M E 320 Professor John M. Cimbala Lecture 38

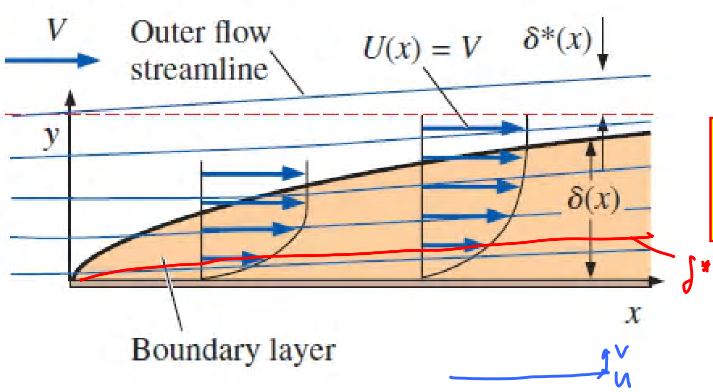
Today, we will:

- Discuss ***displacement thickness*** in a laminar boundary layer
- Discuss the ***turbulent*** boundary layer on a flat plate, and compare with laminar flow
- Talk about boundary layers with ***pressure gradients***

d. Displacement thickness, δ^*

Definition:

δ^* = the distance that a streamline just outside the BL is deflected away from the wall due to the effect of the BL; the distance in which the outer flow is "displaced" away from the wall.



Note: Neither δ nor δ^* are *streamlines*. In fact, streamlines *cross* lines of δ and δ^*.

(+)ve flow in y-direction (very small)

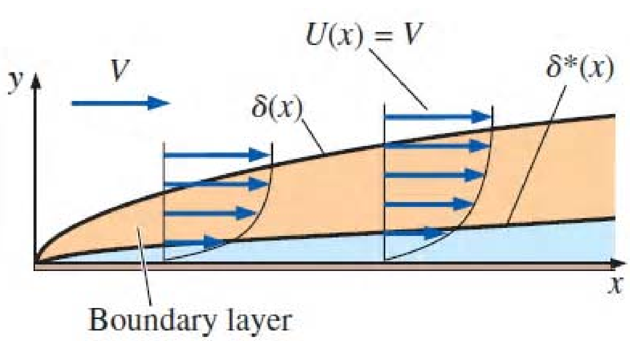
For a laminar BL on a flat plate,

$$\boxed{\frac{\delta^*}{x} = \frac{1.72}{\sqrt{Re_x}}}$$

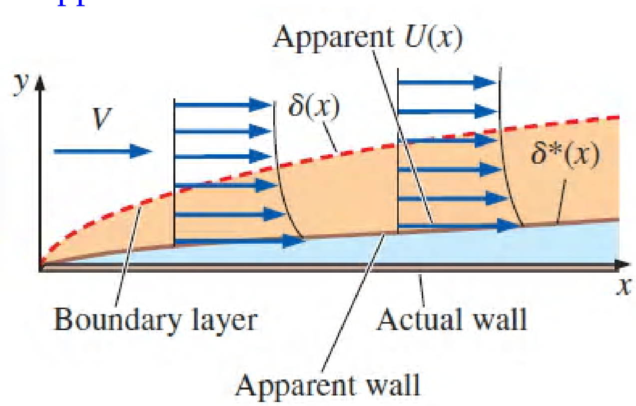
Alternate Definition:

δ^* = the imaginary increase in wall thickness seen by the outer flow, due to the presence of the BL. [The outer flow "feels" like the wall is thicker than it actually is.]

Actual wall case:



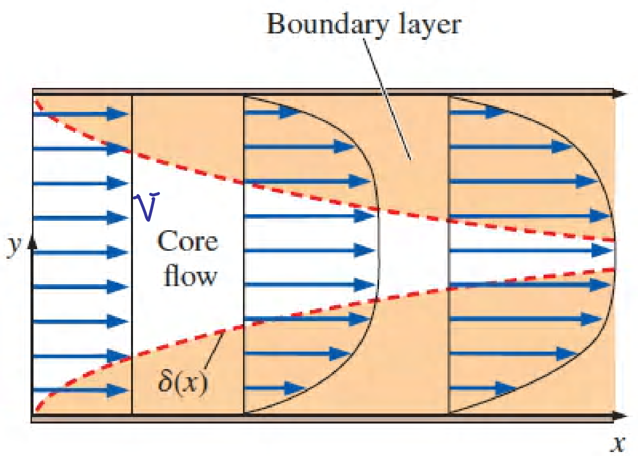
Apparent wall case:



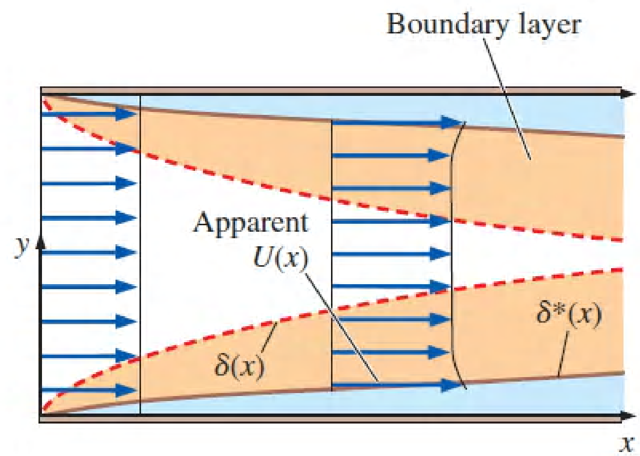
Practical example of the usefulness of displacement thickness: **Wind tunnel design**.

In these exaggerated drawings, as the BL grows along the walls of the wind tunnel, the speed in the core flow $U(x)$ must *increase* because the core flow "feels" like the wind tunnel walls are converging, due to the displacement thickness effect.

Actual wall case:

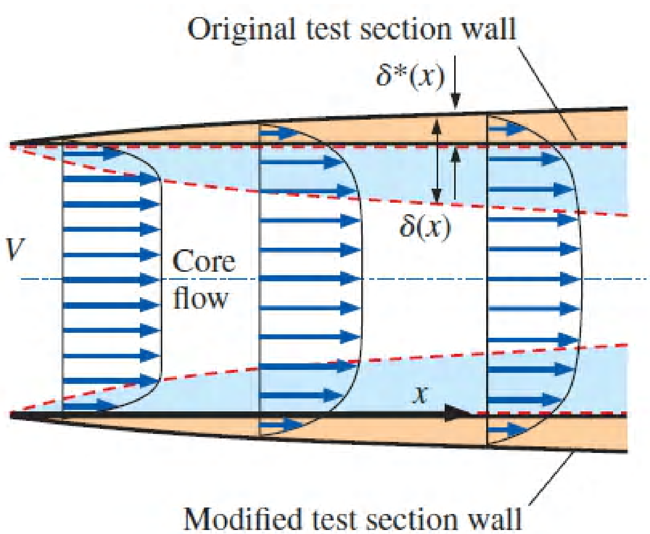


Apparent wall case:

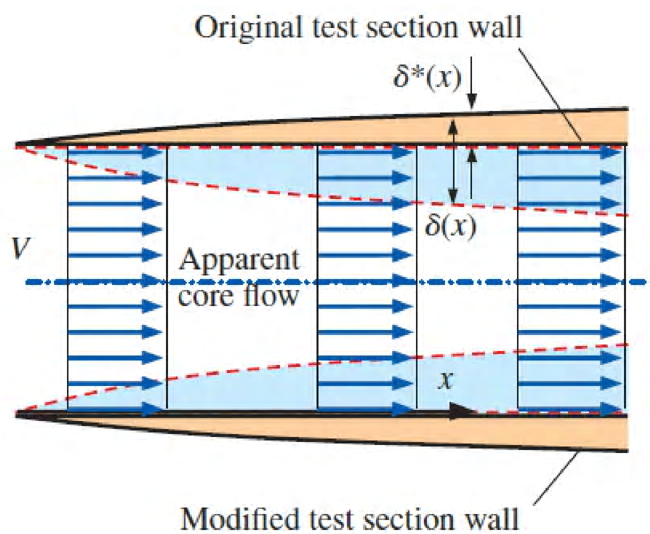


To avoid this effect, and to keep $U(x)$ constant, we would need to make the wind tunnel walls diverge out with downstream distance by the amount of the displacement thickness δ^*:

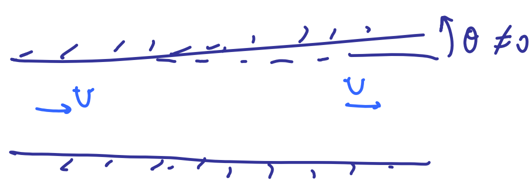
Actual wall case:



Apparent wall case:



Eg. my wind tunnel →



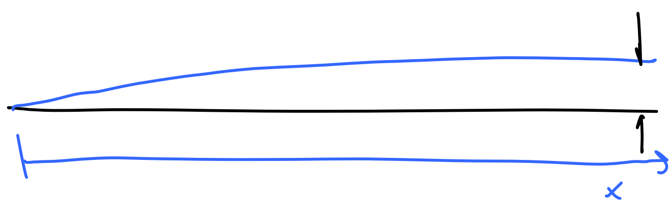
(I made the walls diverge slowly, according to $\delta^*(x)$ so that U remained constant down the entire test section)

- **Step 5** Verify that the BL is thin

$$\boxed{\frac{\delta}{x} = \frac{4.91}{\sqrt{Re_x}}} \longrightarrow \text{ when } \frac{\delta}{x} \ll 1 \text{ the BL is thin}$$

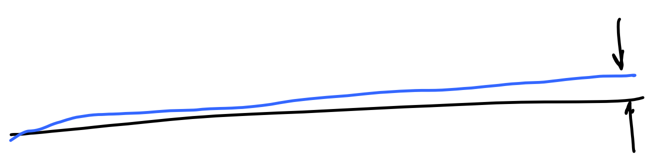
For $\frac{\delta}{x}$ to be small, Re_x must be big.

e.g. $Re \simeq 1000 \ (10^3)$, $\frac{\delta}{x} = \frac{4.91}{\sqrt{1000}} = 0.155$ not really small



$\delta \simeq 16\%$ of x

e.g. $Re = 100{,}000 \ (10^5)$ $\frac{\delta}{x} = \frac{4.91}{\sqrt{100000}} = 0.0155 \simeq 1.6\%$ very small



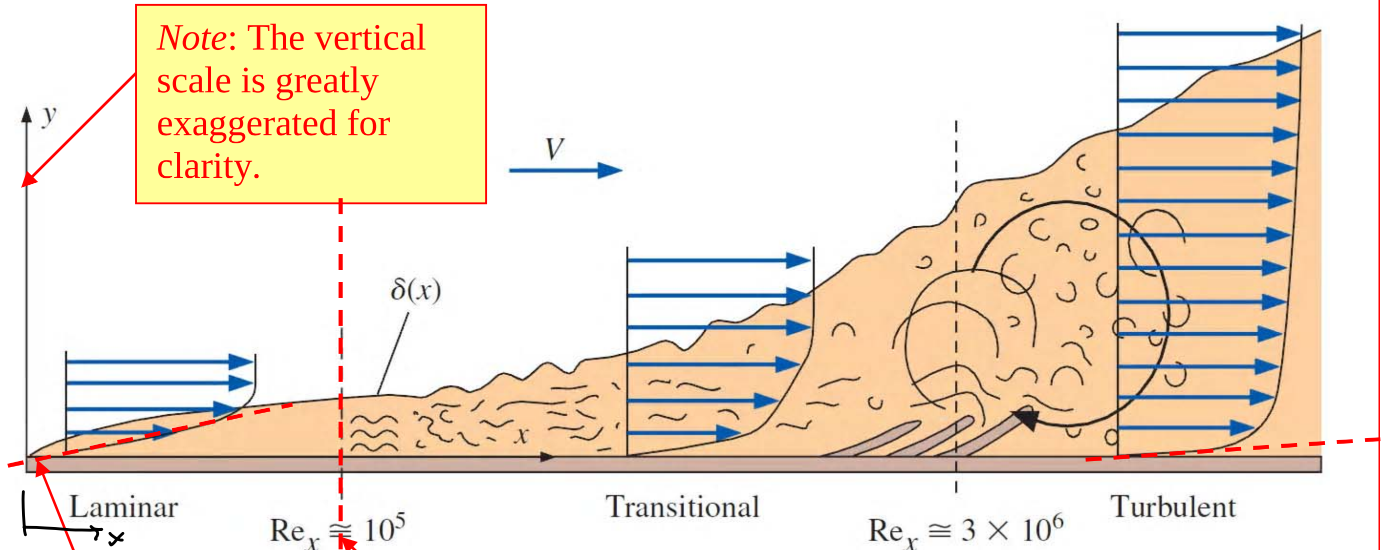
$\delta \simeq 1.6\%$ of x

Problem: · In order for BL approx to hold, Re_x must be big

· But if $Re_x \gtrsim 10^5$, BL begins to transition to turbulent flow

6. Turbulent Boundary Layer on a Flat Plate

Since $Re_x = Ux/\nu$ increases with x (distance down the plate), eventually Re_x gets big enough that the BL transitions from laminar to turbulent. Here is a schematic of the process:



Note: The vertical scale is greatly exaggerated for clarity.

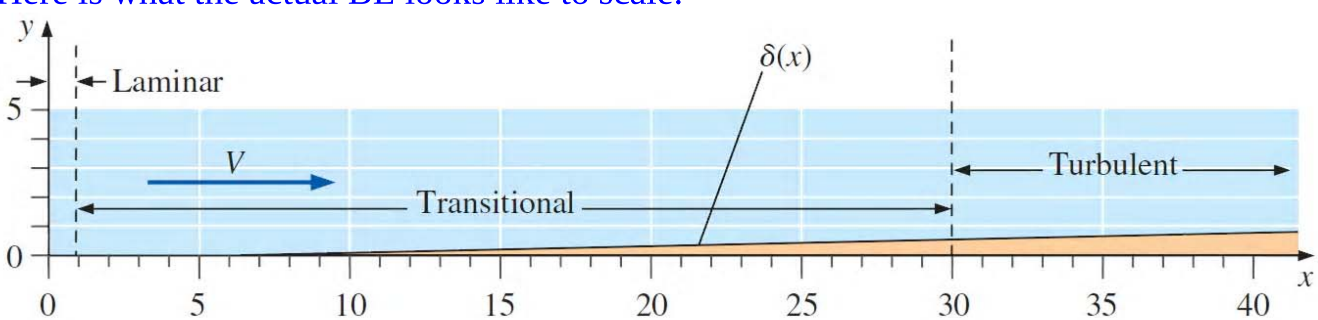
$\tau_w = \mu \partial u / \partial y$ at $y = 0$

$Re_{x,\,cr} = 5 \times 10^5$

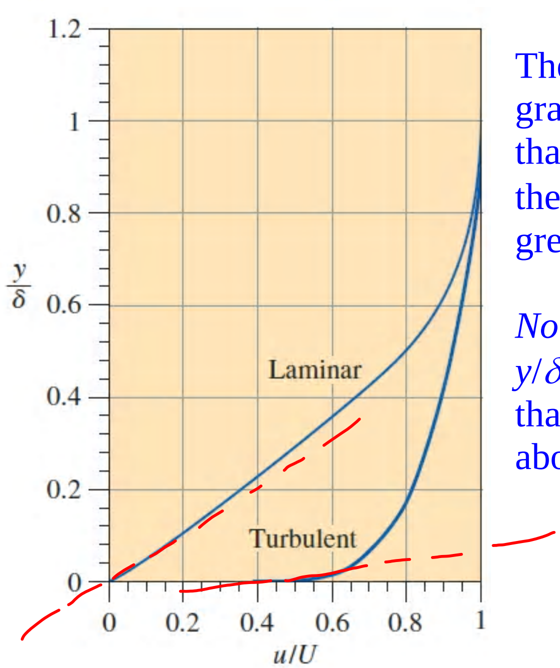
We define the engineering critical Reynolds number:

- $Re_x < Re_{x,\,cr}$ BL most likely laminar
- $Re_x > Re_{x,\,cr}$ BL most likely turbulent

Here is what the actual BL looks like to scale:



$Re_x = 3 \times 10^6$

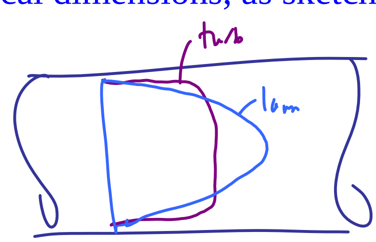


The time-averaged turbulent flat plate (zero pressure gradient) boundary layer velocity profile is much *fuller* than the laminar flat plate boundary layer profile, and therefore has a larger slope $\partial u / \partial y$ at the wall, leading to greater skin friction drag along the wall.

Note: This is a nondimensional plot in terms of u/U vs. y/δ. The actual turbulent BL is of course much *thicker* than the laminar one in physical dimensions, as sketched above.

Turb BL has much higher skin friction

similar to pipe flow



Quantities of interest for the turbulent flat plate boundary layer:

Just as we did for the laminar (Blasius) flat plate boundary layer, we can use these expressions for the velocity profile to estimate quantities of interest, such as the 99% boundary layer thickness δ, the displacement thickness δ^*, the local skin friction coefficient $C_{f,x}$, etc. These are summarized in Table 10-4 in the text.

Column (b) expressions are generally preferred for engineering analysis.

TABLE 10–4

Summary of expressions for laminar and turbulent boundary layers on a smooth flat plate aligned parallel to a uniform stream* (Empirical)

Property	Laminar	(a) Turbulent(†)	(b) Turbulent(‡)
Boundary layer thickness	$\frac{\delta}{x} = \frac{4.91}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta}{x} \cong \frac{0.16}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta}{x} \cong \frac{0.38}{(\mathrm{Re}_x)^{1/5}}$
Displacement thickness	$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta^*}{x} \cong \frac{0.020}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta^*}{x} \cong \frac{0.048}{(\mathrm{Re}_x)^{1/5}}$
Momentum thickness	$\frac{\theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$\frac{\theta}{x} \cong \frac{0.016}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\theta}{x} \cong \frac{0.037}{(\mathrm{Re}_x)^{1/5}}$
Local skin friction coefficient	$C_{f,x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$C_{f,x} \cong \frac{0.027}{(\mathrm{Re}_x)^{1/7}}$	$C_{f,x} \cong \frac{0.059}{(\mathrm{Re}_x)^{1/5}}$

Note that $C_{f,x}$ is the *local* skin friction coefficient, applied at only *one* value of x.

To these we add the integrated ***average skin friction coefficients*** for *one side* of a flat plate of length L, noting that C_f applies to the entire plate from $x = 0$ to $x = L$ (see Chapter 11):

Laminar:
$$C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}} \qquad \mathrm{Re}_L \lesssim 5 \times 10^5 \tag{11–19}$$

Turbulent:
$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \qquad 5 \times 10^5 \lesssim \mathrm{Re}_L \lesssim 10^7 \tag{11–20}$$

Turb. BL from the beginning (x=0)

For cases in which the laminar portion of the plate is taken into consideration, we use:

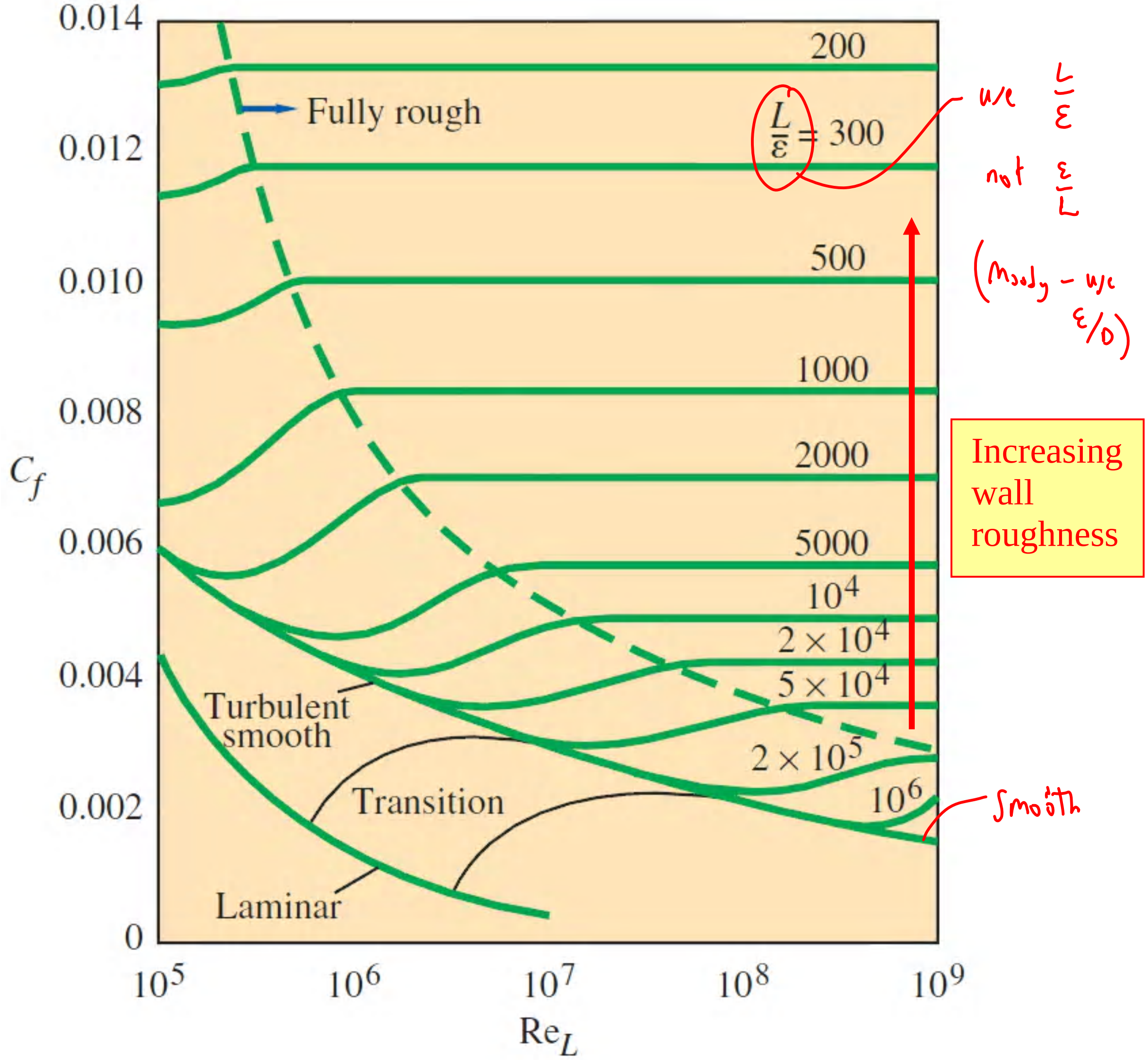
$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5 \times 10^5 \lesssim \mathrm{Re}_L \lesssim 10^7 \tag{11–22}$$

Use this one → more accurate, accounts for the laminar portion of the BL

Turbulent flat plate boundary layers with wall roughness:

Finally, all of the above are for *smooth* flat plates. However, if the plate is *rough*, the average skin friction coefficient C_f increases with roughness ε. This is similar to the situation in pipe flows, in which Darcy friction factor f increases with pipe wall roughness.

BL analogy to the Moody Chart



w/e $\frac{L}{\varepsilon}$ not $\frac{\varepsilon}{L}$

(Moody - w/e ε/D)

smooth

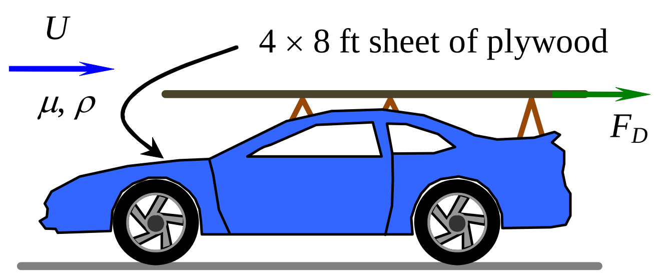
FIGURE 11–31
Friction coefficient for parallel flow over smooth and rough flat plates.

Just as with pipe flows, at high enough Reynolds numbers, the boundary layer becomes "fully rough". For a ***fully rough flat plate turbulent boundary layer*** with average wall roughness height ε,

Fully rough turbulent regime: $$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5} \qquad \textbf{(11–23)}$$

This equation represents the flat portions of Fig. 11-31 that are labeled "Fully rough".

Example: Drag on a sheet of plywood (continued)



Given: Craig buys a 4 × 8 ft sheet of plywood at Lowe's and mounts it on the top of his car. He drives (carefully) at 35 mph ≈ 51.3 ft/s. The air density and kinematic viscosity in English units are $\rho = 0.07518$ lbm/ft^3 and $\nu = 1.632\times10^{-4}$ ft^2/s, respectively.

To do: Estimate δ at the end of the plate ($x = L$) and the drag force on the plate for two cases: (a) smooth plate, turbulent BL, (b) rough plate ($\varepsilon = 0.050$ in.), turbulent BL.

Solution:

We solved this problem previously assuming that the BL remained *laminar*. We first solved for the Reynolds number at the end of the plate, $\mathrm{Re}_x = \mathrm{Re}_L$ (at x = L) = $UL/\nu = 2.515 \times 10^6$. Results: δ = 0.297 in., F_D = 0.165 lbf. Now let's repeat the calculations for a *turbulent* BL.

(a) Smooth ($\varepsilon = 0$) → Use column (b) eqs in Table 10-4

$$\frac{\delta}{x} = \frac{0.38}{(\mathrm{Re}_x)^{1/5}} \Rightarrow \delta = 0.1595 \text{ ft} \to \boxed{\delta \approx 1.9 \text{ inches}}$$

≈ 6.5 × greater than laminar case

$\mathrm{Re}_x = \mathrm{Re}_L$ @ end of plate

Total drag → $F_D = 2\,\frac{1}{2}\rho V^2 C_f A$ (2 sides) (A = 4 ft × 8 ft)

Use the "better, more accurate formula"

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \to \boxed{F_D = 0.63 \text{ lbf}}$$

≈ 4 × greater than laminar case

(b) Rough case ($\varepsilon = 0.05$ in.) $\frac{L}{\varepsilon} = \frac{96''}{0.05''} = 1920$ Look @ figure

$$C_f \approx 0.007 \to \boxed{F_D = 1.4 \text{ lbf}}$$

(2 × the smooth case)

Roughness is very important for turbulent BL's

7. Boundary Layers with Pressure Gradients

So far, flat plate BL $P=\text{const}$, $U=V=\text{const}$

a. Some definitions

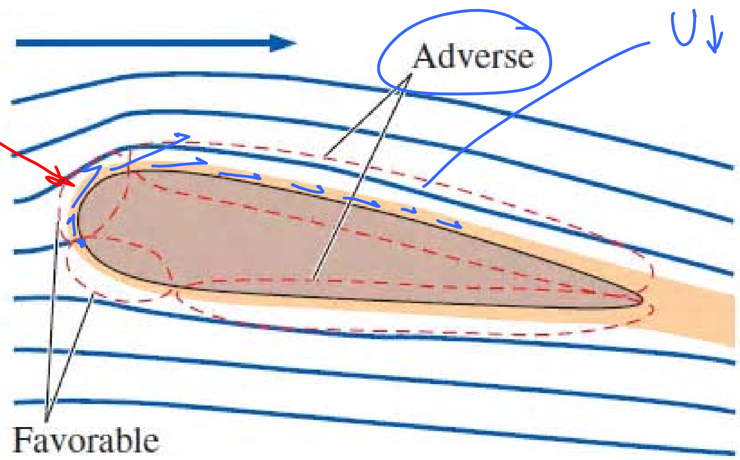
For a flat plate, P = constant, and thus $dP/dx = 0$. We call this a **zero-pressure gradient**.
For most real-life flows, $P \neq$ constant, and thus $dP/dx \neq 0$. Two possibilities:

- If $dP/dx < 0$, $dU/dx > 0$, the flow is *accelerating*. This is a **favorable pressure gradient**.
- If $dP/dx > 0$, $dU/dx < 0$, the flow is *decelerating*. This is an **unfavorable pressure gradient** (also called an **adverse pressure gradient**).

$U\uparrow$, $P\downarrow$ (Favorable)

Stag. pt: $U=0$

$U\downarrow \therefore P\uparrow$



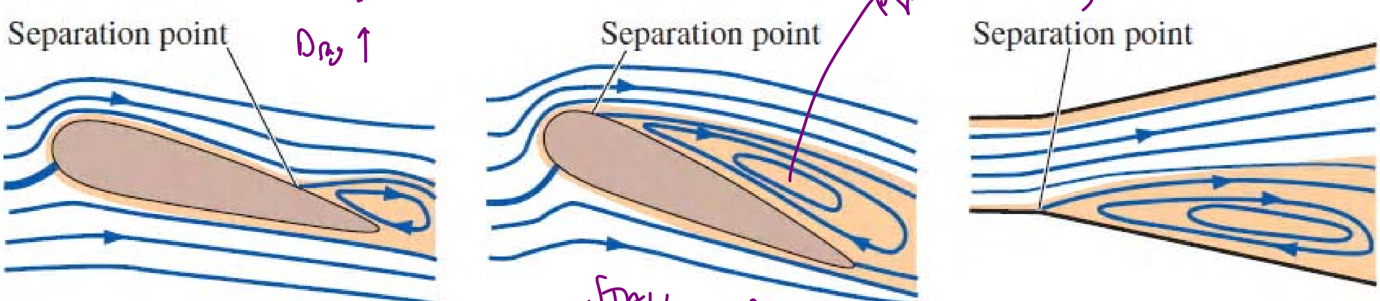
Unfavorable because $P\uparrow$ with x $\therefore$ flow is more likely to separate

Adverse pressure gradients are *unfavorable* in the sense that they are *more likely to separate*.

Examples:

Lift ↓ Drag ↑

(separation bubble)

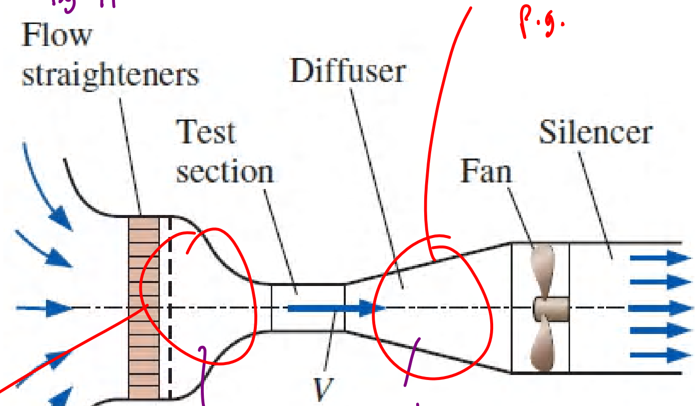


STALL

Lift ↓↓ Drag ↑↑

adverse p.g.

In most practical applications, we design so as to avoid flow separation if possible. For example, in a wind tunnel, the contraction has a *favorable* pressure gradient and flow separation is not likely. Therefore we typically design a very *rapid* contraction. However, the diffuser section has an *unfavorable* (adverse) pressure gradient, and is much more likely to separate. Therefore we design the diffuser with a much *slower* (smaller angle) divergence.



favorable p.g.

contraction → angle can be large

angle must be small

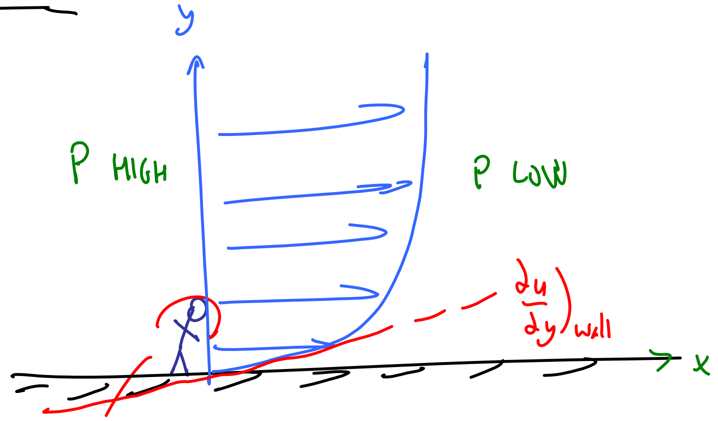
b. Physical explanation for flow separation

Favorable P.G.

$$\boxed{\frac{dp}{dx} < 0}$$

U↑ P↓

(accelerating flow)



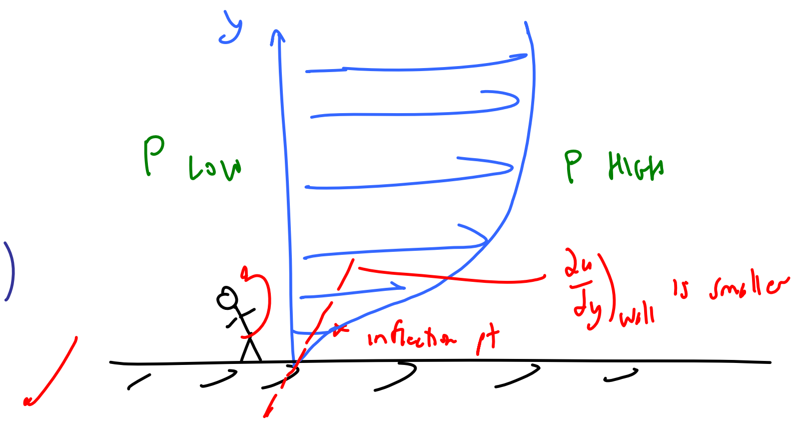
BL likes to "hug" the ground (stay attached)

ADVERSE PG

$$\boxed{\frac{dp}{dx} > 0}$$

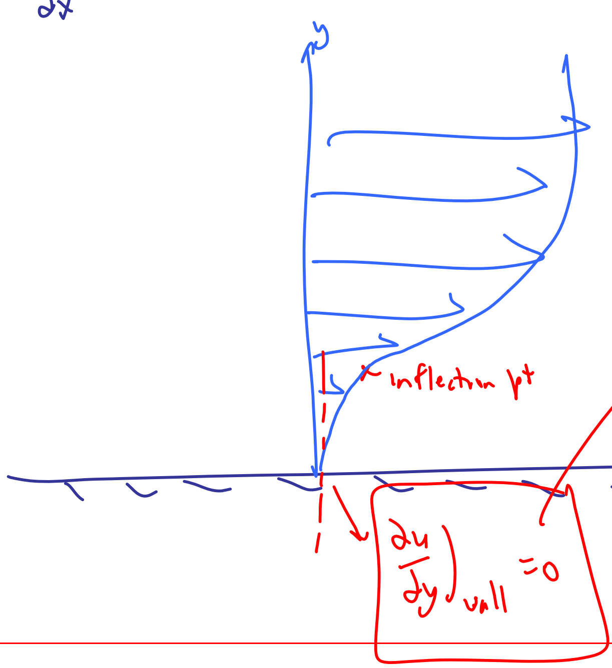
U↓ P↑

(decellerating flow)



BL does not like to "hug" the wall — wants to separate

If $\frac{dp}{dx}$ is big enough, the flow will separate



Let's define this as the separation pt.

→ i.e. where $\left.\frac{\partial u}{\partial y}\right)_{wall} = 0$