Today, we will:

- Discuss dimensional analysis of turbines
- Do an example problem – dimensional analysis with turbines
- Discuss piping networks – how to deal with pipes in series or in parallel

b. Dimensionless parameters in turbine performance

We perform exactly the same dimensional analysis for turbines as we did for pumps. Result:

Dimensionless Parameters: $\boxed{C_Q = \dfrac{\dot{V}}{\omega D^3}}$ Capacity coefficient $\boxed{C_H = \dfrac{gH}{\omega^2 D^2}}$ Head coefficient $\boxed{C_P = \dfrac{bhp}{\rho \omega^3 D^5}}$ Power coefficient

$$\boxed{\eta_{turbine} = \frac{bhp}{\rho \dot{V} g h} = \frac{C_P}{C_Q C_H}} \qquad \left[\eta_{pump} = \frac{\rho \dot{V} g h}{bhp} = \frac{C_Q C_H}{C_P}\right]$$

Re, ε/D these are not important

For turbines C_P is used instead of C_Q as the independent parameter or Π

$$\boxed{\begin{aligned} C_Q &= C_Q(C_P) \\ C_H &= C_H(C_P) \\ \eta_{turbine} &= \eta_{turbine}(C_P) \end{aligned}}$$

[Power is more important in turbines than flow rate – flow rate is more important in pumps]

Example: Scaling up a hydroturbine

Given: An existing hydroturbine (A): Fluid is water at 20°C, D_A = 1.95 m, $\dot{n}_A = 120$ rpm, bhp_A = 220 MW, and $\dot{V}_A = 335\ \text{m}^3/\text{s}$ at $H_A = 72.4$ m. We are designing a new turbine (B) that is geometrically similar, still uses water at 20°C, and $\dot{n}_B = 120$ rpm, but $H_B = 97.4$ m. [Dam B has a higher gross head available than Dam A.]

To do: (a) Calculate D_B and $\dot{V}_B$ for operation of turbine B at a homologous point.
(b) Calculate bhp_B and estimate the turbine efficiency of both turbines.

Solution:

(a) At homologous (dynamically similar) points, the two turbines are dynamically similar. Apply the affinity laws:

$$C_{H,A} = \frac{gH_A}{\omega_A^2 D_A^2} = C_{H,B} = \frac{gH_B}{\omega_B^2 D_B^2} \rightarrow \text{ solve for } D_B = D_A\left(\frac{\omega_A}{\omega_B}\right)\sqrt{\frac{H_B}{H_A}} = D_A\left(\frac{\dot{n}_A}{\dot{n}_B}\right)\sqrt{\frac{H_B}{H_A}}$$

equate head coeffs

Plug in numbers:

$$D_B = (1.95\ \text{m})\left(\frac{120\ \text{rpm}}{120\ \text{rpm}}\right)\sqrt{\frac{97.4\ \text{m}}{72.4\ \text{m}}} = 2.2618\ \text{m} \rightarrow \boxed{D_B = 2.26\ \text{m}}$$

equate capacity coeffs

Similarly, $C_{Q,A} = \dfrac{\dot{V}_A}{\omega_A D_A^3} = C_{Q,B} = \dfrac{\dot{V}_B}{\omega_B D_B^3} \rightarrow \dot{V}_B = \dot{V}_A\left(\dfrac{\omega_B}{\omega_A}\right)\left(\dfrac{D_B}{D_A}\right)^3 = \dot{V}_A\left(\dfrac{\dot{n}_B}{\dot{n}_A}\right)\left(\dfrac{D_B}{D_A}\right)^3$

Plug in numbers:

$$\dot{V}_B = \left(335\ \text{m}^3/\text{s}\right)\left(\frac{120}{120}\right)\left(\frac{2.2618\ \text{m}}{1.95\ \text{m}}\right)^3 = 522.73\ \text{m}^3/\text{s} \rightarrow \boxed{\dot{V}_B = 523\ \text{m}^3/\text{s}}$$

(b) Similarly, $C_{P,A} = \dfrac{bhp_A}{\rho_A \omega_A^{\;3} D_A^{\;5}} = C_{P,B} = \dfrac{bhp_B}{\rho_B \omega_B^{\;3} D_B^{\;5}} \quad \rightarrow \quad bhp_B = bhp_A \left(\dfrac{\rho_B}{\rho_A}\right)\left(\dfrac{\dot{n}_B}{\dot{n}_A}\right)^3 \left(\dfrac{D_B}{D_A}\right)^5$

Plug in numbers: equate power coeffs

$$bhp_B = (1)(1)^3 \left(\frac{2.2618\text{ m}}{1.95\text{ m}}\right)^5 = 461.82\text{ MW} \approx \boxed{462.\text{ MW}}$$

Finally, the efficiency is calculated for each turbine:

$$\eta_{\text{turbine},A} = \frac{bhp_A}{\rho_A g H_A \dot{V}_A} = \frac{220{,}000{,}000\text{ W}}{\left(1000\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)72.4\text{ m}\left(335\dfrac{\text{m}^3}{\text{s}}\right)}\left(\frac{\text{N}\cdot\text{m}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}\right) = 92.5\%$$

$$\eta_{\text{turbine},B} = \frac{bhp_B}{\rho_B g H_B \dot{V}_B} = \frac{461{,}820{,}979\text{ W}}{\left(1000\dfrac{\text{kg}}{\text{m}^3}\right)\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)97.4\text{ m}\left(522.728\dfrac{\text{m}^3}{\text{s}}\right)}\left(\frac{\text{N}\cdot\text{m}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}\right) = 92.5\%$$

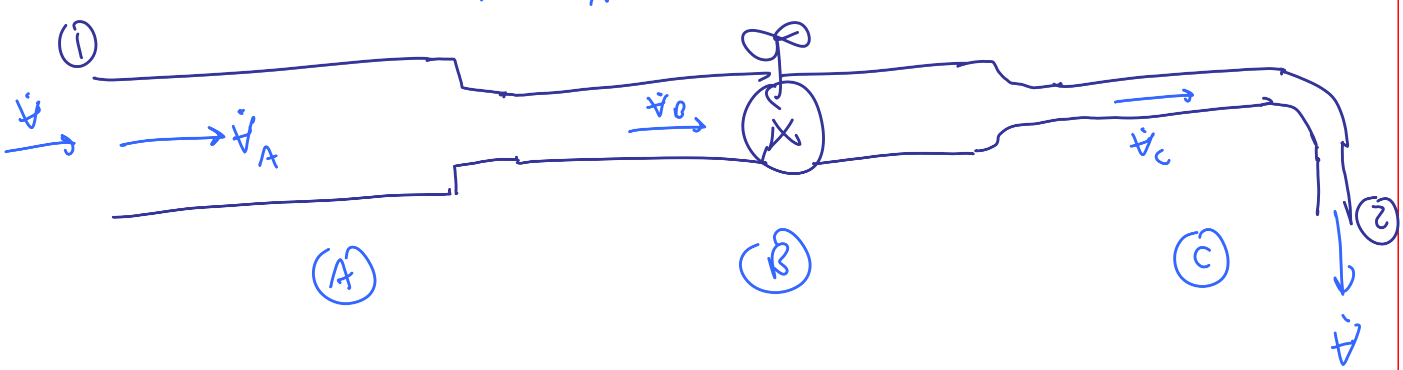
η's are = as they must be.

F. Piping Networks

1. Pipes in Series

— We already know how to do this

$$\dot{V} = \dot{V}_A = \dot{V}_B = \dot{V}_C$$



$V_A \neq V_B \neq V_C$ since $D_A \neq D_B \neq D_C$

$\therefore Re_A \neq Re_B \neq Re_C$, $\left(\dfrac{\varepsilon}{D}\right)_A \neq \left(\dfrac{\varepsilon}{D}\right)_B \neq \left(\dfrac{\varepsilon}{D}\right)_C$

$\rightarrow f_A \neq f_B \overset{?}{\neq} f_C$

But we can apply 1 energy eq from ① to ②:

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + \dots \quad + \underline{\underline{h_L}}$$

$$h_L = \sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + \sum_j K_{L,j} \frac{V_j^2}{2g}$$

Sum over all sections of piping

Sum over all minor losses

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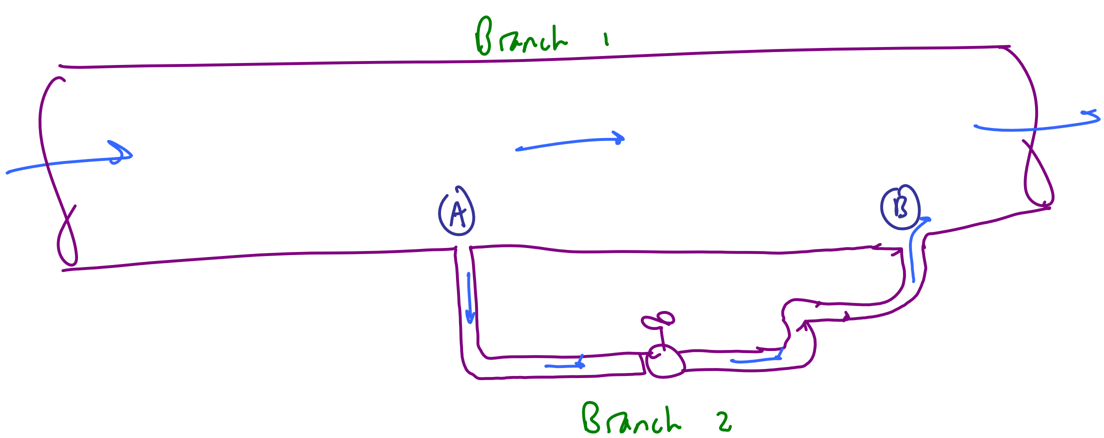
2. <u>Pipes in Parallel</u> – more difficult since $\dot{V}$ not the same in each branch

eg.



Which branch has the larger head loss from A to B?

(A) Branch 1

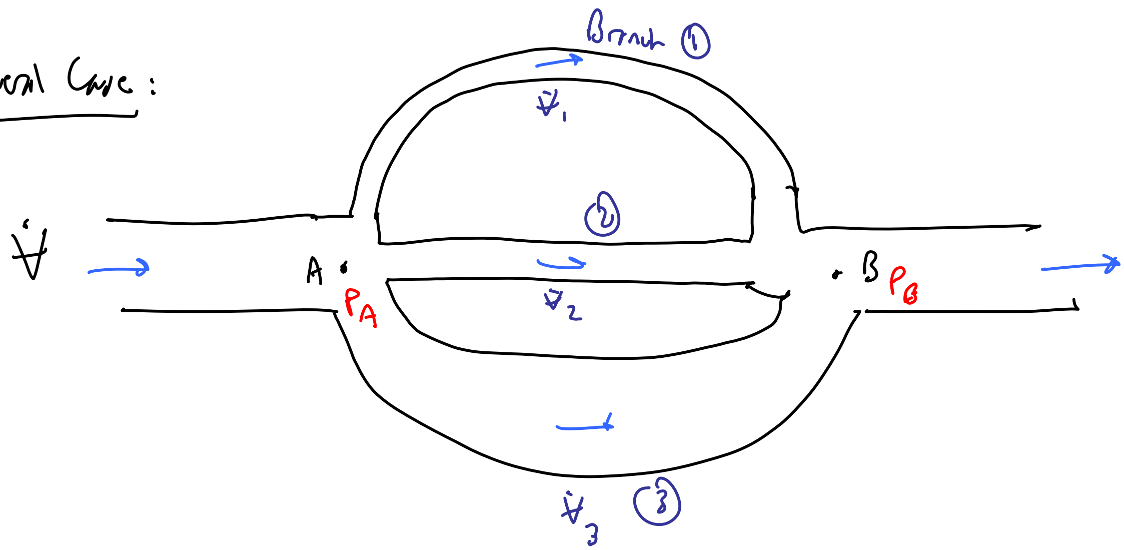
(B) Branch 2

(C) Neither (head loss is the same through either branch)

$$\boxed{h_L = \frac{P_A - P_B}{\rho g}}$$

$\dot{\forall}$ will adjust itself such that ΔP is the same through both branches, $\longrightarrow$ $\dot{\forall}_{Branch\ 2} \ll \dot{\forall}_{Branch\ 1}$

General Case:



$$\dot{\forall} = \dot{\forall}_1 + \dot{\forall}_2 + \dot{\forall}_3 + \dots$$

But since $P_A - P_B$ is the same, same head loss through any section

$\dot{\forall}$'s will automatically adjust to the "correct" $\dot{\forall}$ such that these eqs are met

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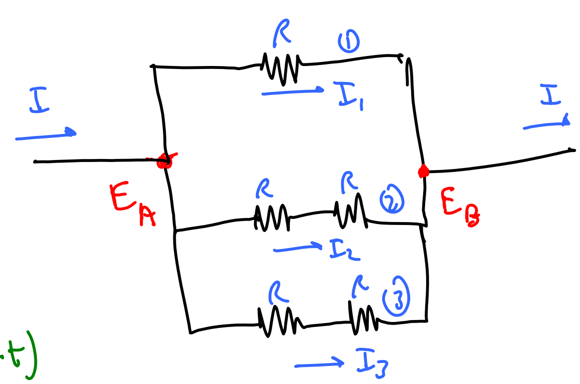
ELECTRICAL ANALOGY:



major & minor losses

$\dot{\forall}$ $\Longleftrightarrow$ I (current)

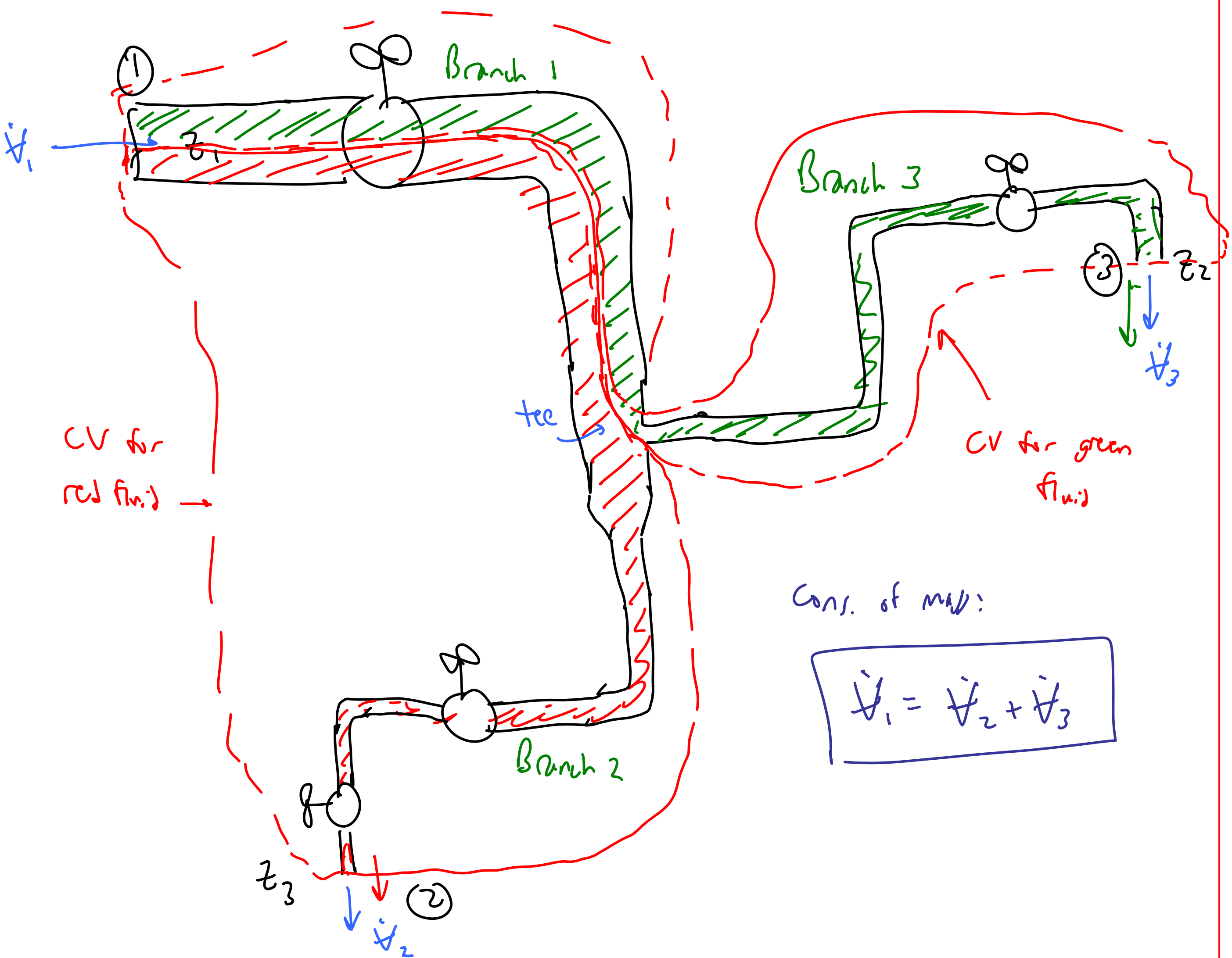
is analogous to

P $\Longleftrightarrow$ E (voltage)

Components of h_L $\Longleftrightarrow$ R (resistance)

$E_A - E_B$ = the same regardless of the branch & the currents will adjust appropriately

3. Complex Piping networks   Qualitative eg.



Cons. of mass:

$$\boxed{\dot{V}_1 = \dot{V}_2 + \dot{V}_3}$$

Cons of energy in head form → Apply to the water flowing through Branch 1 & 2 separately from 1 & 3

From ① to ②:

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + \cancel{h_{pump,u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \cancel{h_{turbine,e}} + h_{L_{1 \to 2}}$$

From ① to ③:

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + \cancel{h_{pump,u}} = \frac{P_3}{\rho g} + \alpha_3 \frac{V_3^2}{2g} + z_3 + \cancel{h_{turbine,e}} + h_{L_{1 \to 3}}$$

Notes: • $h_{L_{1\ to\ 2}}$ includes head losses through Branches 1 & 2

• $h_{L_{1\ to\ 3}}$ " " " " " 1 & 3

• Must use appropriate V's (V_1, V_2, or V_3) to calculate Re through each branch

• Each branch has a different V, Re, ε/D, f (Moody chart), ΣK_L etc.

• Think of the red fluid as two pipes in series, Branches ① & ②

• Think of the green fluid as " " " " " ① & ③

To solve - must solve a set of many simultaneous eq's

$$Re_1 = \frac{\rho V_1 D_1}{\mu} \qquad Re_2 = \frac{\rho V_2 D_2}{\mu} \qquad Re_3 = \frac{\rho V_3 D_3}{\mu}$$

$$f_1 = \text{moody chart}\left(Re_1, \varepsilon_1/D_1\right) \qquad f_2 = \text{Moody chart}\left(Re_2, \varepsilon_2/D_2\right)$$

$$f_3 = \text{moody chart}\left(Re_3, \varepsilon_3/D_3\right)$$

There will be ΣK_{L_1}, ΣK_{L_2}, ΣK_{L_3} & h_{L_1}, h_{L_2}, h_{L_3}

$$\dot{V}_1 = \frac{\pi D_1^2}{4} V_1 \qquad \dot{V}_2 = \frac{\pi D_2^2}{4} V_2 \qquad \dot{V}_3 = \frac{\pi D_3^2}{4} V_3$$

& $\boxed{\dot{V}_1 = \dot{V}_2 + \dot{V}_3}$

☆ MUST SOLVE ALL OF THESE EQUATIONS SIMULTANEOUSLY (I use EES)