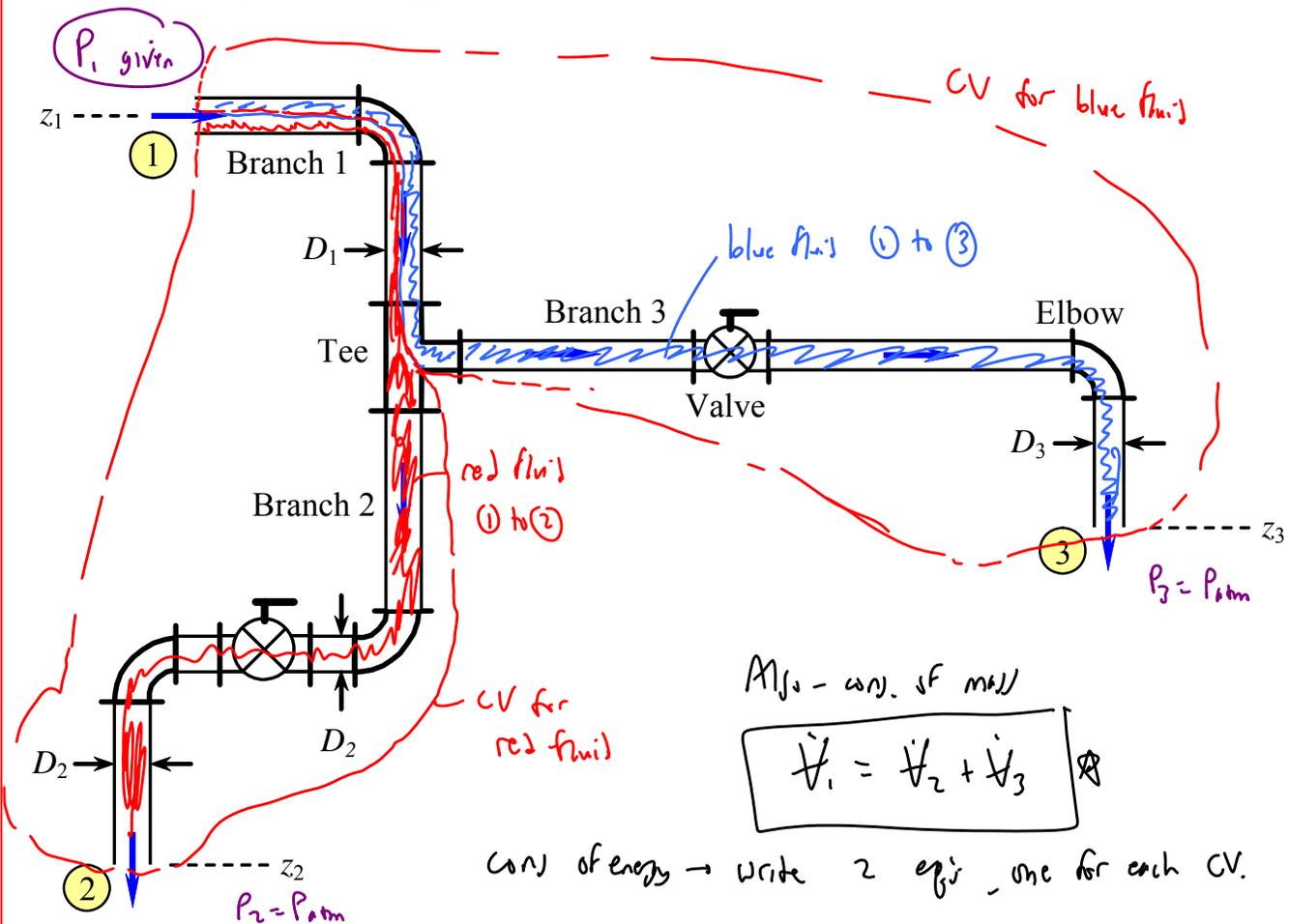


Today, we will:

- Do some examples of complex piping networks (multiple pipes with branches, etc.)
- Briefly mention flow meters and velocity measurement

Complex piping networks – Summary:

- For *each* section of pipe, need to write a separate equation for Re, f, h_L, etc.
- For pipe sections in series, $\dot{V}_1 = \dot{V}_2 = \dot{V}_3$.
- For splitting pipe sections (1 splitting into 2 and 3), $\dot{V}_1 = \dot{V}_2 + \dot{V}_3$
- Write a *separate* energy equation from inlet to outlet for each branch of the network.



When all equations are written, solve simultaneously for all unknowns. If you do everything right, there should be the same number of equations as unknowns.

Given: $P_1, z_1, z_2, z_3, P_2 = P_3 = P_{atm}$, all K_L 's, ϵ 's, L 's of each section, D 's

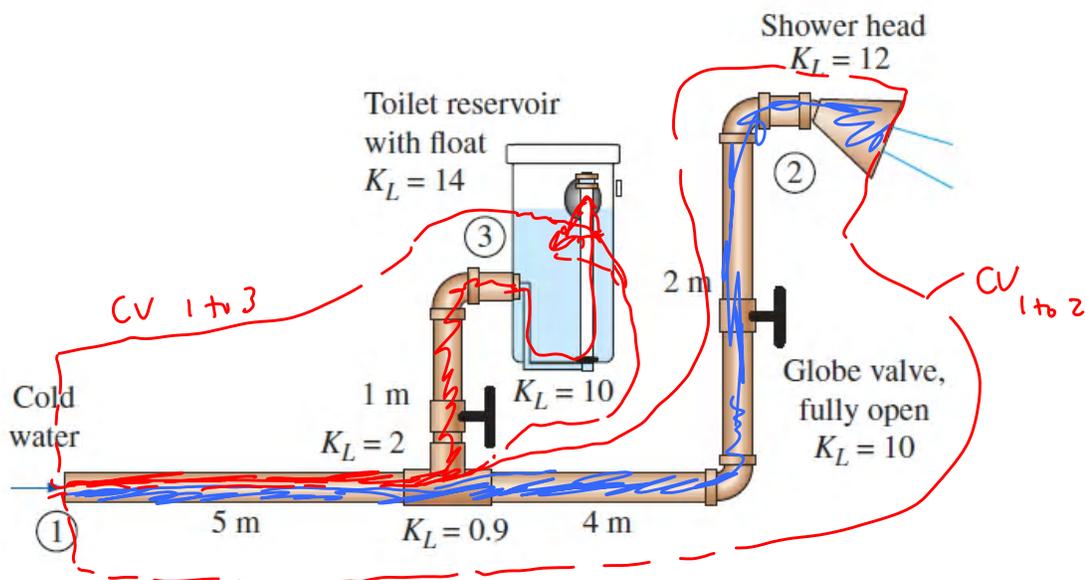
15 Unknowns $\rightarrow V_1, V_2, V_3, \dot{V}_1, \dot{V}_2, \dot{V}_3, f_1, f_2, f_3, Re_1, Re_2, Re_3, h_{L1}, h_{L2}, h_{L3}$

Need 15 eqs \rightarrow 3 Colebrook eqs (or Moody chart), 1 mass eq., 2 energy eqs, 3 eqs for Re, 3 eqs for \dot{V} (sum major + minor losses)

4. Examples

Example: Taking a Shower and Flushing a Toilet (E.g. 8-9, Çengel and Cimbala)

Given: This is a very practical everyday example of a parallel piping network! You are taking a shower. The piping is 1.50-cm copper pipes with threaded connectors as sketched. The gage pressure at the inlet of the system is 200 kPa, and the shower is on. The hot water is from a separate supply – only the cold water system is shown here.



To do:

- Calculate the volume flow rate \dot{V}_2 through the shower head when there is no water flowing through the toilet.
- Calculate the volume flow rate \dot{V}_2 through the shower head when someone flushes the toilet, and water flows into the toilet reservoir.

Solution: (copied from the textbook)

SOLUTION The cold-water plumbing system of a bathroom is given. The flow rate through the shower and the effect of flushing the toilet on the flow rate are to be determined.

Assumptions 1 The flow is steady and incompressible. 2 The flow is turbulent and fully developed. 3 The reservoir is open to the atmosphere. 4 The velocity heads are negligible.

This is a simplifying assumption that may or may not be valid. We should check the validity later.

We consider only the cold water line. The hot water line is separate, and is not connected to the toilet, so the volume flow rate of hot water through the shower remains constant. The cold water, however, is affected by flushing the toilet.

Properties The properties of water at 20°C are $\rho = 998 \text{ kg/m}^3$, $\mu = 1.002 \times 10^{-3} \text{ kg/m}\cdot\text{s}$, and $\nu = \mu/\rho = 1.004 \times 10^{-6} \text{ m}^2/\text{s}$. The roughness of copper pipes is $\epsilon = 1.5 \times 10^{-6} \text{ m}$.

Analysis This is a problem of the second type since it involves the determination of the flow rate for a specified pipe diameter and pressure drop. The solution involves an iterative approach since the flow rate (and thus the flow velocity) is not known.

Part (a) is not a parallel system since no flow through the toilet.

(a) The piping system of the shower alone involves 11 m of piping, a tee with line flow ($K_L = 0.9$), two standard elbows ($K_L = 0.9$ each), a fully open globe valve ($K_L = 10$), and a shower head ($K_L = 12$). Therefore, $\sum K_L = 0.9 + 2 \times 0.9 + 10 + 12 = 24.7$. Noting that the shower head is open to the atmosphere, and the velocity heads are negligible, the energy equation for a control volume between points 1 and 2 simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L$$

We neglect the velocity heads in the energy equation. Alternatively, if $V_1 = V_2$, then these two terms cancel each other out.

$$\frac{P_{1,\text{gage}}}{\rho g} = (z_2 - z_1) + h_L$$

$P_2 = P_{\text{atm}}$, and therefore $P_1 - P_2 = P_{1,\text{gage}}$.

$$h_L = \frac{200,000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 2 \text{ m} = 18.4 \text{ m}$$

Also,

$$h_L = \left(f \frac{L}{D} + \sum K_L \right) \frac{V^2}{2g} \quad \rightarrow \quad 18.4 = \left(f \frac{11 \text{ m}}{0.015 \text{ m}} + 24.7 \right) \frac{V^2}{2(9.81 \text{ m/s}^2)}$$

since the diameter of the piping system is constant. Equations for the average velocity in the pipe, the Reynolds number, and the friction factor are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad V = \frac{\dot{V}}{\pi(0.015 \text{ m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad \rightarrow \quad \text{Re} = \frac{V(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right)$$

Only one Re and one Colebrook equation for Part (a)

$$\rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re} \sqrt{f}} \right)$$

This is a set of four equations with four unknowns, and solving them with an equation solver such as EES gives

Answer to part (a) – no toilet flushing

$$\dot{V} = 0.00053 \text{ m}^3/\text{s}, \quad f = 0.0218, \quad V = 2.98 \text{ m/s}, \quad \text{and} \quad \text{Re} = 44,550$$

Therefore, the flow rate of water through the shower head is **0.53 L/s.**

(b) When the toilet is flushed, the float moves and opens the valve. The discharged water starts to refill the reservoir, resulting in parallel flow after the tee connection. The head loss and minor loss coefficients for the shower branch were determined in (a) to be $h_{L,2} = 18.4 \text{ m}$ and $\sum K_{L,2} = 24.7$, respectively. The corresponding quantities for the reservoir branch can be determined similarly to be

$$h_{L,3} = \frac{200,000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 1 \text{ m} = 19.4 \text{ m}$$

$$\sum K_{L,3} = 2 + 10 + 0.9 + 14 = 26.9$$

The relevant equations in this case are

$$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$$

$$h_{L,2} = f_1 \frac{5 \text{ m}}{0.015 \text{ m}} \frac{V_1^2}{2(9.81 \text{ m/s}^2)} + \left(f_2 \frac{6 \text{ m}}{0.015 \text{ m}} + 24.7 \right) \frac{V_2^2}{2(9.81 \text{ m/s}^2)} = 18.4$$

$$h_{L,3} = f_1 \frac{5 \text{ m}}{0.015 \text{ m}} \frac{V_1^2}{2(9.81 \text{ m/s}^2)} + \left(f_3 \frac{1 \text{ m}}{0.015 \text{ m}} + 26.9 \right) \frac{V_3^2}{2(9.81 \text{ m/s}^2)} = 19.4$$

$$V_1 = \frac{\dot{V}_1}{\pi(0.015 \text{ m})^2/4}, \quad V_2 = \frac{\dot{V}_2}{\pi(0.015 \text{ m})^2/4}, \quad V_3 = \frac{\dot{V}_3}{\pi(0.015 \text{ m})^2/4}$$

$$\text{Re}_1 = \frac{V_1(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}, \quad \text{Re}_2 = \frac{V_2(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}, \quad \text{Re}_3 = \frac{V_3(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2/\text{s}}$$

This is where EES comes in handy – solving all these simultaneous equations! (12 equations and 12 unknowns)

$$\frac{1}{\sqrt{f_1}} = -2.0 \log \left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}_1 \sqrt{f_1}} \right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log \left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}_2 \sqrt{f_2}} \right)$$

$$\frac{1}{\sqrt{f_3}} = -2.0 \log \left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}_3 \sqrt{f_3}} \right)$$

Now we need **three** Colebrook equations – one for each branch!

Solving these 12 equations in 12 unknowns simultaneously using an equation solver, the flow rates are determined to be

Answer to part (b) – with toilet flushing

$$\dot{V}_1 = 0.00090 \text{ m}^3/\text{s}, \quad \dot{V}_2 = 0.00042 \text{ m}^3/\text{s}, \quad \text{and} \quad \dot{V}_3 = 0.00048 \text{ m}^3/\text{s}$$

Therefore, the flushing of the toilet **reduces the flow rate of cold water through the shower by 21 percent** from 0.53 to 0.42 L/s, causing the shower water to suddenly get very hot (Fig. 8–53).

Discussion If the velocity heads were considered, the flow rate through the shower would be 0.43 instead of 0.42 L/s. Therefore, the assumption of negligible velocity heads is reasonable in this case. Note that a leak in a piping system would cause the same effect, and thus an unexplained drop in flow rate at an end point may signal a leak in the system.



FIGURE 8–53

Flow rate of cold water through a shower may be affected significantly by the flushing of a nearby toilet.

EES Solution – Toilet Flushing Example Problem

Part (a) EES Equation window:

"Example 8.9 - Toilet Flushing Problem, Part (a)"

"Constants and properties:"

$g = g\#$

$\rho = 998 \text{ [kg/m}^3\text{]}$

$\mu = 1.002\text{E-3 [kg/m-s]}$

$D = 0.015 \text{ [m]}$

$\epsilon = 1.5\text{e-6 [m]}$

$P_{\text{gage}_1} = 200000 \text{ [N/m}^2\text{]}$

$L_1 = 5 \text{ [m]}$

$L_2 = 6 \text{ [m]}$

$\text{SIGMAK}_{L_1} = 0$

$\text{SIGMAK}_{L_2} = 0.9 + 2*0.9 + 10 + 12$ "tee, two elbows, valve, and shower head"

$\Delta z_{1\text{to}2} = 2 \text{ [m]}$

"Equations to solve"

$P_{\text{gage}_1}/(\rho*g) = \Delta z_{1\text{to}2} + h_{L_1\text{to}2}$

$h_{L_1\text{to}2} = V^2/(2*g) * (f_{1\text{to}2}*(L_1+L_2)/D + \text{SIGMAK}_{L_1} + \text{SIGMAK}_{L_2})$

$A = \pi*D^2/4$

$\dot{V} = V*A$

$Re = V*D*\rho/\mu$

$f_{1\text{to}2} = \text{MoodyChart}(Re,\epsilon/D)$

$\dot{V}_{\text{LPS}} = \dot{V}*\text{CONVERT}(\text{m}^3/\text{s},\text{L/s})$

Part (a) EES Solution:

Unit Settings: SI K kPa kJ molar deg

$A = 0.0001767 \text{ [m}^2\text{]}$

$D = 0.015 \text{ [m]}$

$\Delta z_{1\text{to}2} = 2 \text{ [m]}$

$\epsilon = 0.0000015 \text{ [m]}$

$f_{1\text{to}2} = 0.0217$

$g = 9.807 \text{ [m/s}^2\text{]}$

$h_{L_1\text{to}2} = 18.43 \text{ [m]}$

$L_1 = 5 \text{ [m]}$

$L_2 = 6 \text{ [m]}$

$\mu = 0.001002 \text{ [kg/m-s]}$

$P_{\text{gage}_1} = 200000 \text{ [N/m}^2\text{]}$

$Re = 44576 \text{ [-]}$

$\rho = 998 \text{ [kg/m}^3\text{]}$

$\text{SIGMAK}_{L_1} = 0$

$\text{SIGMAK}_{L_2} = 24.7$

$V = 2.984 \text{ [m/s]}$

$\dot{V} = 0.0005273 \text{ [m}^3/\text{s]}$

$\dot{V}_{\text{LPS}} = 0.5273 \text{ [L/s]}$

No unit problems were detected.

Part (b) EES Equation Window:

|"Example 8.9 - Toilet Flushing Problem, Part (b)"

"Constants and properties:"

$$g = g\#$$

$$\rho = 998 \text{ [kg/m}^3\text{]}$$

$$\mu = 1.002\text{E-}3 \text{ [kg/m-s]}$$

$$D = 0.015 \text{ [m]}$$

$$\epsilon = 1.5\text{e-}6 \text{ [m]}$$

$$P_{\text{gage}_1} = 200000 \text{ [N/m}^2\text{]}$$

$$L_1 = 5 \text{ [m]}$$

$$L_2 = 6 \text{ [m]}$$

$$L_3 = 1 \text{ [m]}$$

$$\text{SIGMAK}_{L_1} = 0$$

$$\text{SIGMAK}_{L_2} = 0.9 + 2 \cdot 0.9 + 10 + 12 \quad \text{"tee, two elbows, valve, and shower head"}$$

$$\text{SIGMAK}_{L_3} = 2 + 10 + 0.9 + 14 \quad \text{"tee, one elbow, valve, and toilet mechanism"}$$

$$\Delta z_{1\text{to}2} = 2 \text{ [m]}$$

$$\Delta z_{1\text{to}3} = 1 \text{ [m]}$$

"Equations to solve"

$$P_{\text{gage}_1}/(\rho \cdot g) = \Delta z_{1\text{to}2} + h_{L_1\text{to}2}$$

$$P_{\text{gage}_1}/(\rho \cdot g) = \Delta z_{1\text{to}3} + h_{L_1\text{to}3}$$

$$h_{L_1\text{to}2} = V_1^2/(2 \cdot g) \cdot (f_1 \cdot L_1/D + \text{SIGMAK}_{L_1}) + V_2^2/(2 \cdot g) \cdot (f_2 \cdot L_2/D + \text{SIGMAK}_{L_2})$$

$$h_{L_1\text{to}3} = V_1^2/(2 \cdot g) \cdot (f_1 \cdot L_1/D + \text{SIGMAK}_{L_1}) + V_3^2/(2 \cdot g) \cdot (f_3 \cdot L_3/D + \text{SIGMAK}_{L_3})$$

$$A = \pi \cdot D^2/4$$

$$\dot{V}_1 = V_1 \cdot A$$

$$\dot{V}_2 = V_2 \cdot A$$

$$\dot{V}_3 = V_3 \cdot A$$

$$\text{Re}_1 = V_1 \cdot D \cdot \rho / \mu$$

$$\text{Re}_2 = V_2 \cdot D \cdot \rho / \mu$$

$$\text{Re}_3 = V_3 \cdot D \cdot \rho / \mu$$

$$f_1 = \text{MoodyChart}(\text{Re}_1, \epsilon / D)$$

$$f_2 = \text{MoodyChart}(\text{Re}_2, \epsilon / D)$$

$$f_3 = \text{MoodyChart}(\text{Re}_3, \epsilon / D)$$

$$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$$

$$\dot{V}_2_{\text{LPS}} = \dot{V}_2 \cdot \text{CONVERT}(\text{m}^3/\text{s}, \text{L}/\text{s})$$

Part (b) EES Solution:

$$A = 0.0001767 \text{ [m}^2\text{]}$$

$$D = 0.015 \text{ [m]}$$

$$\Delta z_{1\text{to}2} = 2 \text{ [m]}$$

$$\Delta z_{1\text{to}3} = 1 \text{ [m]}$$

$$\epsilon = 0.0000015 \text{ [m]}$$

$$f_1 = 0.01943$$

$$f_2 = 0.0228 \text{ [-]}$$

$$f_3 = 0.02212 \text{ [-]}$$

$$g = 9.807 \text{ [m/s}^2\text{]}$$

$$h_{L_1\text{to}2} = 18.43 \text{ [m]}$$

$$h_{L_1\text{to}3} = 19.43 \text{ [m]}$$

$$L_1 = 5 \text{ [m]}$$

$$L_2 = 6 \text{ [m]}$$

$$L_3 = 1 \text{ [m]}$$

$$\mu = 0.001002 \text{ [kg/m-s]}$$

$$P_{\text{gage}_1} = 200000 \text{ [N/m}^2\text{]}$$

$$\text{Re}_1 = 76419 \text{ [-]}$$

$$\text{Re}_2 = 35608 \text{ [-]}$$

$$\text{Re}_3 = 40811 \text{ [-]}$$

$$\rho = 998 \text{ [kg/m}^3\text{]}$$

$$\text{SIGMAK}_{L_1} = 0$$

$$\text{SIGMAK}_{L_2} = 24.7$$

$$\text{SIGMAK}_{L_3} = 26.9$$

$$V_1 = 5.115 \text{ [m/s]}$$

$$V_2 = 2.383 \text{ [m/s]}$$

$$V_3 = 2.732 \text{ [m/s]}$$

$$\dot{V}_1 = 0.0009039 \text{ [m}^3\text{/s]}$$

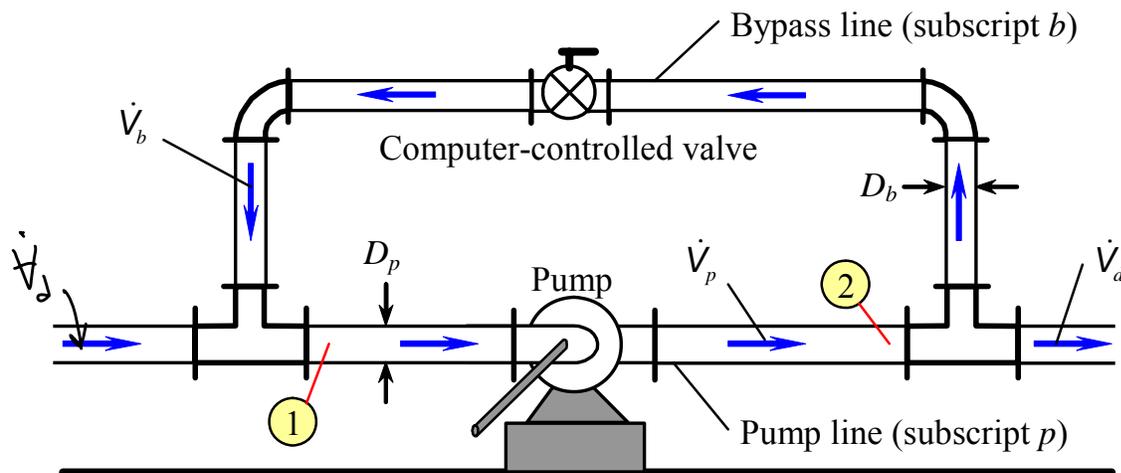
$$\dot{V}_2 = 0.0004212 \text{ [m}^3\text{/s]}$$

$$\dot{V}_{2\text{LPS}} = 0.4212 \text{ [L/s]}$$

$$\dot{V}_3 = 0.0004827 \text{ [m}^3\text{/s]}$$

Example: Parallel Pipe Network and Pump Bypass System

Background: In some applications (e.g., nuclear reactor cooling), it is critical that the volume flow rate of a fluid remains constant, regardless of head changes in the system downstream (within specified limits, of course). One method of ensuring a constant volume flow rate is to install an *oversized* pump to drive the flow. A bypass line is then installed *in parallel* with the pump so that some of the fluid (bypass volume flow rate \dot{V}_b) recirculates through the bypass line as shown. Based on feedback from a downstream volume flowmeter, the bypass valve is then adjusted by a computer to control both \dot{V}_b and the volume flow rate through the pump \dot{V}_p such that the volume flow rate \dot{V}_d downstream of the system remains constant.



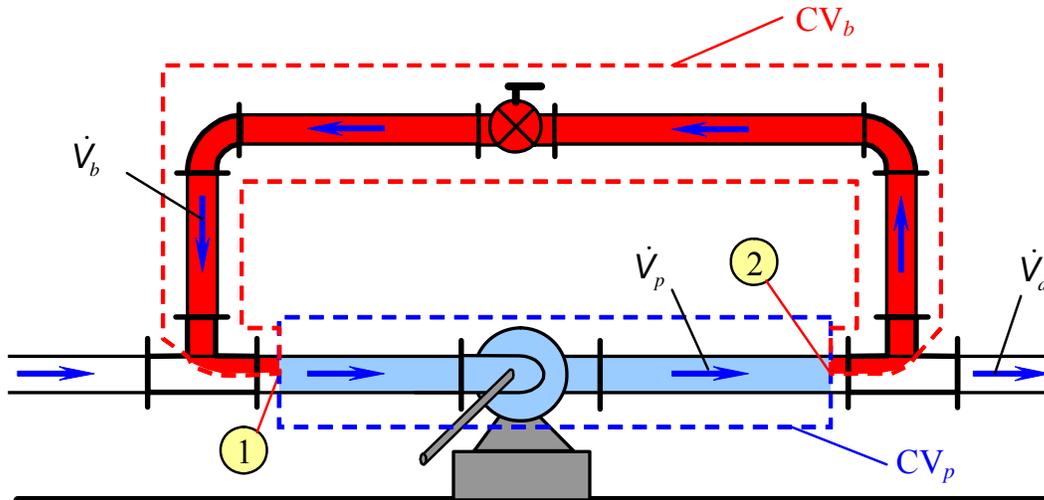
Given: In this particular case, water ($\rho = 998 \text{ kg/m}^3$, $\mu = 1.00 \times 10^{-3} \text{ kg/m}\cdot\text{s}$) needs to be supplied at a constant downstream volume flow rate of $\dot{V}_d = 0.20 \text{ m}^3/\text{s}$. The pump's performance curve is given by the pump manufacturer as $h_{\text{pump,u,supply}} = a(b - c\dot{V}_p^2)$ where $a = 100 \text{ m}$, $b = 1.0$, and $c = 1.0 \text{ s}^2/\text{m}^6$. The units of $h_{\text{pump,u,supply}}$ are [m] and the units of \dot{V}_p are [m^3/s]. The pump line has a diameter $D_p = 0.50 \text{ m}$ and length $L_p = 3.0 \text{ m}$, while the bypass line has a diameter $D_b = 0.50 \text{ m}$ and length $L_b = 5.0 \text{ m}$. All pipes have roughness $\varepsilon = 0.002 \text{ m}$. Minor losses in the system include two flanged 90° bends ($K_L = 0.20$ each) and a gate valve ($0.2 < K_L < \infty$) in the bypass line, and two flanged tees (*line flow* $K_L = 0.20$, and *branch flow* $K_L = 1.00$).

To Do: Calculate and plot how volume flow rates \dot{V}_p , \dot{V}_b , and \dot{V}_d vary with the minor loss coefficient of the valve as it goes from fully open ($K_{L,\text{valve}} = 0.20$) to fully closed ($K_{L,\text{valve}} \rightarrow \infty$).

Solution:

- First, as always, we need to pick a control volume. In this case, we need *two* control volumes since there are two branches in the parallel pipe system. After careful thought (and experience), we decide that the most appropriate control volumes go between points (1) and (2) as labeled on the above sketch. We re-draw the flow system

including the two control volumes. In this parallel pipe system it is useful to imagine that the fluid in the bypass line (CV_b) is colored red, while that in the pump line (CV_p) is colored blue. This is an *artificial* separation of the fluid into the two branches since the fluid mixes because of turbulence. However, it is useful as an aid to drawing the control volumes. Note that CV_p has its inlet at (1) and its outlet at (2), while CV_b has its inlet at (2) and its outlet at (1).



- We apply conservation of mass at either tee:

$$\dot{V}_p = \dot{V}_b + \dot{V}_d$$

Note: This equation couples the two control volumes together. Other than this, we treat the two control volume separately in the analysis below.

- We apply the head form of the energy equation for CV_p from inlet (1) to outlet (2), assuming that the flow at both (1) and (2) is fully developed turbulent pipe flow so that both velocity and kinetic energy correction factor are the same at (1) as at (2):

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine,e}} + h_{L,p}$$

$$V_1 = V_2 \text{ and } \alpha_1 = \alpha_2$$

We call this $h_{\text{pump,u,system}}$ since it is the required pump head for the given piping system.

Or,
$$h_{\text{pump,u,system}} = \frac{P_2 - P_1}{\rho g} + h_{L,p}$$

- We next apply the head form of the energy equation for CV_b from inlet (2) to outlet (1), recognizing again that $V_1 = V_2$ and $\alpha_1 = \alpha_2$, and that there is no pump in this CV:

$$\frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{pump,u}} = \frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{turbine,e}} + h_{L,b}$$

Or,
$$h_{L,b} = \frac{P_2 - P_1}{\rho g}$$

Note: $P_2 - P_1$ is the same regardless of whether we are considering the pump line or the branch line.

- Combining the above two results, we get
$$h_{\text{pump,u,system}} = h_{L,b} + h_{L,p}$$

- We note that since the velocities (and therefore the Reynolds numbers) in the pump line and the bypass line are not the same, we therefore need *two* Colebrook or Churchill equations to solve the problem, one for the pump line and one for the bypass line.
- We write all the equations, and solve them simultaneously [I used EES].

Here is what I typed into the main “Equations Window” of EES:

```
"Constants:"
rho = 998 [kg/m^3]           "density of the fluid"
mu = 1.00e-3 [kg/(m*s)]    "viscosity of the fluid"
g = g#                       "gravitational constant, a predefined constant in EES"
V_dot_d = 0.20 [m^3/s]      "desired downstream volume flow rate (to be kept constant)"
L_p = 3.0 [m]               "pipe length for the pump line"
L_b = 5.0 [m]               "pipe length for the bypass line"
D_p = 0.5 [m]               "pipe diameter for the pump line"
D_b = 0.5 [m]               "pipe diameter for the bypass line"
epsilon_p = 0.002 [m]       "average roughness height of the pump line pipe"
epsilon_b = 0.002 [m]       "average roughness height of the bypass line pipe"
SigmaK_p = 0                 "minor losses in the pump line (there are none)"
SigmaK_b = 2*1.0 + 2*0.2 + K_L_valve "minor losses in the bypass line (two branch tees, two elbows, and one valve)"
  "K_L_valve = 0.4"          "remove the quotes in the equation to the left to test at one K_L_valve"
```

"NOTE: The above equation for SigmaK_b is the only place in the Equations Window in which variable K_L_valve appears. This is because we are going to construct a parametric table in which K_L_valve varies over a range of values. All the other variables will then be solved by EES for each value of K_L_valve in the parametric table, and a plot will be generated."

```
"Pump performance curve (supply curve), as provided by the pump manufacturer:"
a = 100 [m]                 "pump performance constant 1"
b = 1.0                     "pump performance constant 2"
c = 1.0 [s^2/m^6]           "pump performance constant 3"
h_pump_u_supply = a*(b-c*V_dot_p^2) "useful pump head supplied by the pump"
```

```
"Equations:"
V_dot_p = V_dot_b + V_dot_d "conservation of mass equation"
V_dot_p = V_p*PI*D_p^2/4    "volume flow rate in the pump line"
V_dot_b = V_b*PI*D_b^2/4    "volume flow rate in the bypass line"
h_L_p = (f_p*L_p/D_p + SigmaK_p)*V_p^2/(2*g) "irreversible head loss in the pipe line"
h_L_b = (f_b*L_b/D_b + SigmaK_b)*V_b^2/(2*g) "irreversible head loss in the bypass line"
h_pump_u_system = h_L_b + h_L_p "useful pump head required for the system"
Re_p = rho*D_p*V_p/mu       "Reynolds number in the pipe line"
Re_b = rho*D_b*V_b/mu       "Reynolds number in the bypass line"
h_pump_u_system = h_pump_u_supply | "conservation of energy equation (combination of two CVs)"
1/sqrt(f_p) = -2.0*log10(epsilon_p/D_p/3.7 + 2.51/Re_p/sqrt(f_p)) "Colebrook equation for pump line"
1/sqrt(f_b) = -2.0*log10(epsilon_b/D_b/3.7 + 2.51/Re_b/sqrt(f_b)) "Colebrook equation for bypass line"
```

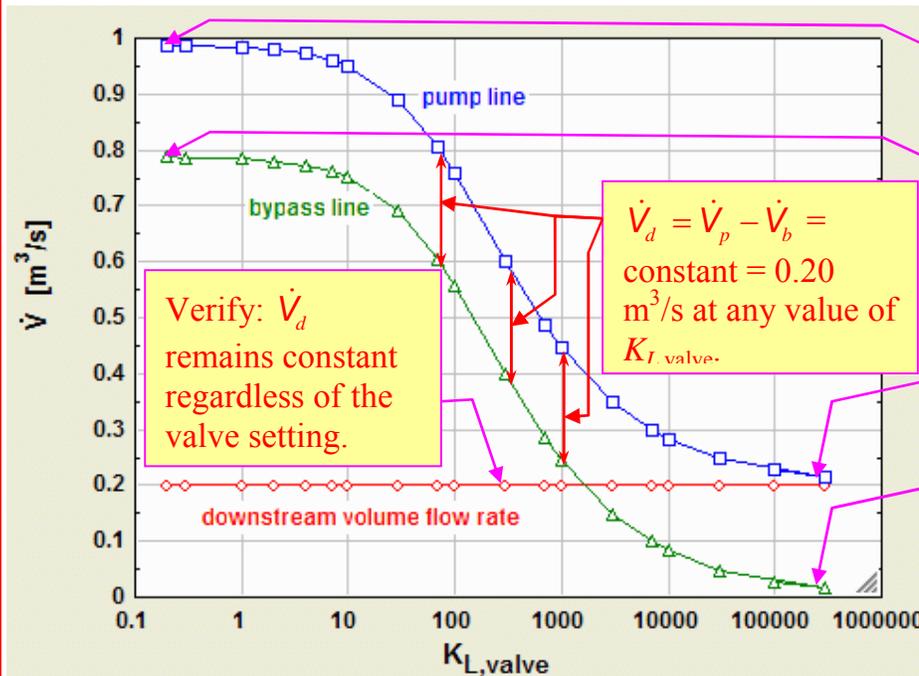
I created a parametric table, and selected four of the variables (Table-New Parametric Table):

1..20	1 $K_{L, \text{valve}}$	2 \dot{V}_p [m ³ /s]	3 \dot{V}_d [m ³ /s]	4 \dot{V}_b [m ³ /s]
Run 1	0.2	0.987	0.2	0.787
Run 2	0.3	0.9866	0.2	0.7866
Run 3	1	0.9838	0.2	0.7838
Run 4	2	0.9799	0.2	0.7799
Run 5	4	0.9722	0.2	0.7722
Run 6	7	0.9611	0.2	0.7611
Run 7	10	0.9505	0.2	0.7505
Run 8	30	0.8901	0.2	0.6901
Run 9	70	0.8045	0.2	0.6045
Run 10	100	0.7584	0.2	0.5584
Run 11	300	0.5997	0.2	0.3997
Run 12	700	0.4865	0.2	0.2865
Run 13	1000	0.4458	0.2	0.2458
Run 14	3000	0.3487	0.2	0.1487
Run 15	7000	0.2991	0.2	0.09915
Run 16	10000	0.2834	0.2	0.08337
Run 17	30000	0.2486	0.2	0.04862
Run 18	300000	0.2155	0.2	0.0155
Run 19	100000	0.2268	0.2	0.02678
Run 20	300000	0.2155	0.2	0.0155

I specified the values of $K_{L, \text{valve}}$, ranging from the minimum of 0.2 (fully open gate valve) to a very large number (valve nearly closed).

The rest of the values are automatically calculated and filled in when you click on the green arrow at the upper left.

Finally, I plotted all three of the dependent volume flow rates as functions of $K_{L, \text{valve}}$:



When the valve is fully open, we get the maximum flow rate through both the pump and bypass lines.

When the valve is nearly closed, there is hardly any flow through the bypass line – nearly all the flow is through the pump line.