse turbines***: Fluid is sent through a nozzle that then impinges on the rotating blades, called buckets. Compared to reaction turbines, impulse turbines require higher head, and work with a lower volume flow rate.
- The most common example is the ***Pelton wheel turbine***.

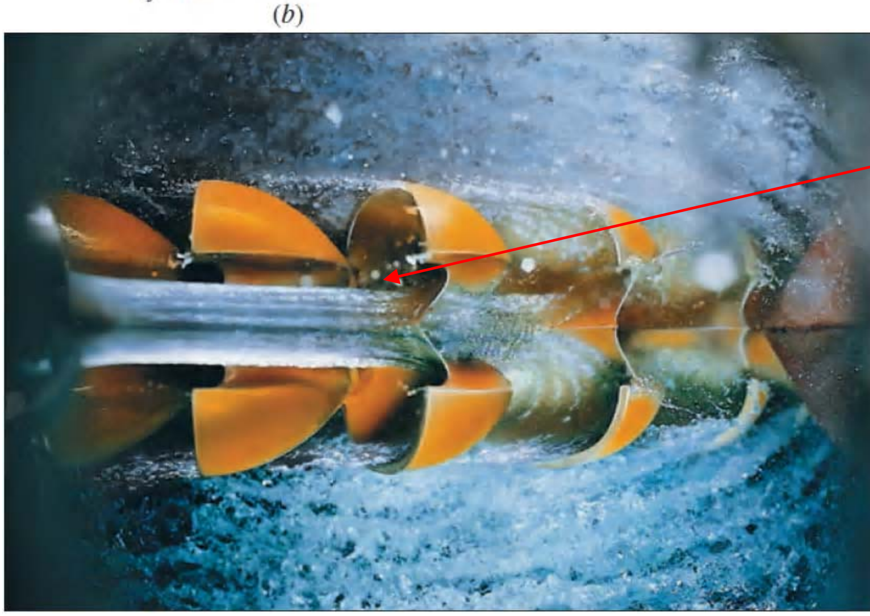


Water flows out of a nozzle at very high speed to rotate the buckets.

FIGURE 14–82

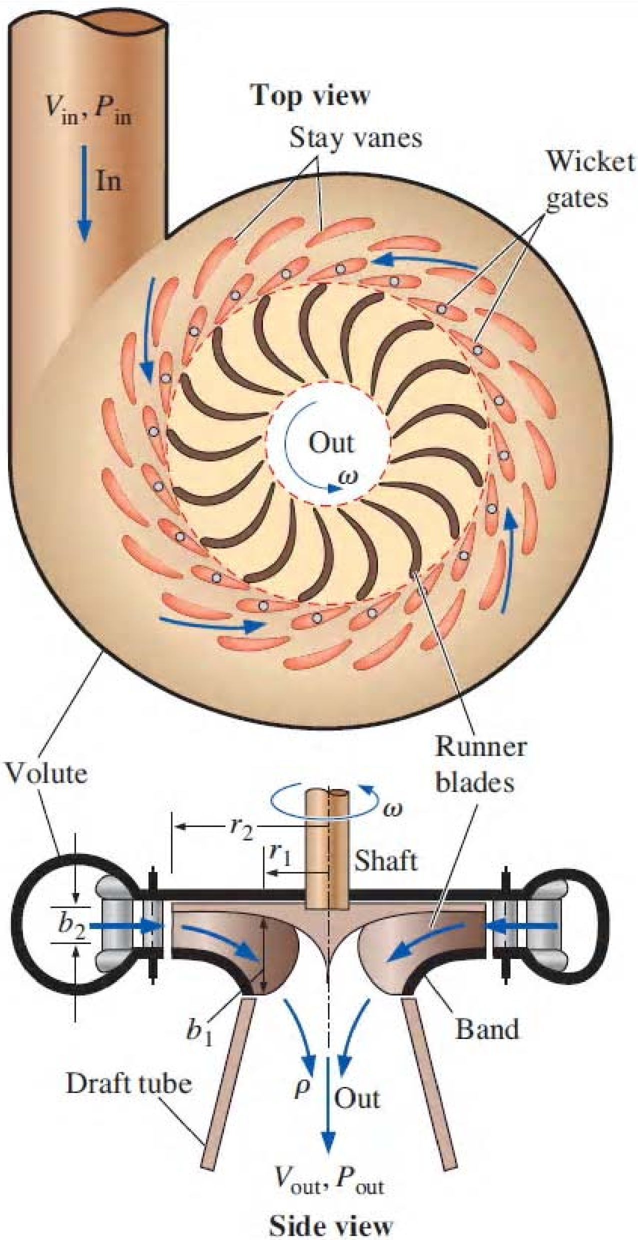
Schematic diagram of a Pelton-type *impulse turbine*; the turbine shaft is turned when high-speed fluid from one or more jets impinges on buckets mounted to the turbine shaft. (*a*) Side view, absolute reference frame, and (*b*) bottom view of a cross section of bucket n, rotating reference frame.

The source of the water is usually from either a natural or man-made reservoir at much higher elevation, so that it has high momentum to transfer to the buckets.



The buckets are shaped carefully to take advantage of the high momentum water jet, and are designed to turn around the water nearly 180° for maximum transfer of momentum from the water jet to the rotating turbine wheel.

- ***Reaction turbines***: Instead of using water jets, reaction turbines fill a *volute* with swirling water that rotates the runner blades. Compared to impulse turbines, reaction turbines require a lower head, and work with a higher volume flow rate. They are used primarily for electricity production (hydroelectric dams).



The ***stay vanes*** are fixed guide vanes that induce swirl to the water.

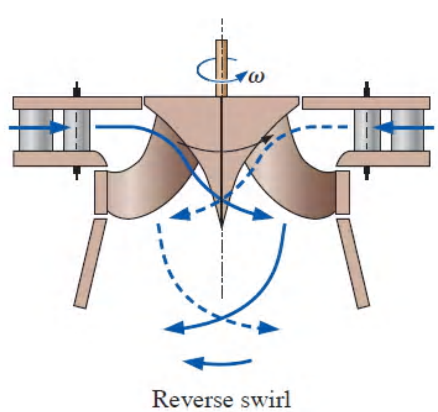
The ***wicket gates*** are adjustable vanes that control the volume flow rate through the turbine. They can usually be completely closed in order to shut off flow to the turbine.

FIGURE 14–87

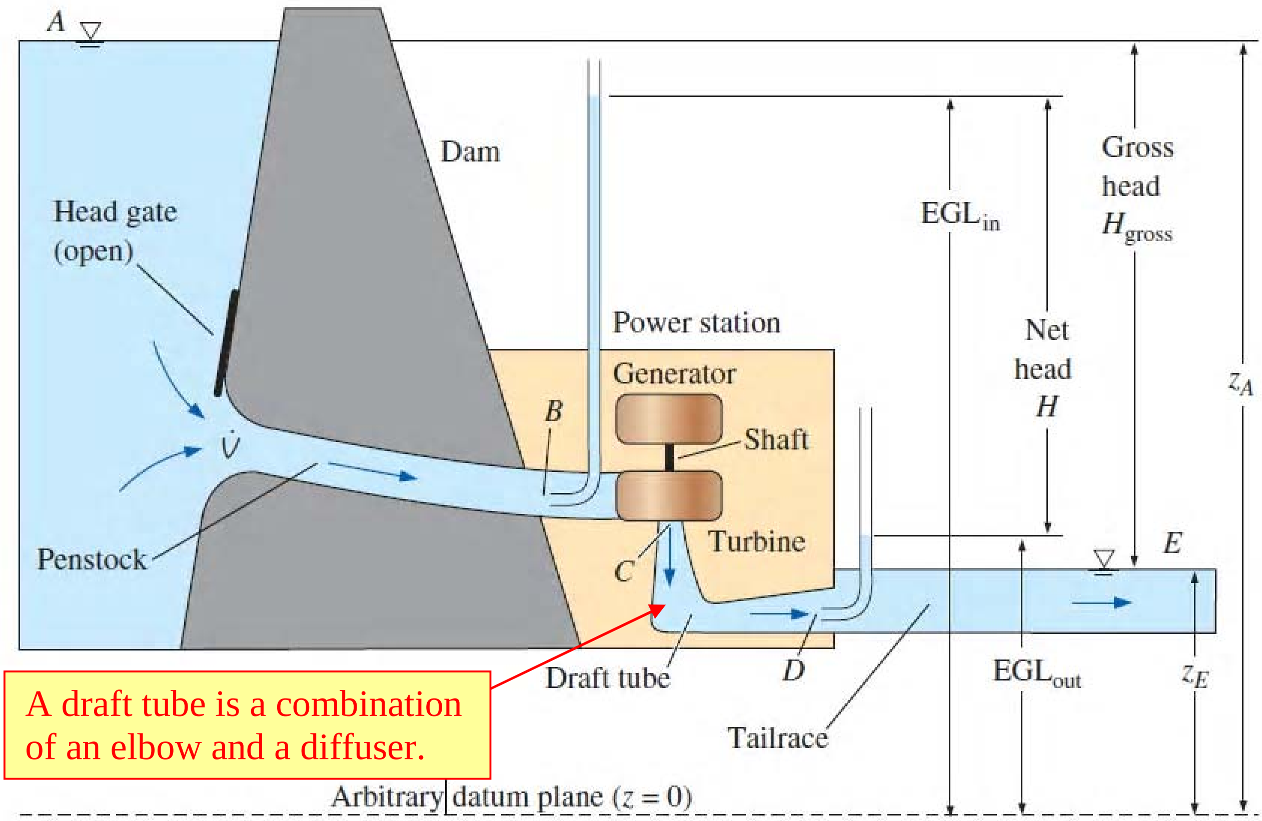
A *reaction turbine* differs significantly from an impulse turbine; instead of using water jets, a *volute* is filled with swirling water that drives the runner. For hydroturbine applications, the axis is typically vertical. Top and side views are shown, including the fixed *stay vanes* and adjustable *wicket gates*.

There are various types of hydroturbine designs, as discussed in the text: radial flow, mixed flow, propeller mixed flow, and propeller axial flow.

- In some modern Francis mixed-flow hydroturbines, the flow exiting the turbine swirls in a direction *opposite* to that of the runner itself. This is called ***reverse swirl***, and is designed to extract the maximum possible momentum from the water, similar to how a Pelton wheel turbine bucket turns the water nearly 180° around.



- Here is a typical setup for a hydroelectric plant that produces electricity with a hydroturbine and generator. Note that ***net head*** *H* across the turbine is measured from just upstream of the turbine to just downstream of the draft tube, while ***gross head*** H_{gross} is measured from the upstream reservoir surface to the downstream tailrace surface.



- ***Turbine efficiency***:

Turbine efficiency:
$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\text{water horsepower}}} = \frac{\text{bhp}}{\rho g H \dot{V}} \qquad (14\text{–}44)$$

- The efficiency of a turbine is the reciprocal of the efficiency of a pump!

Efficiency is always defined as

$$\eta = \text{efficiency} = \frac{\text{actual output}}{\text{required input}}$$

Thus, for a pump,

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{water horsepower}}}{\dot{W}_{\text{shaft}}} = \frac{\rho g H \dot{V}}{\text{bhp}}$$

and for a turbine,

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\text{water horsepower}}} = \frac{\text{bhp}}{\rho g H \dot{V}}$$

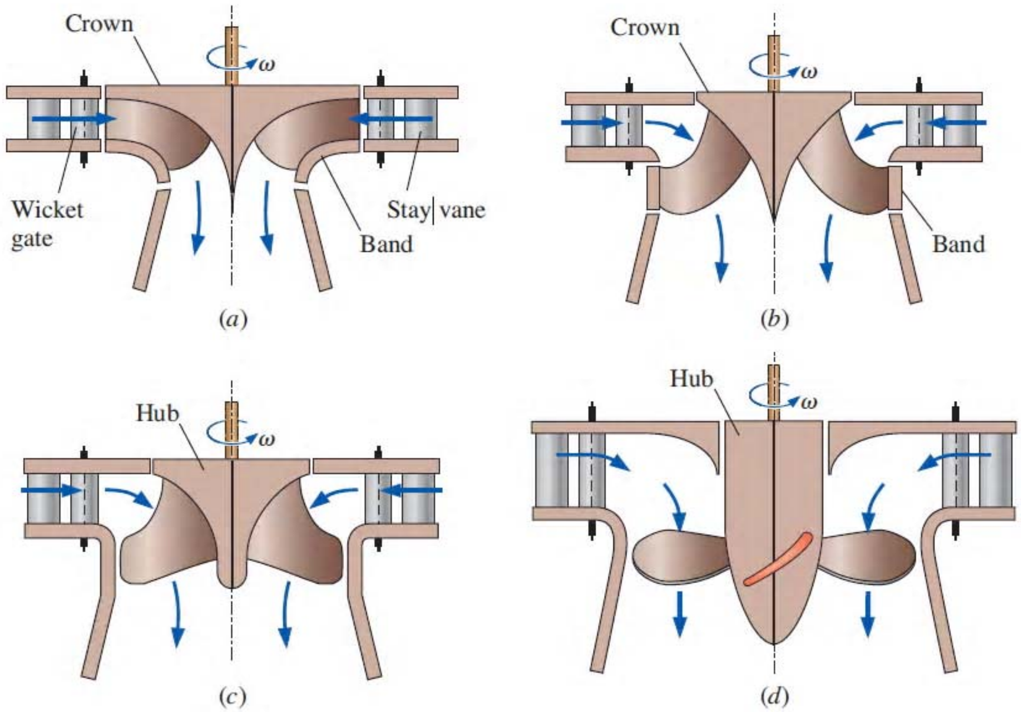
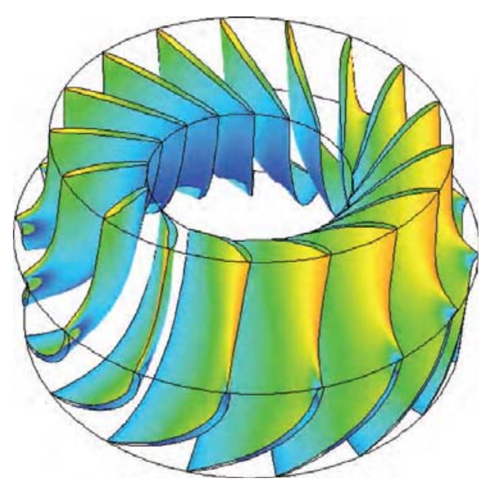


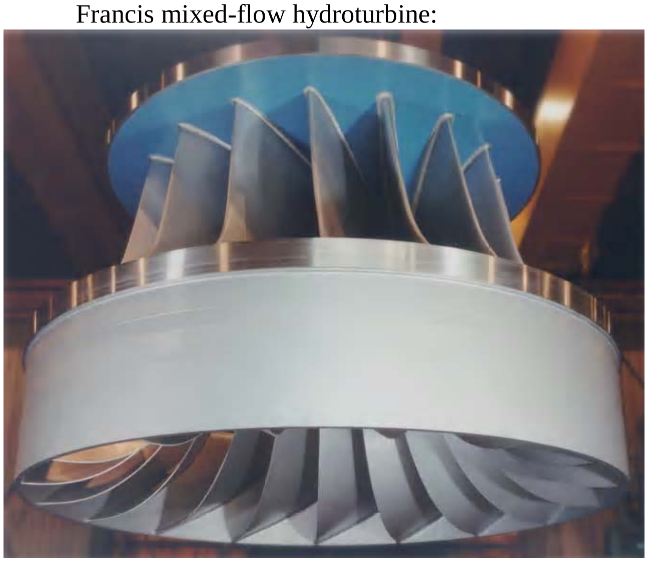
FIGURE 14–88
The distinguishing characteristics of the four subcategories of reaction turbines: (*a*) *Francis radial flow*, (*b*) *Francis mixed flow*, (*c*) *propeller mixed flow*, and (*d*) *propeller axial flow*. The main difference between (*b*) and (*c*) is that Francis mixed-flow runners have a *band* that rotates with the runner, while propeller mixed-flow runners do not. There are two types of propeller mixed-flow turbines: *Kaplan turbines* have adjustable pitch blades, while *propeller turbines* do not. Note that the terminology used here is not universal among turbomachinery textbooks nor among hydroturbine manufacturers.

Francis radial-flow hydroturbine:

CFD calculations, Francis mixed-flow



Francis mixed-flow hydroturbine:



Propeller hydroturbine (5-bladed):