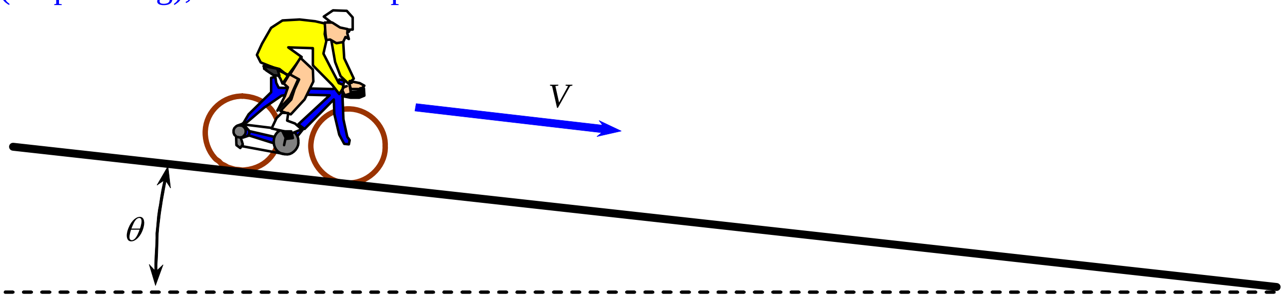
**Today, we will**:

- Finish external flow; some examples of lift and drag (finish Chapter 11)
- Start Chapter 12 – Compressible Flow

## Example – Drag on a Bicycle Rolling Down a Hill

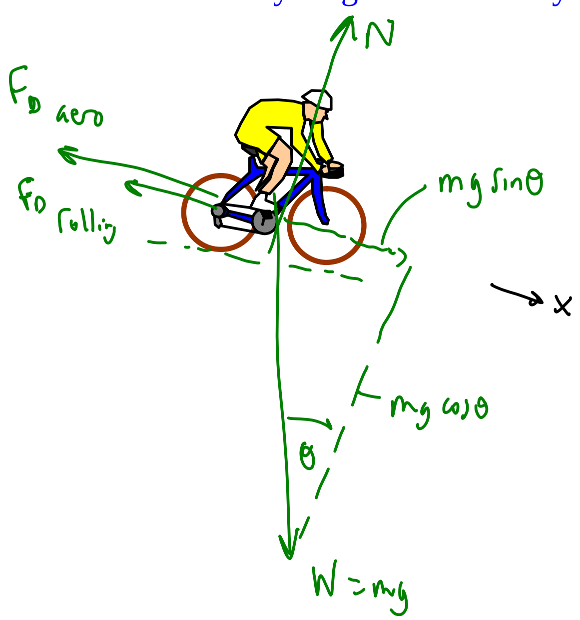
**Given**: A person coasts a bicycle down a long hill with a slope of 5° in order to measure the drag area of the bike and rider. The mass of the bike is 7.0 kg, the mass of the rider is 70.0 kg, and the rolling resistance of the bike is measured separately – it is 19.0 N. When the rider coasts down the hill (no pedaling), the terminal speed is 10.1 m/s.




(**a**) **To do**: Calculate the drag area $C_DA$ of the rider/bicycle combination in m².

**Solution**: (to be completed in class)

First draw a free-body diagram of the bicycle and rider, showing all forces acting.




No accel. → $\sum \vec{F} = 0$

in x-direction

$$F_{D_{aero}} + F_{D_{rolling}} = mg\sin\theta$$

$$\downarrow$$

$$\frac{1}{2}\rho V^2 C_DA + F_{D_{rolling}} = mg\sin\theta$$

Solve for this

$$C_DA = \text{"drag area"} = \frac{m g \sin\theta - F_{D,rolling}}{\frac{1}{2}\rho V^2}$$

5°

70 + 7 = 77 kg

1.204 kg/m³

$$C_D A = \frac{(70+7\ \text{kg})(9.807\ \text{m/s}^2)\left[\frac{\text{N}}{\text{kg}\cdot\text{m/s}^2}\right]\sin(5^\circ) - 19.0\ \text{N}}{\frac{1}{2}(1.204\ \text{kg/m}^3)\left(10.1\ \frac{\text{m}}{\text{s}}\right)^2\left[\frac{\text{N}}{\text{kg}\cdot\text{m/s}^2}\right]}$$

$$= \boxed{C_D A = 0.763\ \text{m}^2}$$

**(b) To do**: Calculate how much power in Watts (to the wheel) it would take for the person to ride this bike on a level road at the same speed (10.1 m/s).

$\rho = 1.204\ \text{kg/m}^3$ $\quad m = 77\ \text{kg}$

**Solution**: (to be completed in class)

Equation: $\dot{W} = \mu_{\text{rolling}} W V + \frac{1}{2}\rho V^3 C_D A$

$F_{D,\text{rolling}} = 19.0\ \text{N}$

#'s $= \boxed{665.\ \text{W}}$

Details:

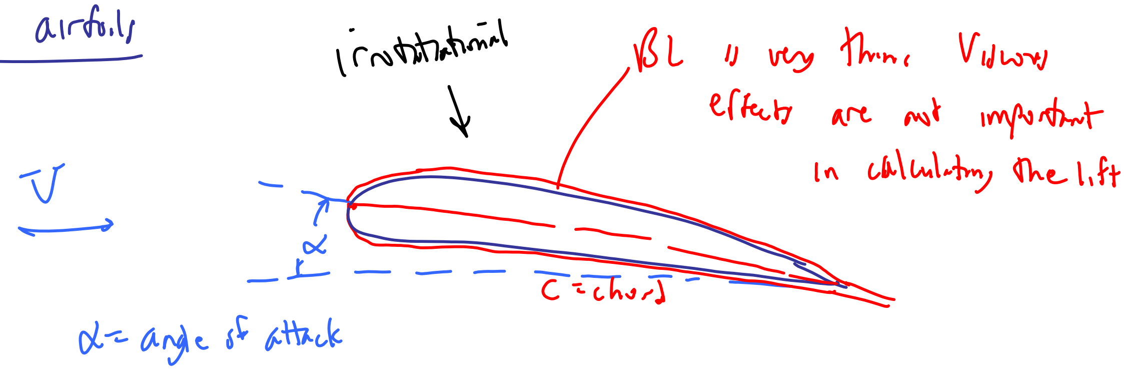
$$\dot{W} = F_{D,\text{rolling}} \cdot V + \frac{1}{2}\rho V^3 C_D A$$

$$= \left\{(19.0\ \text{N})\left(10.1\ \frac{\text{m}}{\text{s}}\right) + \frac{1}{2}\left(1.204\ \frac{\text{kg}}{\text{m}^3}\right)\left(10.1\ \frac{\text{m}}{\text{s}}\right)^3(0.763\ \text{m}^2)\left[\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right]\right\}\left[\frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}}\right]$$

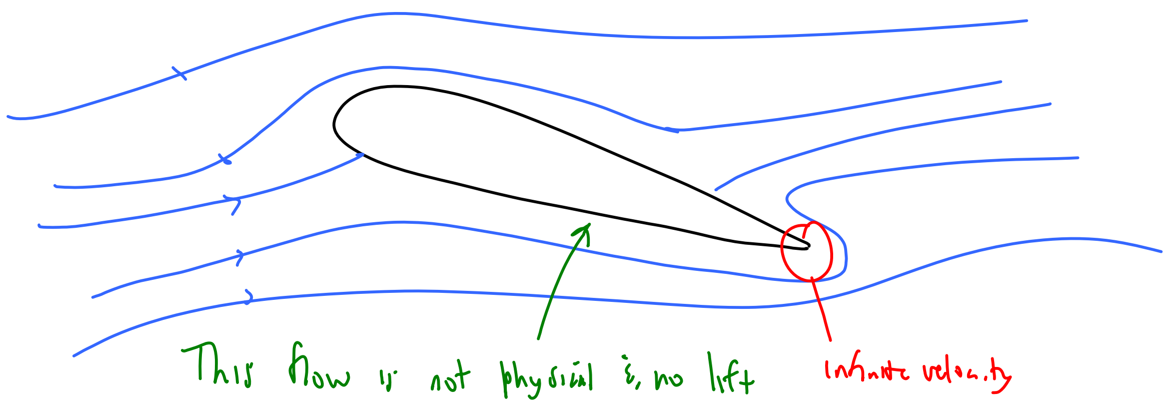
$$= \boxed{665.\ \text{W}}$$

## B. Lift on Bodies

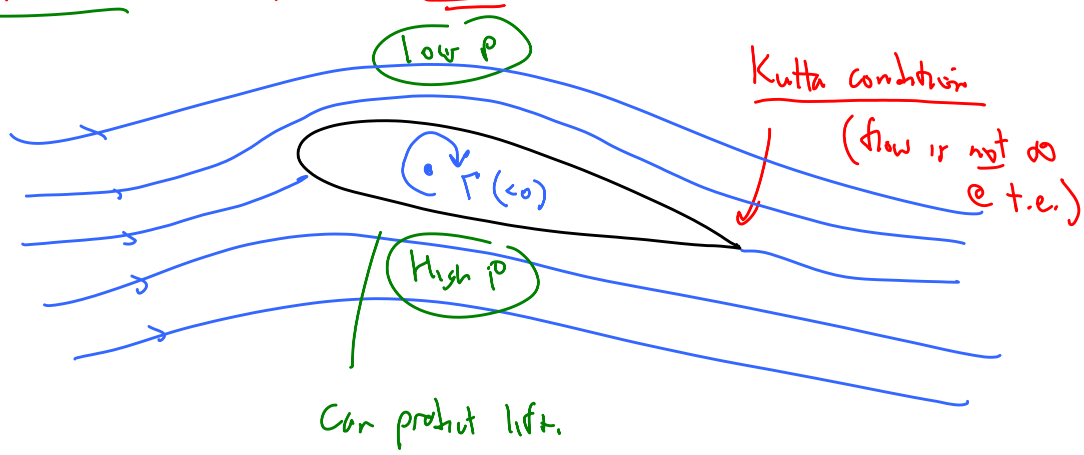
### 1. 2-D airfoils

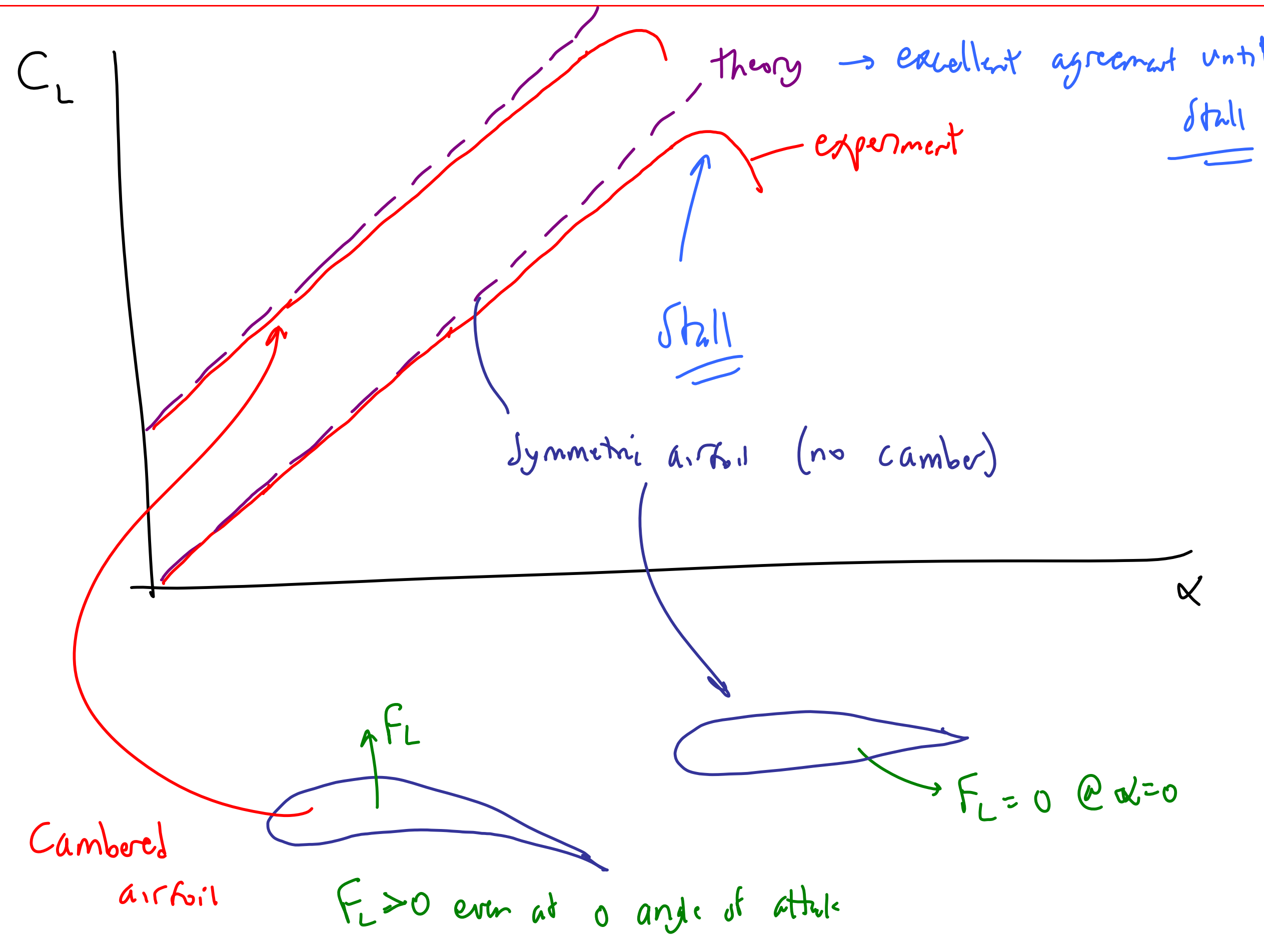



We can use irrotational (potential) flow to predict the lift




Fix this problem: superpose a vortex also






No drag for 2-D airfoils (via potential flow theory)

↓

Must add a BL & solve the BL eqs to predict the drag.

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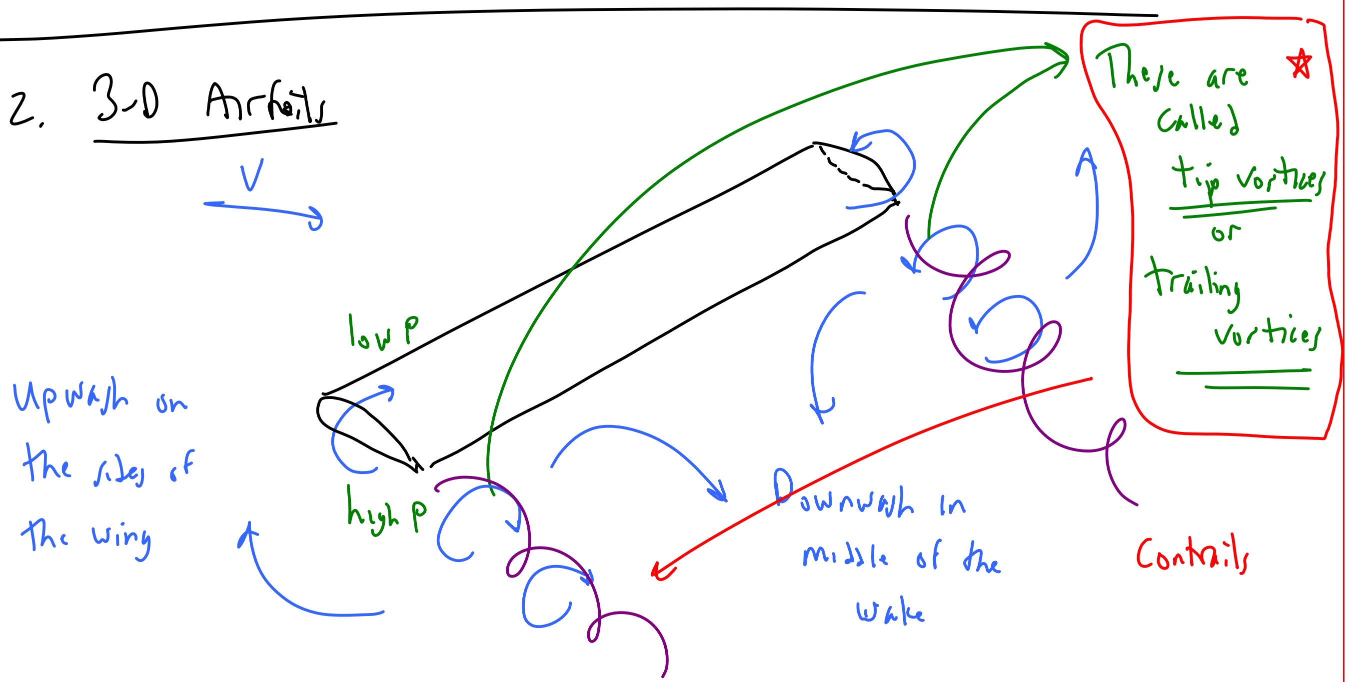
2. <u>3-D Airfoils</u>



The trailing vortices cause additional <u>drag</u> (waste power)

$\boxed{\text{INDUCED DRAG}}$ ✳ induced by the lift

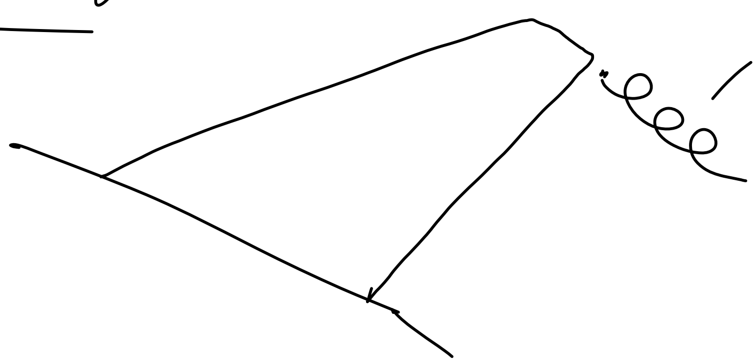
Also leads to <u>accidents</u> (Small plane gets caught in the tip vortices of a big plane)

Can take advantage of the upwash — e.g. geese & jet fighters in formation

See pics of geese & jets in formation "V" pattern

How can we <u>reduce</u> the tip vortices?

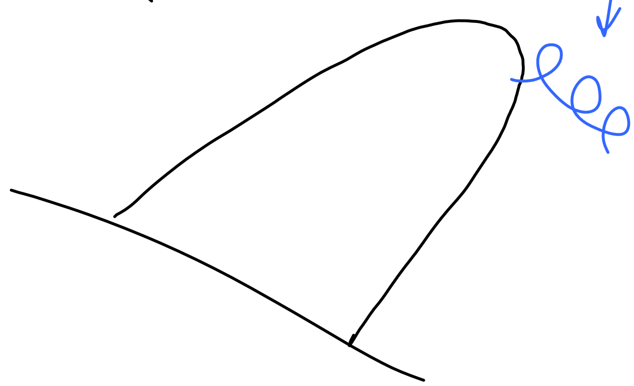
- <u>Taper the wings</u>



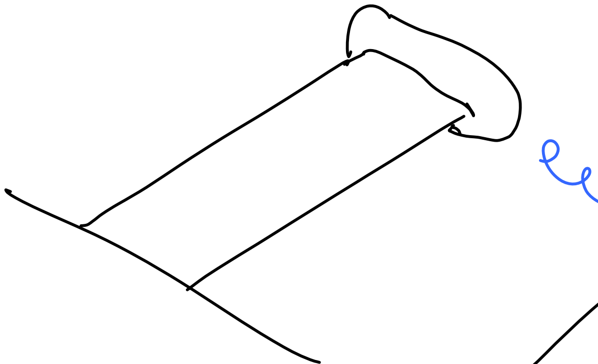
t.e. vortex

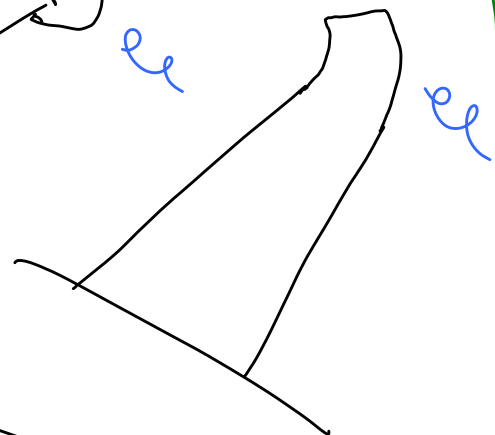
tip vortex is weaker

- <u>Round off the tip</u>

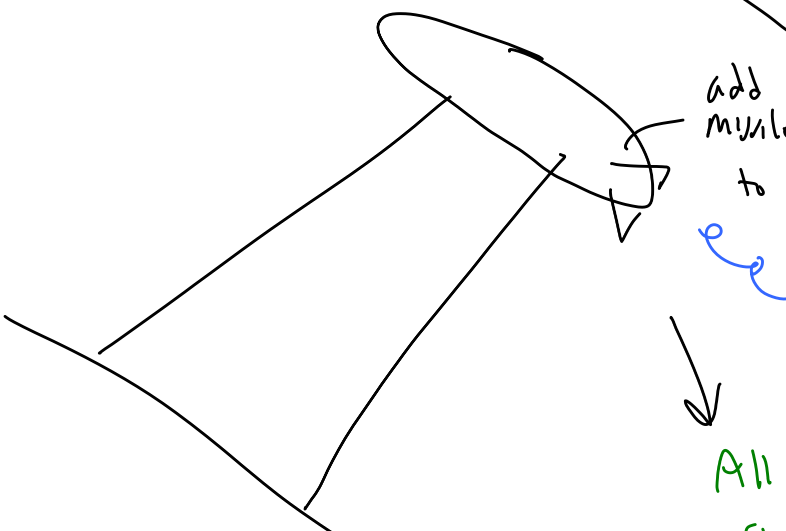


- Add endplates or winglets





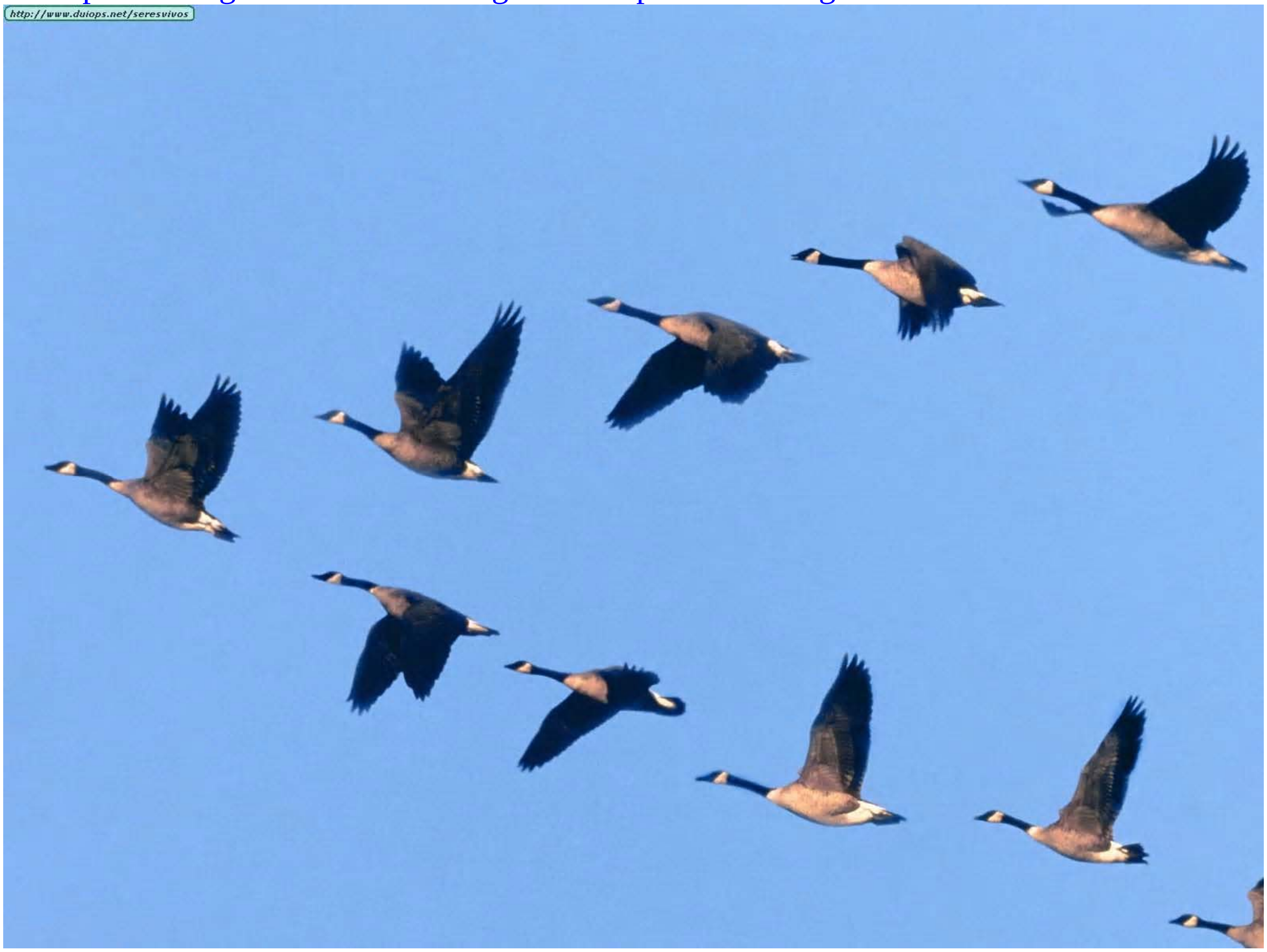
[modern airplanes use this]



add a missile or fuel tank to act like a wing tip

All of these reduce the strength of the tip vortices & reduce induced drag (but they do not totally eliminate induced drag)

Example: Each goose takes advantage of the upwash of the goose in front of him.




Fighter pilots also do the same thing:

# X. Ch. 12 Intro. to Compressible Flow

## A. Intro

– incompressible is good approx for liquids
& gases at low speed

### 1. Definitions & Review

a. Mach Number $\boxed{Ma = \frac{V}{c}}$ ☆ most important dimensionless parameter

speed of sound

recall, if $Ma \lesssim 0.3$ → "incompressible" → $< 5\%$ error in neglecting $\Delta\rho$

at room temperature

$$\boxed{\begin{aligned} c \simeq\ & 1100\ \text{ft/s} \\ & 750\ \text{mph} \\ & 335\ \text{m/s} \end{aligned}}$$

0.3 c (incompressible limit)

330 ft/s

225 mph

100.5

→ $\simeq \boxed{100\ \text{m/s}}$

Regimes:

$Ma \lesssim 0.3$ – "incompressible"

$0.3 \lesssim Ma < 1.0$ – Subsonic

$Ma > 1.0$ – Supersonic

$Ma$ near 1.0 → transonic (some subsonic & some supersonic regions in the flow)

$Ma = 1.0$ – Sonic

$Ma \gtrsim 3$ – hypersonic

b. Thermo. relations for an ideal gas

$$\boxed{P = \rho R T}$$ eq of state

$R$ = specific gas const

$$\boxed{R = \frac{R_u}{M}}$$ — universal gas const ($R_u$) — molecular weight ($M$)

$$\boxed{k = \frac{c_p}{c_v}}$$ = ratio of specific heats

★ $\boxed{k = 1.4}$ for air = constant

[some books use $\gamma$ instead of $k$]

$c_p$ = specific heat @ const Pressure
$c_v$ = " " " " volume

$$\boxed{R = c_p - c_v}$$

$$\boxed{c = \sqrt{kRT}}$$ for an ideal gas

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For isentropic flow (neglect friction & other losses)

$$\boxed{\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}} \qquad \boxed{\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{k-1}}}$$

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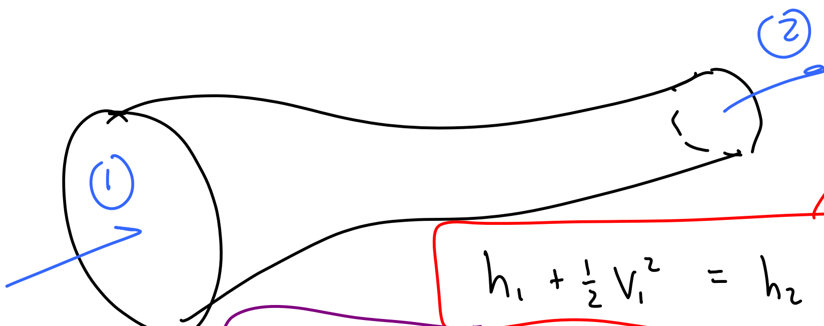
Energy eq. for flow through a duct (cons. of energy)




★ $$\boxed{h_1 + \tfrac{1}{2}V_1^2 = h_2 + \tfrac{1}{2}V_2^2}$$

$h_0$ = constant in a duct flow with no heat transfer

$$\boxed{h_0 = \text{stagnation enthalpy} \equiv h + \tfrac{1}{2}V^2}$$

ideal gas $\rightarrow$ $\boxed{h = c_p T}$ (+ const)

$T_0$ = stagnation Temperature $\rightarrow$ The temp you get when the flow is brought to rest isentropically

$$\boxed{h_0 = C_p T_0}$$

stagnation enthalpy

stagnation temperature

So, for flow in a duct,

$$h_0 = h + \frac{1}{2}V^2 = \text{constant}$$

$$C_p T_0 = C_p T + \frac{1}{2}V^2 = \text{constant}$$

$\div C_p T$ & use $c_p = \dfrac{kR}{k-1}$ & $c = \sqrt{kRT}$

some algebra

$$\boxed{\frac{T_0}{T} = 1 + \frac{k-1}{2} Ma^2}$$ ☆

Relationship between $T_0$ & $T$ for flow of an ideal gas