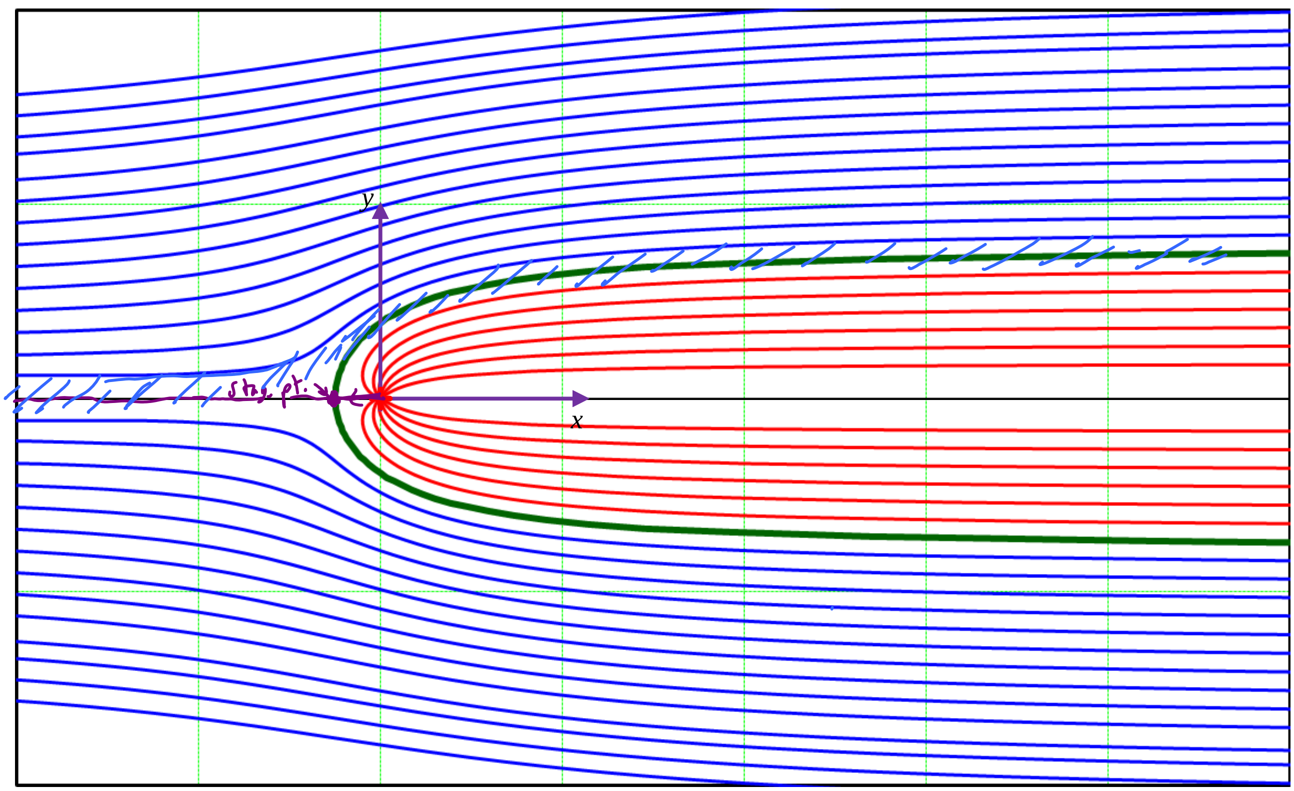
Today, we will:

- Continue examples of superposition of irrotational flows – flow over a circular cylinder
- Start discussing the last approximation of Chapter 10: **The Boundary Layer Approx.**

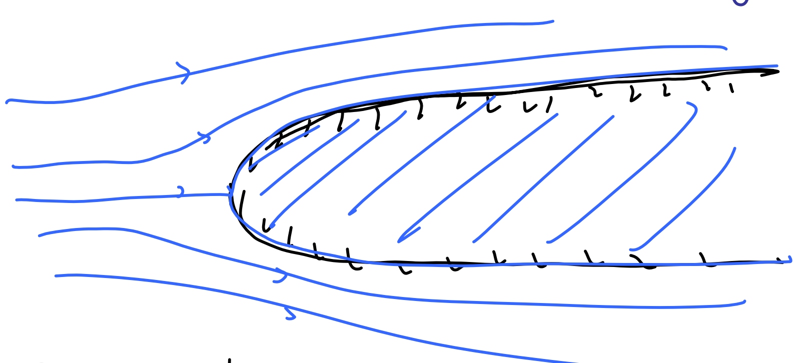
Recall, the Rankine half-body:



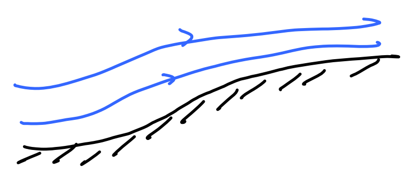
USEFULNESS OF THIS APPROX. SOLUTION:

We can think of any streamline as a solid wall in irrotational flow

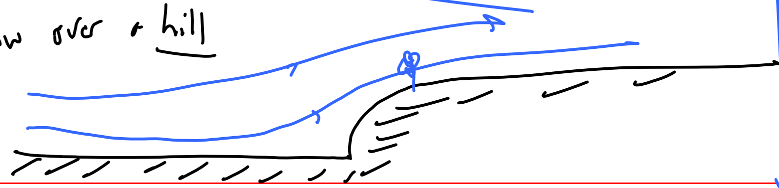
e.g. Flow at the nose of a body, like a wing



Flow over a rounded hill



e.g. Flow over a hill



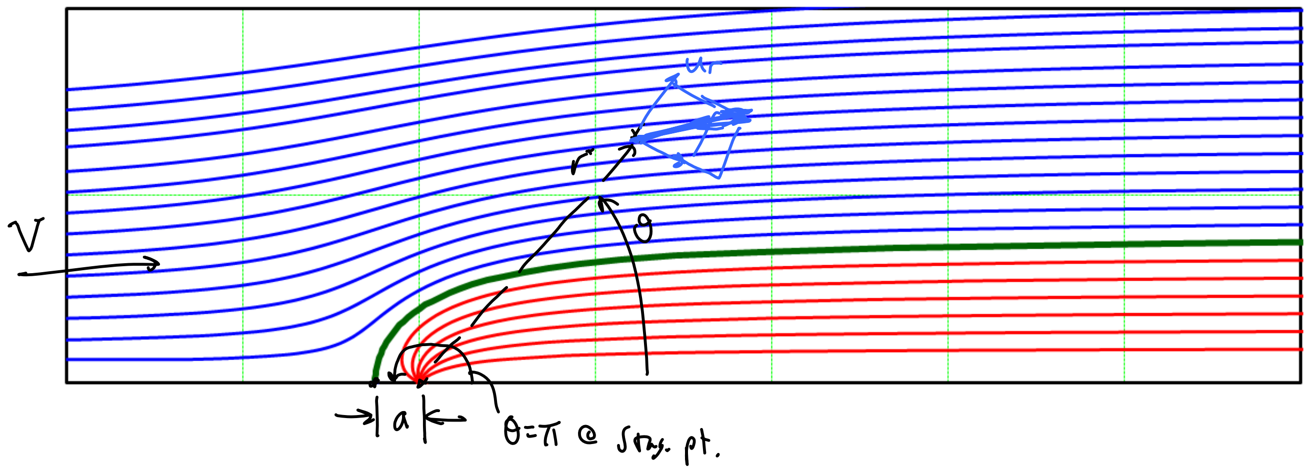
Example: Rankine Half-Body

Given: A Rankine half-body is constructed using a horizontal freestream of velocity V = 5.0 m/s and line source at the origin of strength $\frac{\dot{V}}{L} = 2.5\pi \frac{\text{m}^2}{\text{s}}$. The stream function is

$$\psi = Vr\sin\theta + \frac{1}{2\pi}\frac{\dot{V}}{L}\theta$$

To do: Generate expressions for u_r and u_θ, and calculate the distance a (the distance between the origin and the stagnation point).

Solution:



$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}Vr\cos\theta + \frac{1}{r}\frac{1}{2\pi}\frac{\dot{V}}{L} \qquad u_\theta = -\frac{\partial\psi}{\partial r} = -V\sin\theta$$

$$\boxed{u_r = V\cos\theta + \frac{1}{2\pi r}\frac{\dot{V}}{L}} \qquad \boxed{u_\theta = -V\sin\theta}$$

Stag. pt. is when $\vec{V} = 0 \longrightarrow u_r = 0,\ u_\theta = 0$

$\theta = \pi$ @ stag. pt. $\qquad V = 5.0$ m/s $\qquad \frac{\dot{V}}{L} = 2.5\pi \frac{m^2}{s}$

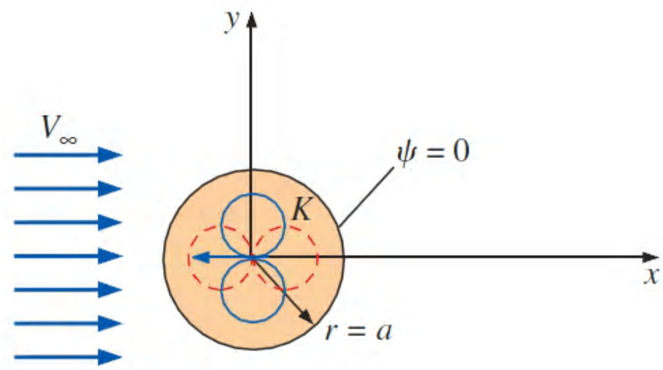
& $r = a$ @ stag. pt.

$$u_r = 0 \longrightarrow -V\cos\theta = \frac{1}{2\pi a}\frac{\dot{V}}{L}$$

@ $\theta = \pi$
$= -1$

Solve for a $\longrightarrow \boxed{a = \frac{\dot{V}/L}{2\pi V}} \longrightarrow$ #'s, $\boxed{a = 0.25 \text{ m}}$

b. Example of superposition: Flow over a circular cylinder



Given: Superpose a uniform stream of velocity V_∞ and a doublet of strength K at the origin.

To do: Plot streamlines, and discuss the flow that results from this superposition.

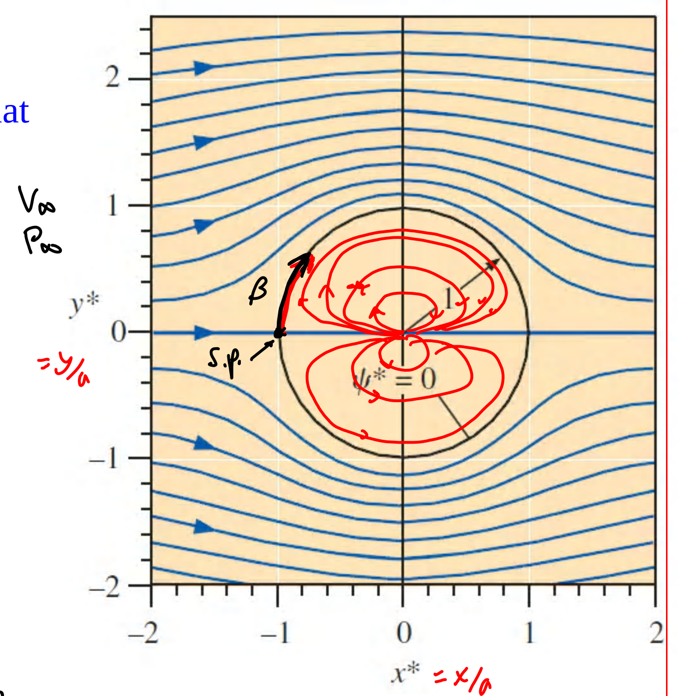
Solution:

- We simply add up the stream functions for the two building block flows: $\psi = \psi_{\text{freestream}} + \psi_{\text{doublet}} = V_\infty y - K\dfrac{\sin\theta}{r}$.
- But we know that $\boxed{y = r\sin\theta}$, thus, $\boxed{\psi = V_\infty r\sin\theta - K\dfrac{\sin\theta}{r}}$.
- For "convenience", and with hindsight, we choose to set $\psi = 0$ at $r = a$.
  [It turns out that radius *a* is a special radius that becomes the radius of the circle.]
- Set $r = a$ in our equation for the stream function:

$$0 = V_\infty a\sin\theta - K\frac{\sin\theta}{a} \quad \rightarrow \quad \boxed{K = V_\infty a^2}.$$

- Then our final expression for ψ becomes

$$\boxed{\psi = V_\infty \sin\theta\left(r - \frac{a^2}{r}\right)}.$$



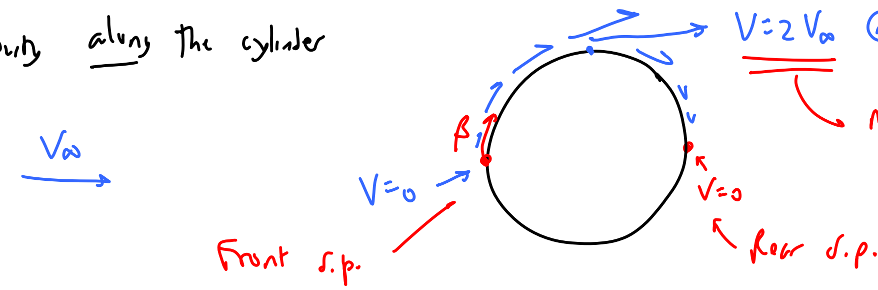
- Plot streamlines: [we plot nondimensionally, setting $x^*=x/a$ and $y^*=y/a$]
- From our equation for ψ above, we can calculate the velocity field from the definition of ψ, i.e., $u_r = \dfrac{1}{r}\dfrac{\partial\psi}{\partial\theta}$ $\quad u_\theta = -\dfrac{\partial\psi}{\partial r}$. See text for details. On the cylinder ($r = a$),

$$\boxed{u_r = 0 \qquad u_\theta = -2V_\infty \sin\theta}$$

Bernoulli w/o gravity → $P + \frac{1}{2}\rho V^2 = \text{const} = P_\infty + \frac{1}{2}\rho V_\infty^2$

- We can also define the **pressure coefficient**, $\boxed{C_p = \dfrac{P - P_\infty}{\frac{1}{2}\rho V_\infty^{\,2}} = 1 - \dfrac{V^2}{V_\infty^{\,2}}}$.
- *On the cylinder*, it turns out that $\boxed{C_p = 1 - 4\sin^2\beta}$, where β is the angle from the nose.

· Plot velocity along the cylinder

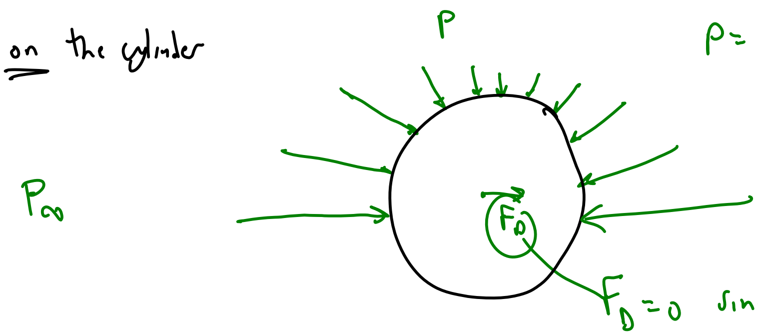


$V = 2V_\infty$ @ $\beta = 90°$

Max speed is at the top (& bottom) of the cylinder

· Plot pressure on the cylinder

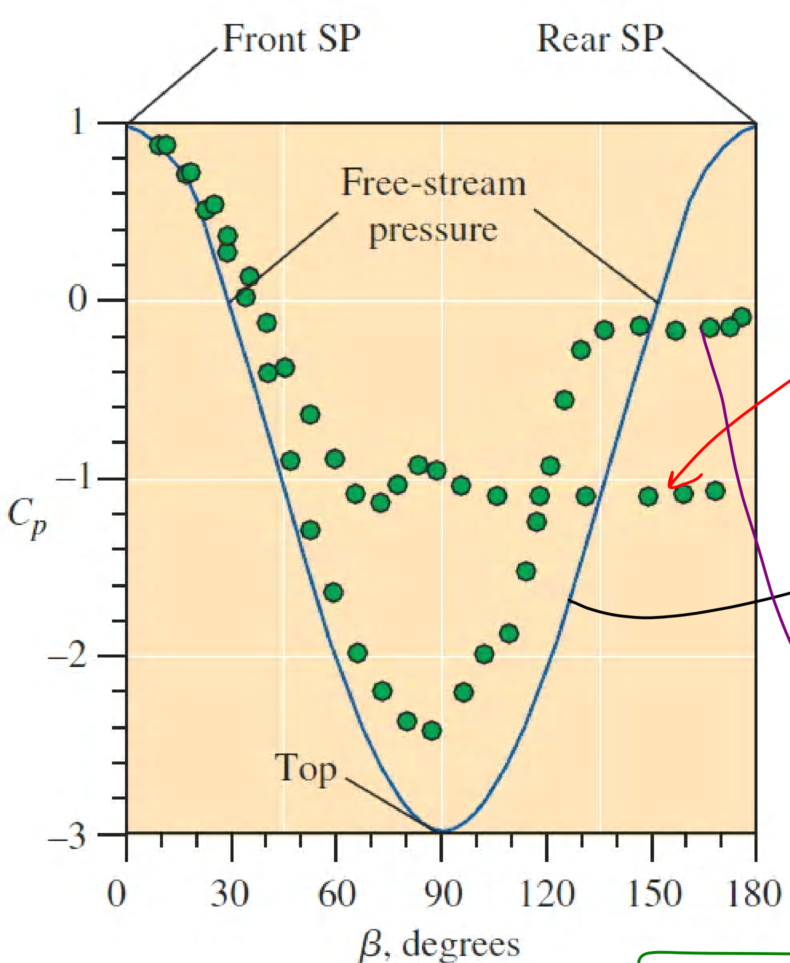
(Bernoulli)



$P = \min$ @ $\beta = 90°$

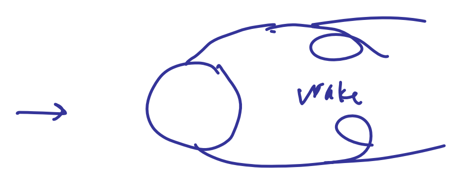
$P = \max$ @ stag. pt.

$F_D = 0$ since pressure is symmetric



$\boxed{\text{D'Alembert's Paradox} \rightarrow \text{Drag} = 0 \text{ for } \underline{\text{any}} \text{ non-lifting body in irrotational flow}}$

Laminar flow



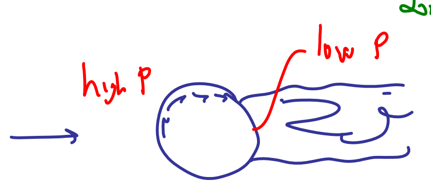
Irrotational flow approx

Turbulent flow

Pressure drag dominates

low P

high P



skin friction dominates

$$\boxed{F_D = F_{D_{\text{skin friction}}} + F_{D_{\text{pressure}}}}$$

(τ shear stress)

F. The Boundary Layer Approximation

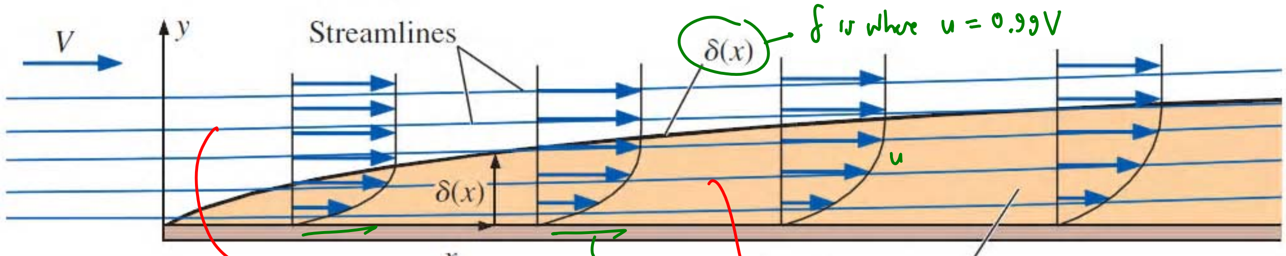
1. Introduction

Definition: A ***boundary layer*** is a thin layer in which viscous effects and vorticity are significant, and cannot be ignored.

Flow is rotational

Examples

- BL on a flat plate aligned with the freestream flow (we show top side only):



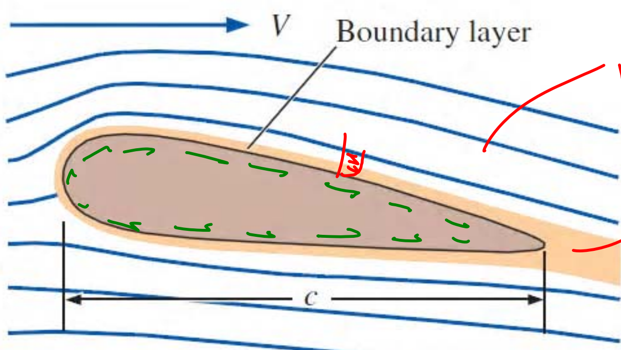
δ is where $u = 0.99V$

τ_w

irrotational flow outside BL

rotational flow inside BL

- BL on an airfoil:



irrotational region

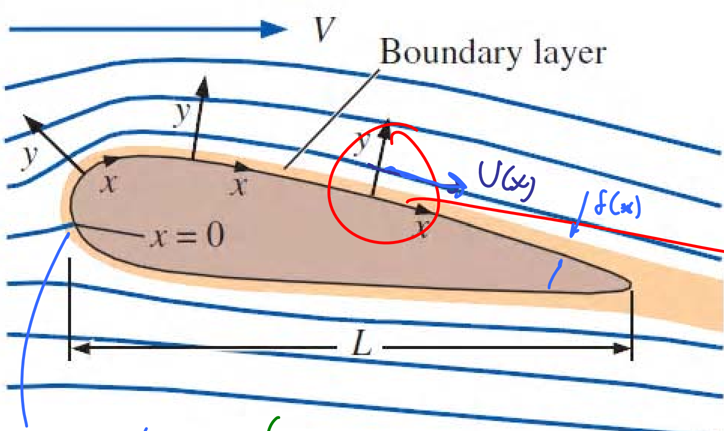
BL, rotational

Comments

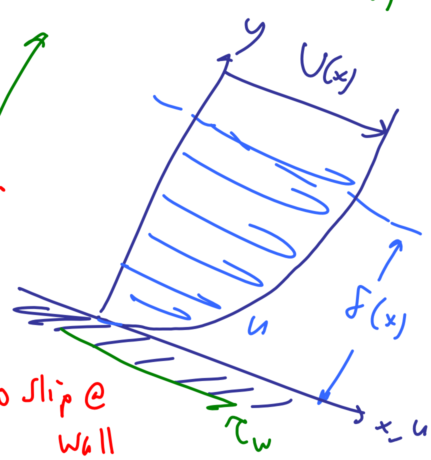
- N-S eq solves everything – but it is too hard to solve w/o CFD
- Euler eq. or Potential flow is a good approx. <u>outside</u> of the BL
- The BL bridges the gap between irrotational analysis & a full N-S analysis

2. The Boundary Layer Coordinate System

In a 2-D flow, we let *x* = distance along the wall, and *y* = distance normal to the wall.



☆ Local coordinate system (x, y)



Stag. pt. $(u = v = 0)$

X starts @ stag. pt., but y is locally ⊥ to the wall

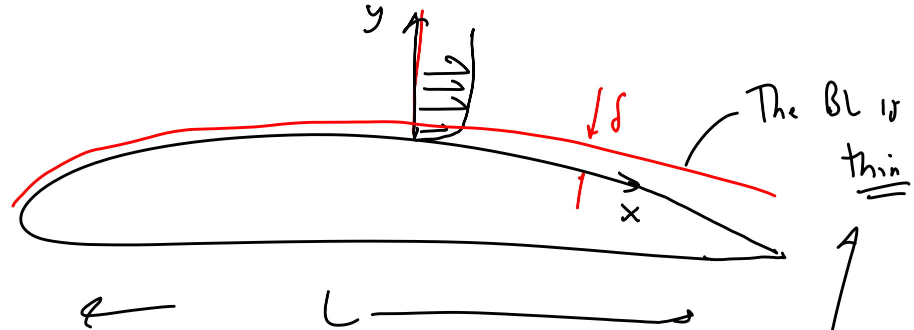
No slip @ wall

τ_w

3. The BL equations → use 2 length scales instead of 1.

use L for $\frac{\partial}{\partial x}$ terms

use δ for $\frac{\partial}{\partial y}$ terms ☆



$$\boxed{\frac{\partial}{\partial x} \sim \frac{1}{L}} \;☆ \qquad \boxed{\frac{\partial}{\partial y} \sim \frac{1}{\delta}} \;☆ \qquad \boxed{\delta \ll L}$$

See Text For Derivation

The BOUNDARY LAYER EQUATIONS: (In x,y BL coordinates, steady, no gravity)

Continuity
$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

x-mom
$$\boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\cancel{\frac{\partial^2 u}{\partial x^2}} + \nu\frac{\partial^2 u}{\partial y^2}}$$

Small! — Ignore this term

y-mom
$$u\cancel{\frac{\partial v}{\partial x}} + v\cancel{\frac{\partial v}{\partial y}} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\cancel{\frac{\partial^2 v}{\partial x^2}} + \nu\cancel{\frac{\partial^2 v}{\partial y^2}} \Rightarrow \boxed{\frac{\partial p}{\partial y} \approx 0} \;☆$$

Small   Small   Small   Small