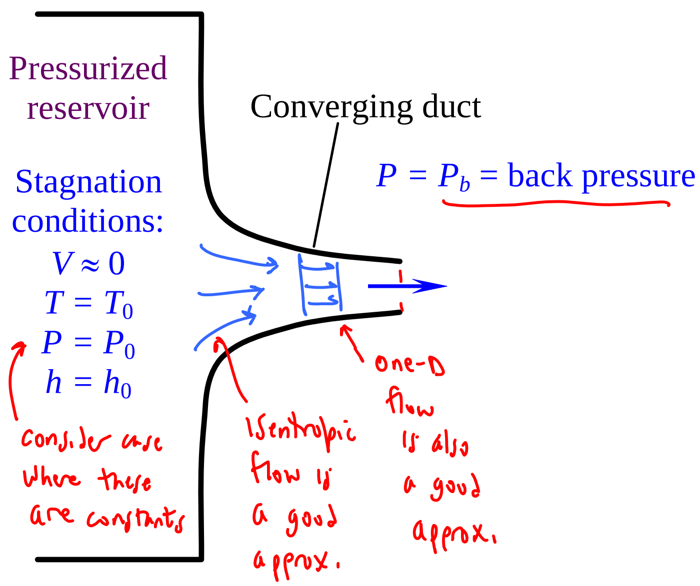
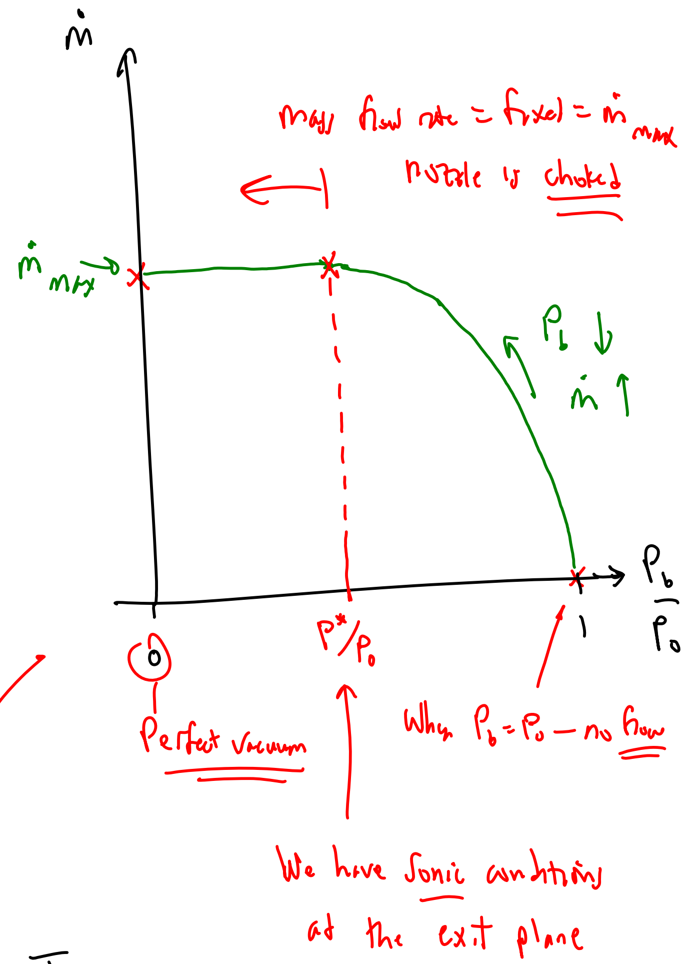
Today, we will:

- Finish discussing choked flow and its implications
- Discuss shock waves, particularly *normal* shock waves

Consider a large tank with a converging duct that *ends at the throat* (no diverging part):





Let's vary P_b & see what happens

When the flow is CHOKED, we can calc. T

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} Ma^2 \quad \rightarrow \quad @ \ Ma = 1 \ (\text{sonic}), \quad \frac{T_0}{T} = 1.2 \ (\text{for air})$$

$$\therefore \ T = \frac{T_0}{1.2} \quad \text{if } T_0 = 300 \text{ K} \rightarrow T = \frac{300}{1.2} = 250 \text{ K} \cong -23^\circ\text{C}$$

cold!

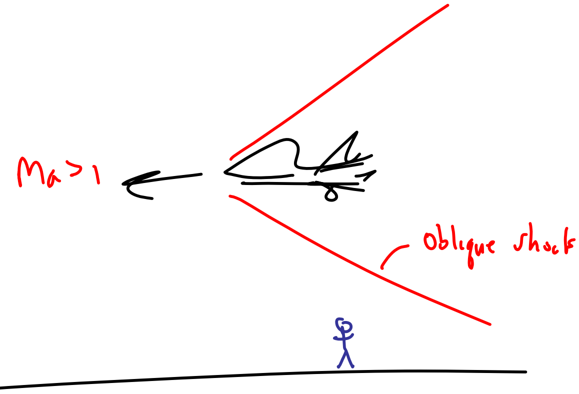
★ If we add a diverging section, we can get supersonic flow

(This is the only way to achieve supersonic flow in a wind tunnel, for example)

B. Shock Waves → a strong pressure wave

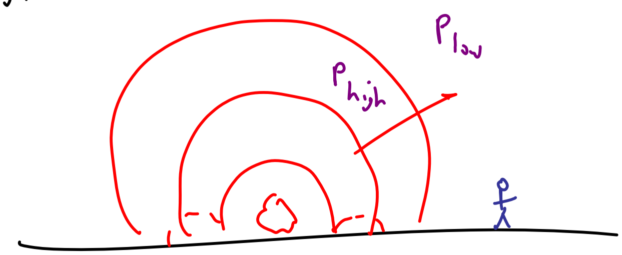
e.g.'s

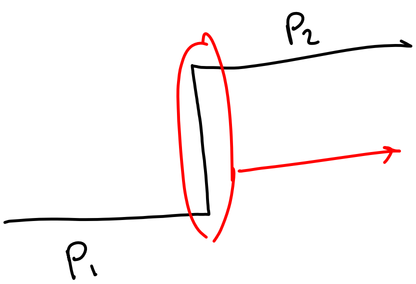
sonic boom



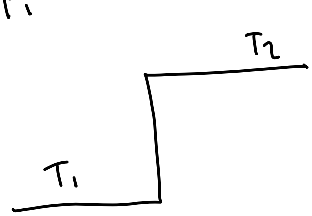
whip cracking (towel)

bomb blast



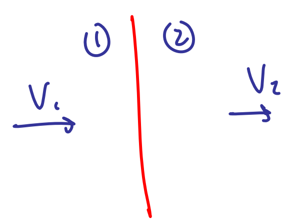


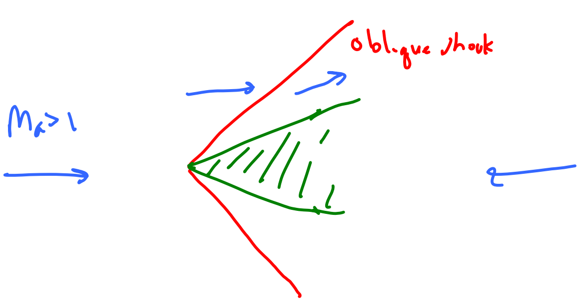
Thickness of shock is very small ~ 1 μm (10^{-6} m in air)



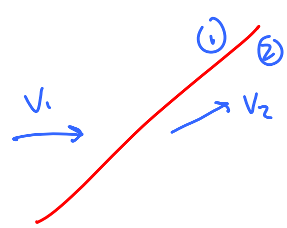
Classification of shock waves

1) normal shock



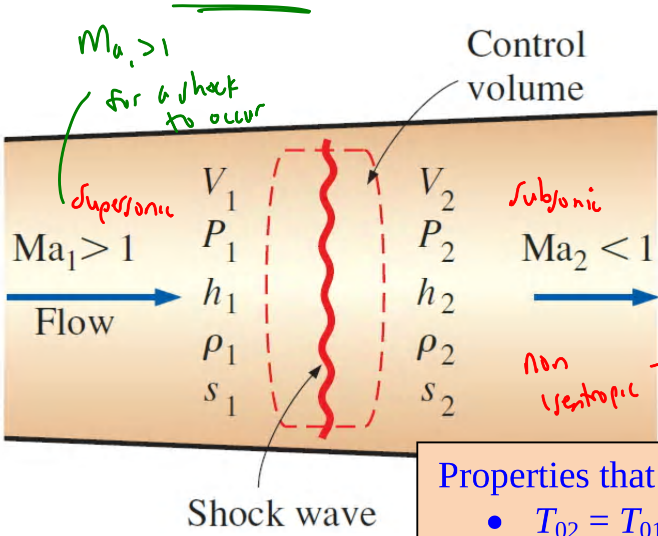


2) Oblique shock



The "dime" analogy

Consider a *stationary* normal shock wave (as in a supersonic wind tunnel)

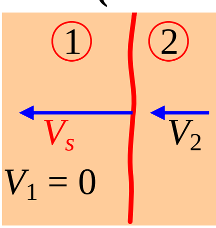


Properties that ***increase*** across the shock:
- $P_2 > P_1$
- $T_2 > T_1$, thus:
  - o $c_2 > c_1$
  - o $h_2 > h_1$
- $\rho_2 > \rho_1$
- $s_2 > s_1$
- $A_2^* > A_1^*$

Properties that ***decrease*** across the shock:
- $Ma_2 < Ma_1$
- $P_{02} < P_{01}$
- $\rho_{02} < \rho_{02}$
- $V_2 < V_1$

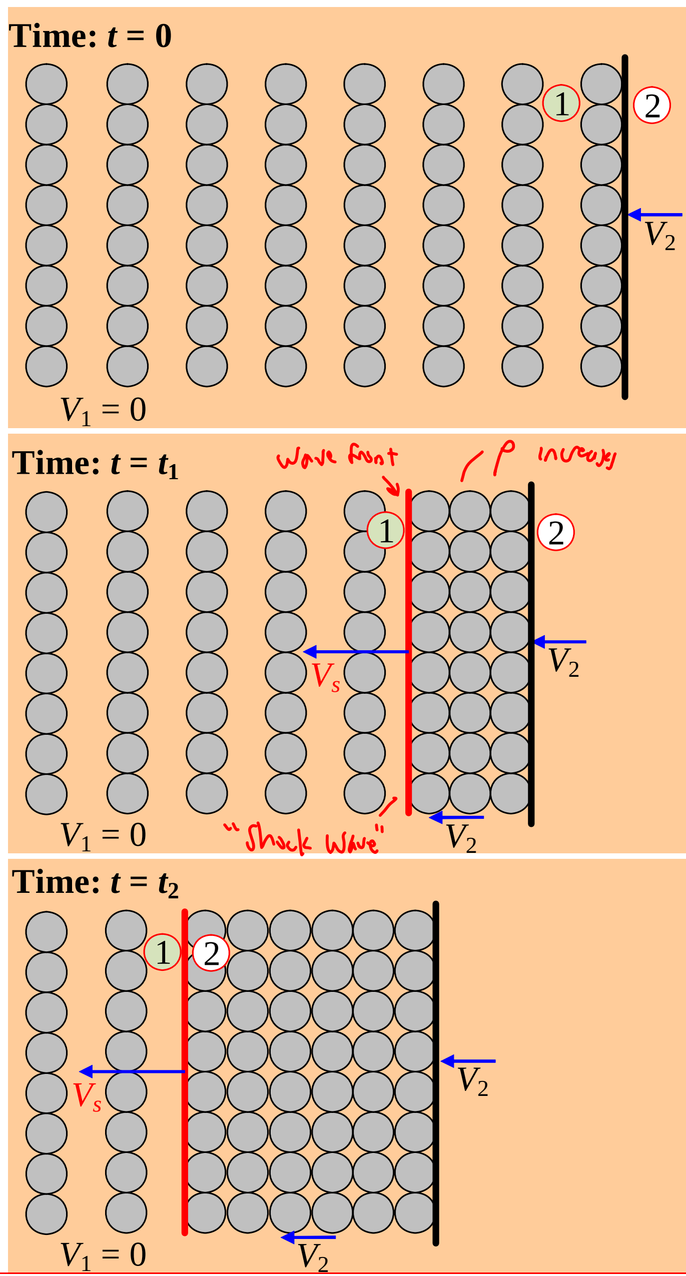
Properties that ***stay the same*** across the shock:
- $T_{02} = T_{01}$
- $h_{02} = h_{01}$

Consider instead a *moving* normal shock wave (as in a blast wave from an explosion)



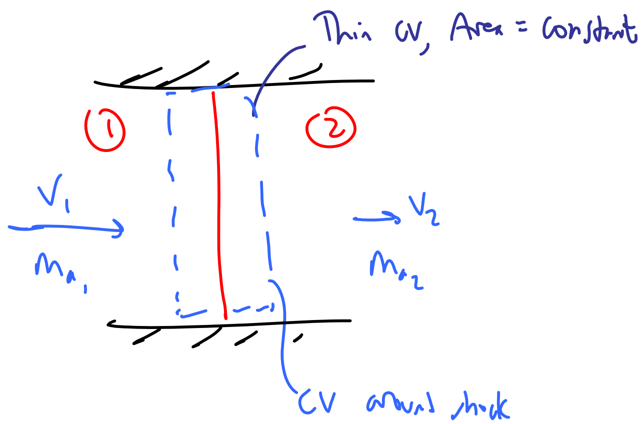
- The shock is moving into quiescent air (region 1)
- In this frame of reference we define $Ma_1 = V_s / c_1$
- The shock wave travels into region 1 at supersonic speed ($Ma_1 > 1$)
- The air behind the shock (region 2) follows at a slower speed

The "dime analogy" (model the moving shock as rows of dimes that pile up when pushed by a rod or "piston" as sketched; three sequential times):



Comments:
- The vertical red line is analogous to a shock wave: $V_1 = 0$, $V_s > V_2$, $\rho_2 > \rho_1$ (there is sudden increase in density, and the "wave front" of dimes moves faster than the piston).
- The dimes in region 1 don't "know" anything is happening until the shock hits them.
- Similarly in a shock wave in air, the air in region 1 does not "know" anything is happening until the shock wave hits it.

Eq'ns for a normal shock (stationary)



Cons. of mass $\boxed{\rho_1 V_1 = \rho_2 V_2}$ $(A_1 = A_2)$

x-mom $\boxed{P_1 - P_2 = \rho_2 V_2^2 - \rho_1 V_1^2}$

Cons. of energy $\boxed{h_0 = h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2 = \text{const}}$

$\boxed{h_{0_1} = h_{0_2} \;,\; T_{0_1} = T_{0_2}}$

Ideal gas $\boxed{P = \rho R T}$, $\boxed{h = c_p T}$, $\boxed{h_0 = C_p T_0}$

$R = \text{const.} \rightarrow$ $\boxed{\dfrac{P_1}{\rho_1 T_1} = \dfrac{P_2}{\rho_2 T_2}}$

In compressible flow analysis, the key is to use $\boxed{\text{RATIOS}}$ ☆

e.g. $$\frac{T_2}{T_1} = \frac{T_2}{T_{0_2}} \; \frac{T_{0_2}}{T_{0_1}} \; \frac{T_{0_1}}{T_1}$$

$$\left[1 + \frac{k-1}{2} Ma_2^2\right]^{-1} \left[1\right] \left[1 + \frac{k-1}{2} Ma_1^2\right]$$

across a normal shock

$$\boxed{\frac{T_2}{T_1} = \frac{1 + \frac{k-1}{2} Ma_1^2}{1 + \frac{k-1}{2} Ma_2^2}}$$

Similarly for $\frac{P_2}{P_1}$, $\frac{\rho_2}{\rho_1}$, etc. across a shock wave

(See next pg)

Normal Shock Equations (1 = upstream, 2 = downstream of stationary shock):

$$T_{01} = T_{02}$$

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - k + 1}}$$

$$\frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{2k\mathrm{Ma}_1^2 - k + 1}{k + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)}$$

$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1 + \mathrm{Ma}_2^2(k-1)/2}{1 + \mathrm{Ma}_1^2(k-1)/2}\right]^{(k+1)/[2(k-1)]}$$

$$\frac{P_{02}}{P_1} = \frac{(1 + k\mathrm{Ma}_1^2)[1 + \mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\mathrm{Ma}_2^2}$$

TABLE A–14

One-dimensional normal shock functions for an ideal gas with $k = 1.4$

For air

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.8929
1.1	0.9118	1.2450	1.1691	1.0649	0.9989	2.1328
1.2	0.8422	1.5133	1.3416	1.1280	0.9928	2.4075
1.3	0.7860	1.8050	1.5157	1.1909	0.9794	2.7136
1.4	0.7397	2.1200	1.6897	1.2547	0.9582	3.0492
1.5	0.7011	2.4583	1.8621	1.3202	0.9298	3.4133
1.6	0.6684	2.8200	2.0317	1.3880	0.8952	3.8050
1.7	0.6405	3.2050	2.1977	1.4583	0.8557	4.2238
1.8	0.6165	3.6133	2.3592	1.5316	0.8127	4.6695
1.9	0.5956	4.0450	2.5157	1.6079	0.7674	5.1418
2.0	0.5774	4.5000	2.6667	1.6875	0.7209	5.6404
2.1	0.5613	4.9783	2.8119	1.7705	0.6742	6.1654
2.2	0.5471	5.4800	2.9512	1.8569	0.6281	6.7165
2.3	0.5344	6.0050	3.0845	1.9468	0.5833	7.2937
2.4	0.5231	6.5533	3.2119	2.0403	0.5401	7.8969
2.5	0.5130	7.1250	3.3333	2.1375	0.4990	8.5261
2.6	0.5039	7.7200	3.4490	2.2383	0.4601	9.1813
2.7	0.4956	8.3383	3.5590	2.3429	0.4236	9.8624
2.8	0.4882	8.9800	3.6636	2.4512	0.3895	10.5694
2.9	0.4814	9.6450	3.7629	2.5632	0.3577	11.3022
3.0	0.4752	10.3333	3.8571	2.6790	0.3283	12.0610
4.0	0.4350	18.5000	4.5714	4.0469	0.1388	21.0681
5.0	0.4152	29.000	5.0000	5.8000	0.0617	32.6335
∞	0.3780	∞	6.0000	∞	0	∞

The bigger Ma_1 is, the stronger the shock

Notes about normal shocks

1) shocks are <u>only</u> from supersonic to subsonic

2) shocks cause <u>compression</u> $P_2 > P_1$ $T_2 > T_1$ $\rho_2 > \rho_1$

3) Total pressure $\underline{P_{0_2} < P_{0_1}}$ but $\underline{T_{0_1} = T_{0_2}}$

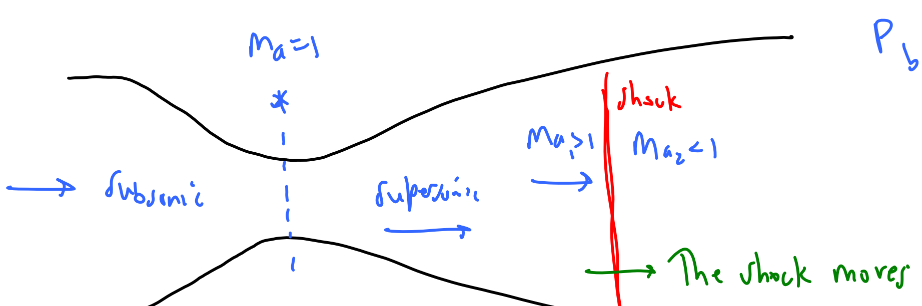
4) Shock waves are not isentropic $\Delta s > 0$

5) "Weak shocks" → $Ma_1 \approx 1.0$ (e.g. 1.0001)
↘ <u>sound wave</u> or acoustic wave

6) "Strong shocks" → $Ma_1 \gtrsim 2$

We can generate ^stationary shocks in the lab using a converging-diverging nozzle

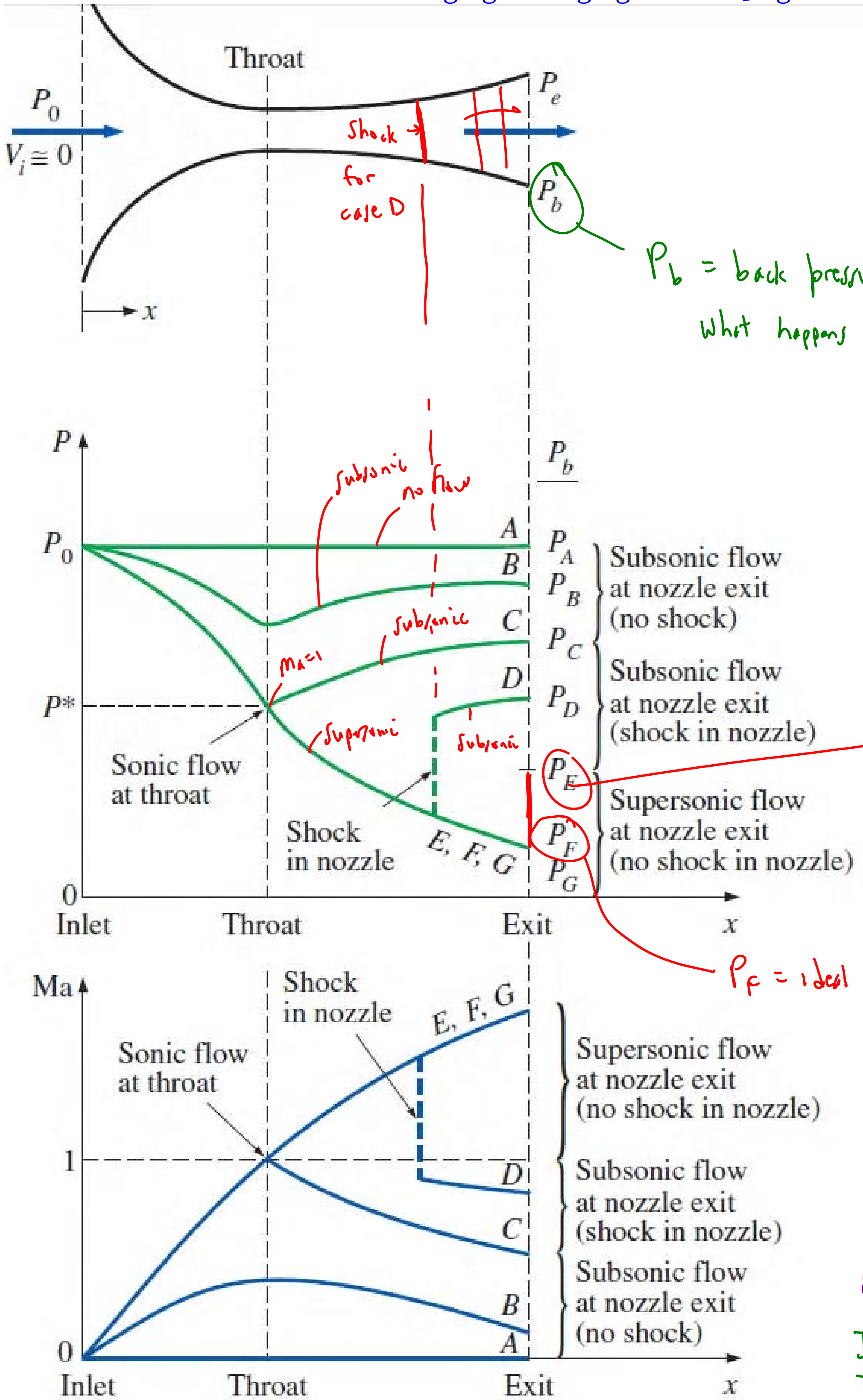
CONVERGING - DIVERGING NOZZLE



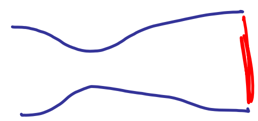
→ The shock moves farther downstream as back pressure P_b is decreased

(See next pg. for full description of P_b decreased)

Effect of Back Pressure in a Converging-Diverging Nozzle: [Fig. 12-22 in Ed 2 of textbook]



P_b = back pressure. We look at what happens as P_b decreases



Shock @ exit plane

P_F = ideal or design case

If $P_F < P_b < P_E$

The flow is overexpanded

get shock diamonds →

If $P_b < P_F$

The flow is underexpanded

get flare out →

(See pictures) next pg.

Overexpanded nozzles:

Example – High speed jet aircraft

Given: The SR-71 travels at Ma = 3.2 at 24 km altitude (80,000 ft).

To do: Calculate its air speed.

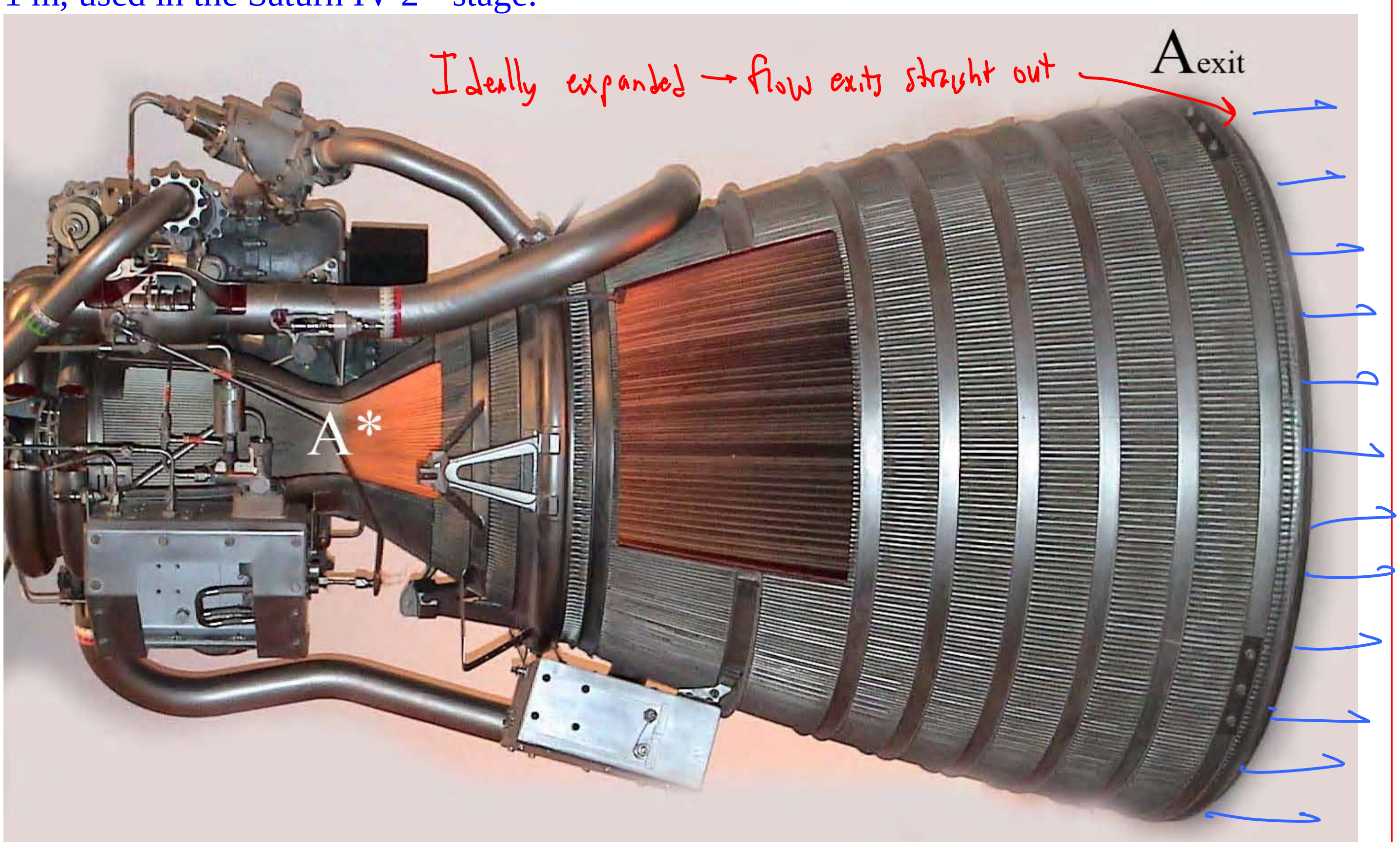
Solution:

- From Table A-11E, T at 24 km altitude is -69.7ºF = 217 K.
- Using $k = 1.4$ and $R_{air} = 287\ \mathrm{m^2/(s^2 \cdot K)}$, $c = (kRT)^{1/2} = 295$ m/s.
- Thus, $V = \mathrm{Ma} \cdot c = 3.2(295\ \mathrm{m/s}) = \boxed{944\ \mathrm{m/s}}$ (= 2110 mph).

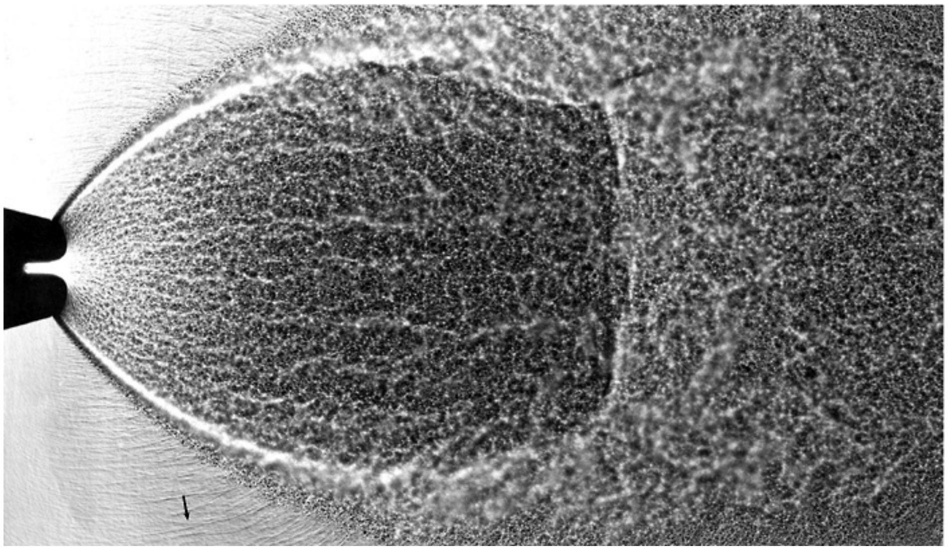
Shock Diamonds ("tiger tail"):

Example of a Rocket Engine:

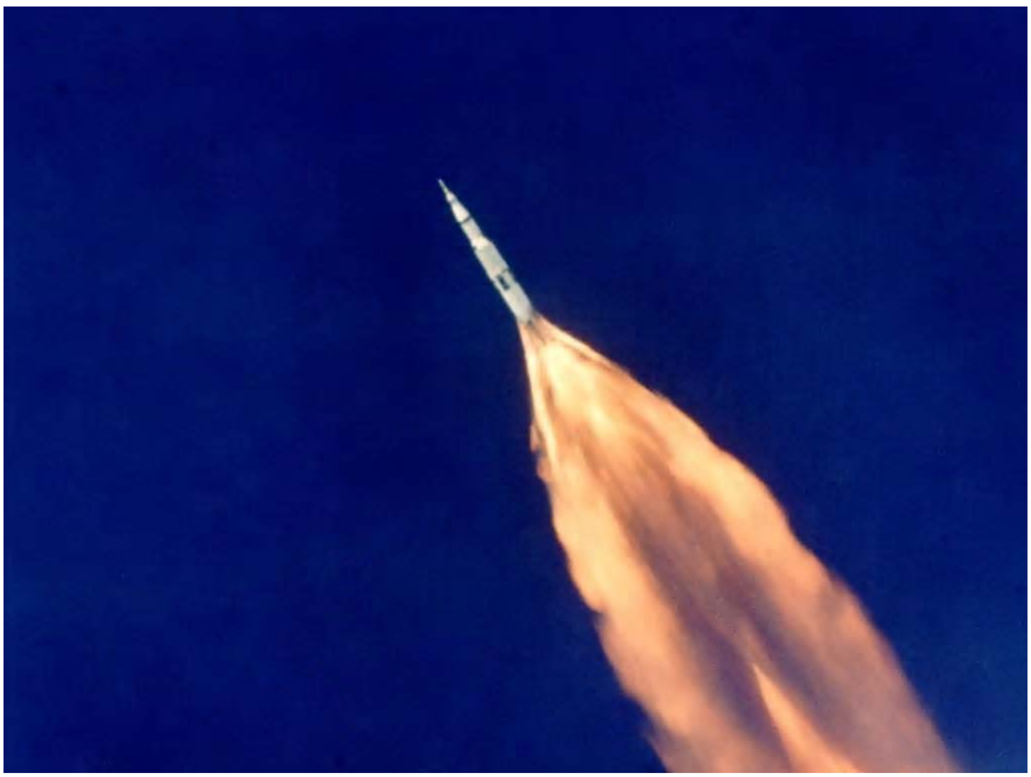
Pratt &Whitney RL-10 rocket motor designed for a specific Ma_{exit} (photographed at the National Air & Space Museum). 1960-vintage, $\mathrm{Ma}_{exit} = 5$, $k = 1.33$, thrust = 15,000 lbf, $D_e \sim 1$ m, used in the Saturn IV 2nd stage.



Underexpanded Nozzles:



FLARE OUT



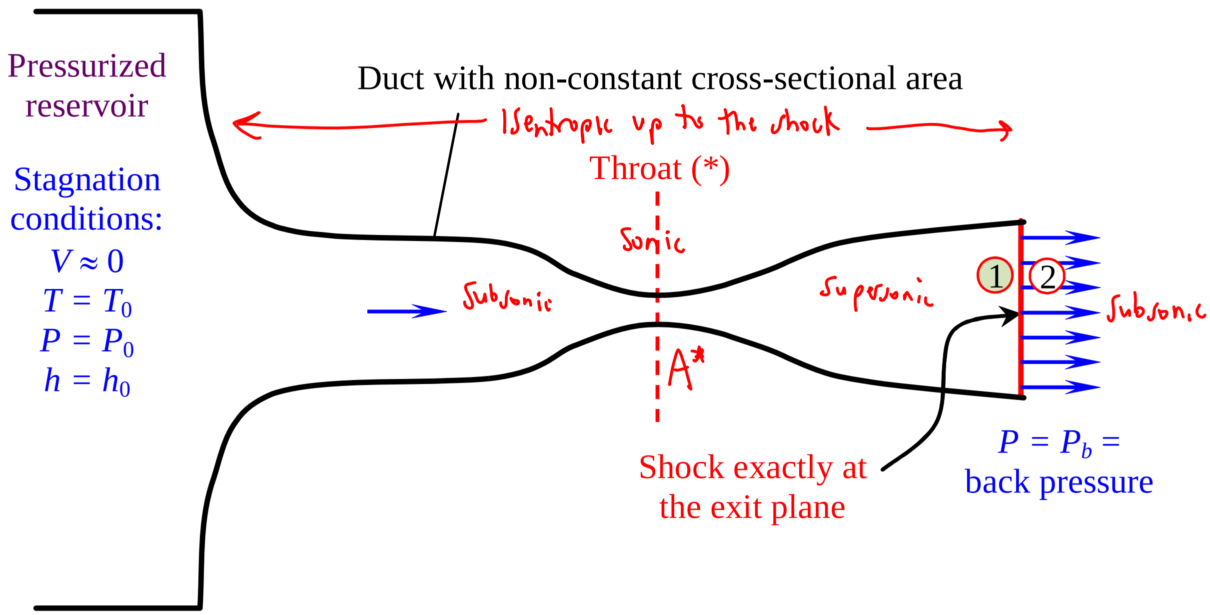
Underexpanded

momentum flux is lower than it should be

$P_{ambient}$ = too low $\boxed{P_b < P_F}$

Example – Normal shock at Ma = 3.0

Given: A large tank has upstream stagnation properties T_0 = 1000 K and P_0 = 1.00 MPa. A converging/diverging nozzle accelerates air isentropically from the tank to Ma = 3.0 just before the exit. Right at the exit plane is a normal shock wave as sketched.



To do: Calculate the pressure, temperature, and density upstream (1) and downstream (2) of the shock.

Solution:

- From Table A-13 at Ma_1 = 3.0,
  A/A^* = 4.2346, P/P_0 = 0.0272, T/T_0 = 0.3571, and ρ/ρ_0 = 0.0760. (isentropic flow)
- From Table A-14 at Ma_1 = 3.0,
  Ma_2 = 0.4752, P_2/P_1 = 10.3333, T_2/T_1 = 2.679, and ρ_2/ρ_1 = 3.8571. (across the shock)
- The rest of the problem to be completed in class.

Use isentropic relations through the whole duct up to 1, just before the shock:

$$P_1 = \left(\frac{P_1}{P_0}\right) P_0 = 0.0272\,(1.00\text{ MPa}) = 0.0272\text{ MPa} = \boxed{27.2\text{ kPa} \cong P_1}$$

Similarly $T_1 = \dfrac{T_1}{T_0} T_0 = 0.3571\,(1000\text{ K}) = 357.1\text{ K} \rightarrow \boxed{T_1 = 357\text{ K}}$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{27{,}200\text{ N/m}^2}{\left(287\,\frac{\text{m}^2}{\text{s}^2\text{K}}\right)(357.1\text{ K})}\left(\frac{\text{kg m}}{\text{s}^2\cdot\text{N}}\right) = 0.26540\,\frac{\text{kg}}{\text{m}^3} \rightarrow \boxed{\rho_1 = 0.265\,\frac{\text{kg}}{\text{m}^3}}$$

Now use shock relations to get properties @ 2, just after the shock:

$$P_2 = \frac{P_2}{P_1} P_1 = (10.3333)(27.2 \text{ kPa}) = 281.1 \text{ kPa} \rightarrow \boxed{P_2 = 281. \text{ kPa}}$$

$$T_2 = \frac{T_2}{T_1} T_1 = (2.679)(357.1 \text{ K}) = 956.67 \text{ K} \rightarrow \boxed{T_2 = 957. \text{ K}}$$

$$\rho_2 = \frac{\rho_2}{\rho_1} \rho_1 = (3.8571)\left(0.26540 \frac{\text{kg}}{\text{m}^3}\right) = 1.0237 \frac{\text{kg}}{\text{m}^3} \rightarrow \boxed{\rho_2 = 1.02 \frac{\text{kg}}{\text{m}^3}}$$

or, use ideal gas law. Try it! – should get same answer

THE END !