Today, we will:

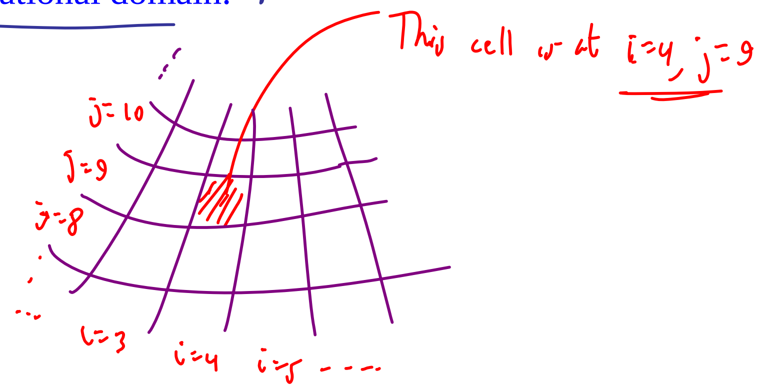
- Continue our brief introduction to: **CFD** (Chapter 15)
- Begin Chapter 10 – Approximate Solutions of the N-S Equation

2. CFD solution procedure

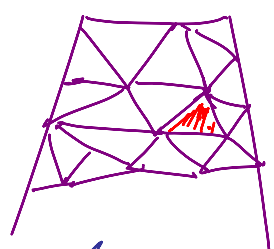
- Step 1: Choose a computational domain. & Specify or create a mesh (grid)

- Types of grids

  - Structured → can identify rows & columns

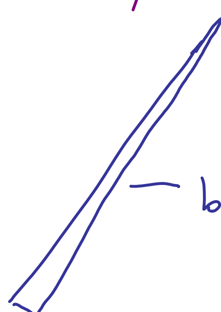


  - Unstructured – cannot identify rows & columns



- Quality of mesh is critical

  e.g. skewness



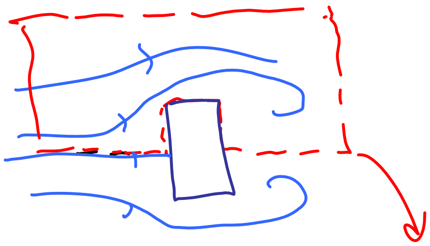
— bad – high skewness

— very good (little skewness

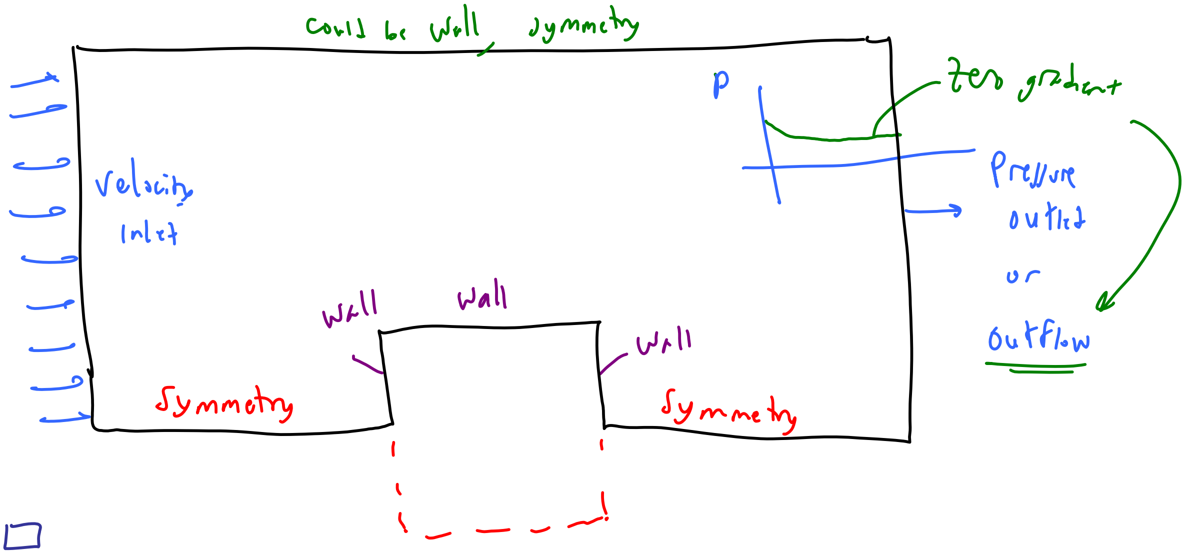
- Step 2 Specify Boundary Conditions on every edge of the comp. domain

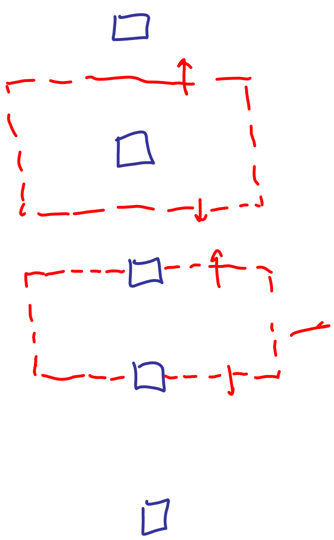
Types:
- Wall (no slip condition)
- inlet (specify velocity or pressure)
- outlet (pressure outlet, outflow → forces slopes → 0)
- Symmetry
- periodic, etc.

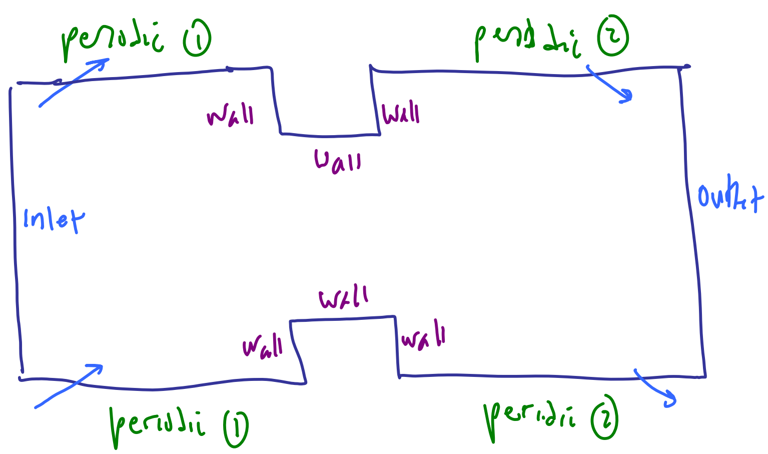
Eg. <u>Flow over a block</u>



If symmetric, we can get away with <u>half</u> of geometry



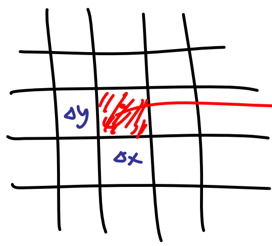




- <u>Step 3</u> – Specify fluid properties, etc.
- <u>Step 4</u> – Specify numerical options & solution options [typ. use defaults]
- <u>Step 5</u> – Specify initial conditions or initial guesses for all variables

Step 6 – Solve the equations

Computer solves cons. eqs: cons of mass, cons. of momentum, [cons. of energy] + turbulent eqs



Solve cons. eqs at each cell **iteratively**

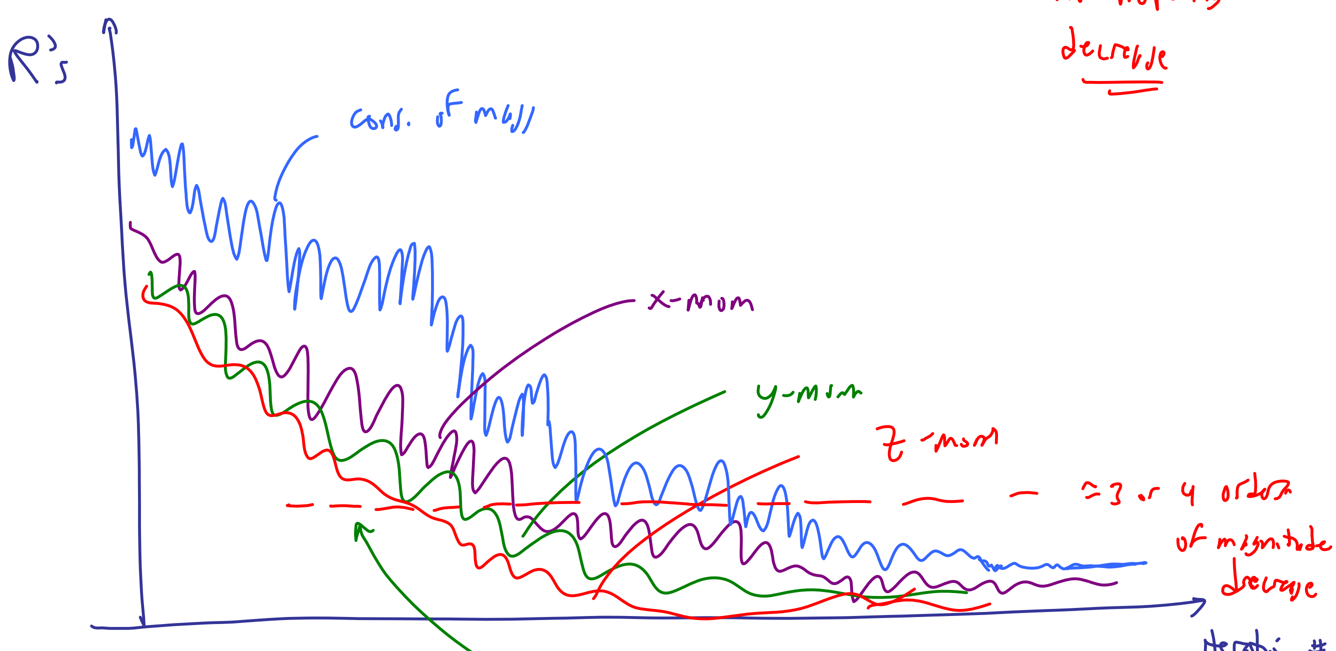
Residual → the error associated with an equation

eg → cont. eq. → $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

crude approx. → $$\frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta y} + \frac{\Delta w}{\Delta z} = R \rightarrow R \neq 0$$

↓

Iterate & R will hopefully **decrease**



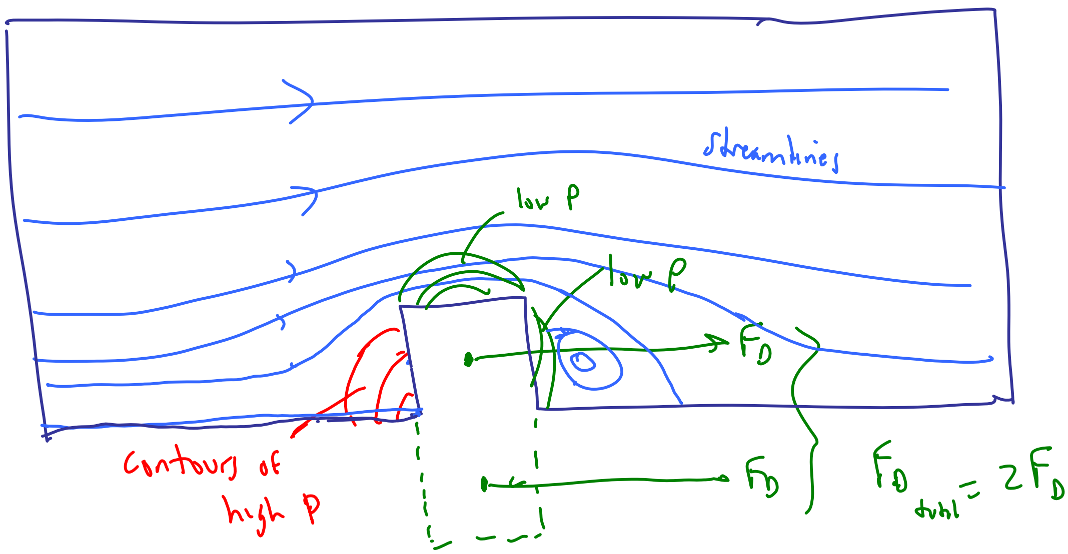
Pick some level of convergence to call it "converged"

Step 7 – Post processing

e.g. Plots – contour plots (velocity magnitude, pressure, etc.)

– Velocity vectors

– Streamlines



Step 8 Calculate global properties (Drag force, – C_D = drag coeff.)

3. Examples (see McGraw-Hill website for the textbook) ed. 3

OR, for Ex 1 or 2, see the DVD that comes with the book

VIII. APPROXIMATE SOLUTIONS OF THE NAVIER-STOKES EQUATION

A. Introduction

We have three ways to solve the differential equations of fluid flow:

→ 1. Analytically (Chapter 9) [solve exactly, but only for very simple problems]
→ 2. Numerically (Chapter 15) [use CFD on a computer to solve for thousands of cells]
→ 3. Approximately (Chapter 10) [ignore some terms in the N-S equation, then solve]

B. Nondimensionalization of the Equations of Motion

↓

Goal – re-write the eqs of motion in nondimensional form

Why? So we can properly compare the order of magnitude of terms & see which ones we can ignore

1. Consider only incompressible flow

Continuity eq. $\boxed{\vec{\nabla} \cdot \vec{V} = 0}$

where $\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$

$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

$\vec{x} = x\vec{i} + y\vec{j} + z\vec{k}$

Introduce scaling parameters & nondimensional forms of these quantities

· Let $\boxed{L = \text{characteristic length scale}}$

* quantities are nondimensional

Let $\boxed{\vec{x}^* = \frac{\vec{x}}{L}}$ → $x^* = \frac{x}{L}$ $y^* = \frac{y}{L}$ $z^* = \frac{z}{L}$

Let $\boxed{\vec{V}^* = \frac{\vec{V}}{V}}$ where $\boxed{V = \text{characteristic velocity scale (typ. the free stream)}}$

$u^* = \frac{u}{V}$, $v^* = \frac{v}{V}$, $w^* = \frac{w}{V}$

Gradient operator? $\{\vec{\nabla}\} = \left\{\frac{1}{L}\right\}$ set $\boxed{\vec{\nabla}^* = L\vec{\nabla}}$

non-dimensional form of $\vec{\nabla}$

Continuity eq.

$u = u^* \cdot V$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$x = x^* \cdot L$

$$\frac{\partial(u^* V)}{\partial(x^* L)} + \frac{\partial(v^* V)}{\partial(y^* L)} + \frac{\partial(w^* V)}{\partial(z^* L)} = 0$$

V & L are constant. so,

$$\frac{\cancel{V}}{\cancel{L}}\underbrace{\left[\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*}\right]}=0$$

using definition of dot product

$$\left(\frac{\partial}{\partial x^*}, \frac{\partial}{\partial y^*}, \frac{\partial}{\partial z^*}\right)\cdot\left(u^*, v^*, w^*\right) = 0$$

$$\vec{\nabla}^* \cdot \vec{V}^* = 0 \longrightarrow \boxed{\vec{\nabla}^* \cdot \vec{V}^* = 0}$$

Nondimensionalized continuity eq.

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2. Navier-Stokes eq. → do same thing

[only pressure differences matter, not absolute pressure]

Introduce some more scaling parameters:

let $\boxed{f = \text{some characteristic frequency}}$ $\left\{\frac{1}{t}\right\}$

let $\boxed{P_0 - P_\infty = \text{" " pressure difference}}$ $\left\{\frac{m}{Lt^2}\right\}$

need $\boxed{g = \text{gravitational constant (scaling parameter)}}$

Plug into N-S eq. & nondimensionalize it → Nondimensionalized N-S eq.