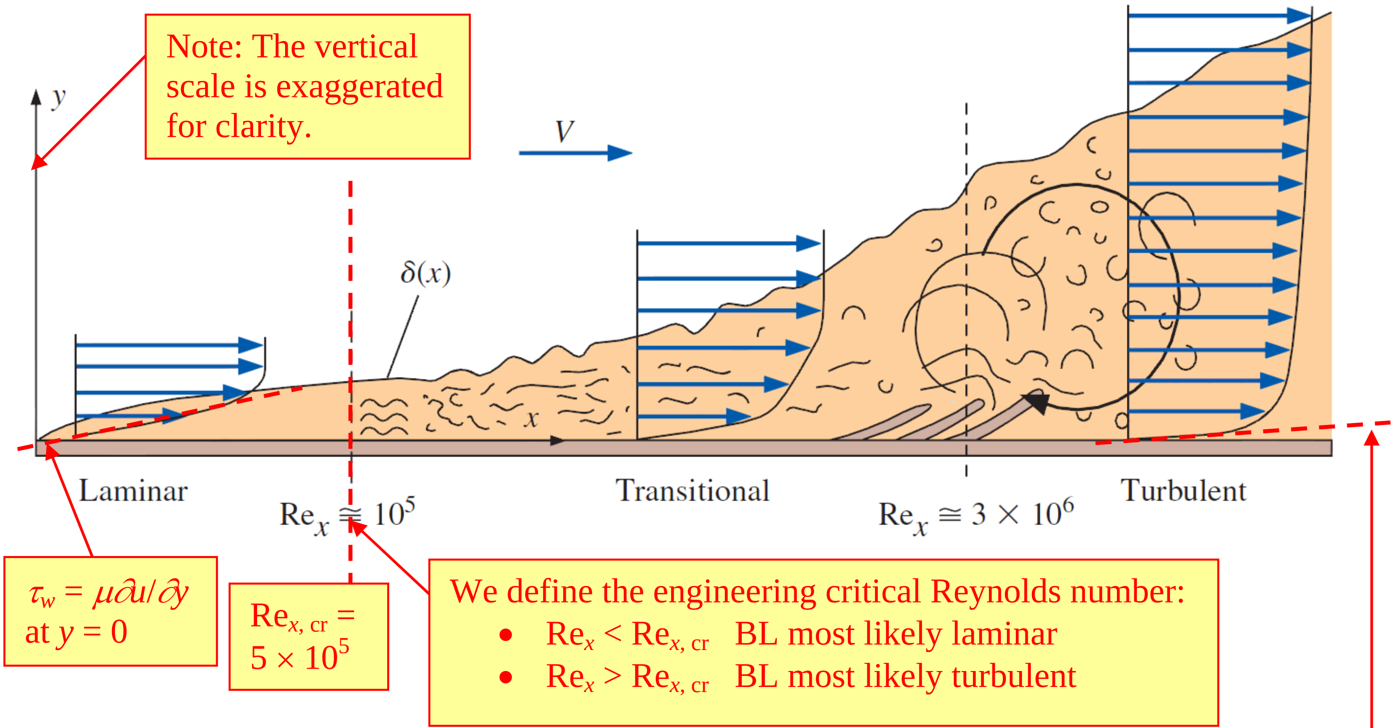
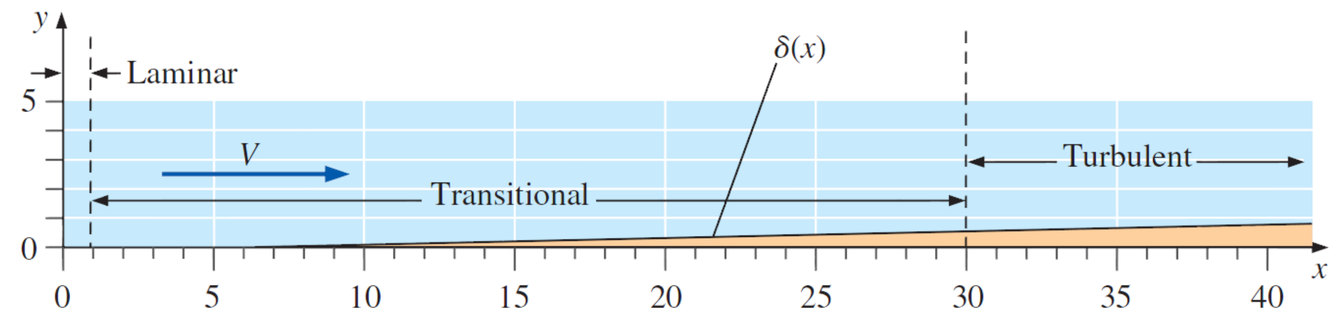
The Turbulent Flat Plate Boundary Layer (Section 10-6, Çengel and Cimbala)



Here is what the actual BL looks like to scale:



The turbulent flat plate boundary layer velocity profile:

The time-averaged turbulent flat plate (zero pressure gradient) boundary layer velocity profile is much *fuller* than the laminar flat plate boundary layer profile, and therefore has a larger slope $\partial u/\partial y$ at the wall, leading to greater skin friction drag along the wall.

There are three common empirical relationships for the turbulent flat plate boundary layer velocity profile:

- **The log law**:

The log law:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{y u_*}{\nu} + B \qquad \textbf{(10–83)}$$

where

Friction velocity:

$$u_* = \sqrt{\frac{\tau_w}{\rho}} \qquad \textbf{(10–84)}$$

- **Spalding's law of the wall**:

$$\frac{yu_*}{\nu} = \frac{u}{u_*} + e^{-\kappa B}\left[e^{\kappa(u/u_*)} - 1 - \kappa(u/u_*) - \frac{[\kappa(u/u_*)]^2}{2} - \frac{[\kappa\,(u/u_*)]^3}{6}\right] \quad \textbf{(10–85)}$$

- **The one-seventh-power law**:

$$\frac{u}{U} \cong \left(\frac{y}{\delta}\right)^{1/7} \quad \text{for } y \le \delta, \qquad \rightarrow \qquad \frac{u}{U} \cong 1 \quad \text{for } y > \delta \qquad \textbf{(10–82)}$$

Quantities of interest for the turbulent flat plate boundary layer:

Just as we did for the laminar (Blasius) flat plate boundary layer, we can use these expressions for the velocity profile to estimate quantities of interest, such as the 99% boundary layer thickness δ, the displacement thickness δ^*, the local skin friction coefficient $C_{f,x}$, etc. These are summarized in Table 10-4 in the text.

Column (b) expressions are generally preferred for engineering analysis.

TABLE 10–4

Summary of expressions for laminar and turbulent boundary layers on a smooth flat plate aligned parallel to a uniform stream*

Property	Laminar	(a) Turbulent(†)	(b) Turbulent(‡)
Boundary layer thickness	$\frac{\delta}{x} = \frac{4.91}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta}{x} \cong \frac{0.16}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta}{x} \cong \frac{0.38}{(\mathrm{Re}_x)^{1/5}}$
Displacement thickness	$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta^*}{x} \cong \frac{0.020}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta^*}{x} \cong \frac{0.048}{(\mathrm{Re}_x)^{1/5}}$
Momentum thickness	$\frac{\theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$\frac{\theta}{x} \cong \frac{0.016}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\theta}{x} \cong \frac{0.037}{(\mathrm{Re}_x)^{1/5}}$
Local skin friction coefficient	$C_{f,x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$C_{f,x} \cong \frac{0.027}{(\mathrm{Re}_x)^{1/7}}$	$C_{f,x} \cong \frac{0.059}{(\mathrm{Re}_x)^{1/5}}$

Note that $C_{f,x}$ is the *local* skin friction coefficient, applied at only *one* value of x.

To these we add the integrated ***average skin friction coefficients*** for *one side* of a flat plate of length L, noting that C_f applies to the entire plate from $x = 0$ to $x = L$ (see Chapter 11):

Laminar: $$C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}} \qquad \mathrm{Re}_L \lesssim 5 \times 10^5 \qquad \textbf{(11–19)}$$

Turbulent: $$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \qquad 5 \times 10^5 \lesssim \mathrm{Re}_L \lesssim 10^7 \qquad \textbf{(11–20)}$$

For cases in which the laminar portion of the plate is taken into consideration, we use:

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5 \times 10^5 \lesssim \mathrm{Re}_L \lesssim 10^7 \qquad \textbf{(11–22)}$$

Turbulent flat plate boundary layers with wall roughness:

Finally, all of the above are for *smooth* flat plates. However, if the plate is *rough*, the average skin friction coefficient C_f increases with roughness ε. This is similar to the situation in pipe flows, in which Darcy friction factor f increases with pipe wall roughness.

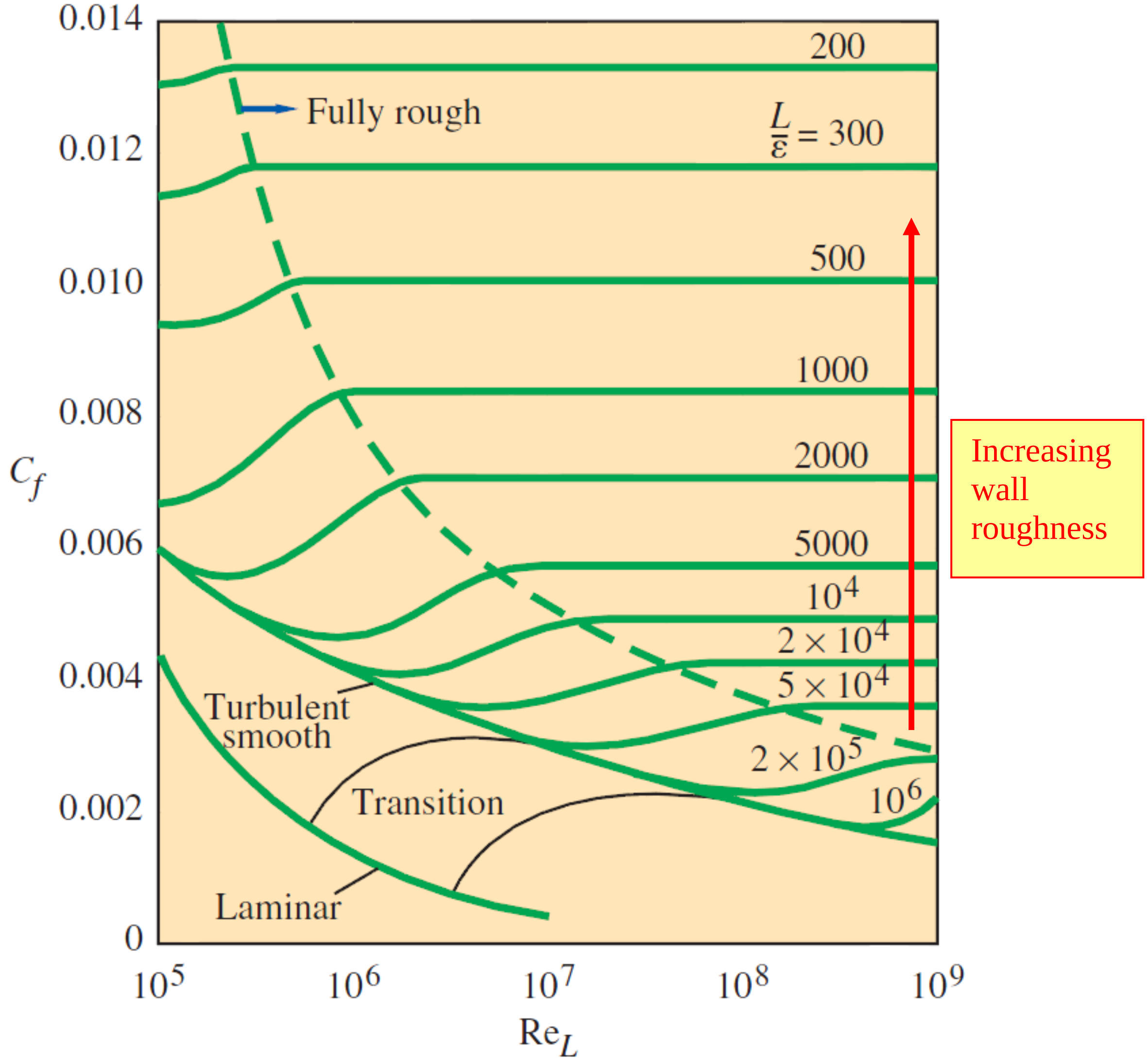


FIGURE 11–31

Friction coefficient for parallel flow over smooth and rough flat plates.

Just as with pipe flows, at high enough Reynolds numbers, the boundary layer becomes "fully rough". For a ***fully rough flat plate turbulent boundary layer*** with average wall roughness height ε,

Fully rough turbulent regime:

$$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5} \qquad \textbf{(11–23)}$$

This equation represents the flat portions of Fig. 11-31 that are labeled "Fully rough".