

Compressible Flow in a Converging-Diverging Nozzle

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Latest revision: 19 January 2012

Nomenclature

Symbols

a	speed of sound: $a = (kRT)^{1/2}$ for an ideal gas
A	cross-sectional flow area
A^*	critical cross-sectional flow area, at throat when the flow is choked
C_p	specific heat at constant pressure
C_v	specific heat at constant volume
k	ratio of specific heats: $k = C_p/C_v$ ($k = 1.4$ for air)
\dot{m}	mass flow rate through the pipe
Ma	Mach number: $Ma = V/a$
n	index of refraction
P	static pressure
Q or \dot{V}	volume flow rate
R	specific ideal gas constant: for air, $R = 287. \text{ m}^2/(\text{s}^2\cdot\text{K})$
t	time
T	temperature
V	mean flow velocity
\mathcal{V}	volume
x	flow direction
y	coordinate normal to flow direction, usually normal to a wall
z	elevation in vertical direction
α	deflection angle for a light beam
ρ	density of the fluid (for water, $\rho \approx 998 \text{ kg/m}^3$ at room temperature)

Subscripts

b	back (downstream of converging-diverging nozzle)
e	exit (at the exit plane of the converging-diverging nozzle)
i	ideal (frictionless)
t	throat
0 or T	stagnation or total (at a location where the flow speed is nearly zero); e.g. $P_0 =$ stagnation pressure

Educational Objectives

1. Conduct experiments to illustrate phenomena that are unique to compressible flow, such as choking and shock waves.
2. Become familiar with a compressible flow visualization technique, namely the schlieren optical technique.

Equipment

1. supersonic wind tunnel with a converging-diverging nozzle
2. Singer diaphragm flow meter, Model A1-800
3. pressure taps and thermocouples at various locations in the wind tunnel, with control panel; thermocouples are type K, chromel-alumel.
4. microphones, amplifier, and speaker system
5. Heise absolute pressure gauge (psi units)
6. Omega Engineering thermocouple-based digital thermometer, model 871
7. schlieren flow visualization system:
 - a. two 70-mm diameter \times 500-mm focal length lenses
 - b. LED light source with condenser lens and slit
 - c. two-color filter (replaces the knife edge for color schlieren capability)
 - d. HD digital camera and HD monitor
 - e. assorted mirrors, traverses and mounts
8. stopwatch

Background

A. Compressible Flow, Choking, and Converging-Diverging Nozzles

In all of the other laboratory experiments conducted in this course, compressibility effects are negligible. In other words, for low speed flows of liquids or gases, density may be assumed constant, without significant loss of accuracy. At very high flow speeds, however, compressibility effects become important, and in fact *dominate* the flow field.

The most important parameter in compressible flows is Mach number, $Ma = V/a$ where V is the flow velocity and a is the speed of sound [$a = (kRT)^{1/2}$ for an ideal gas]. If the Mach number is small (less than about 0.3), compressibility effects are unimportant. For $0.3 < Ma < 1.0$, the flow is still *subsonic* (meaning velocity V is lower than the speed of sound) but compressibility effects (i.e. changes in density ρ) are important. At $Ma = 1.0$, the flow is called *sonic*, since $V = a$. At Mach numbers greater than 1.0, the flow is called *supersonic*, since the velocity is greater than the speed of sound. Bullets fired from a gun travel at supersonic speeds, as do certain jet aircraft. When Mach number becomes very large (greater than about 3.0), the flow is called *hypersonic*. An example of hypersonic flow is encountered when the space shuttle re-enters earth's atmosphere and descends to earth. The proposed National Aerospace Plane was to travel at Mach numbers over 20!

The facility used in this lab experiment is a small wind tunnel, driven by a large compressor located in the basement of Reber Building. The pressure and flow rate produced by the compressor are high enough that a continuous supersonic flow can be sustained in the wind tunnel test section. A converging-diverging nozzle is utilized to provide supersonic flow. It is suggested that you review the basics of compressible flow before proceeding (see Reference 1).

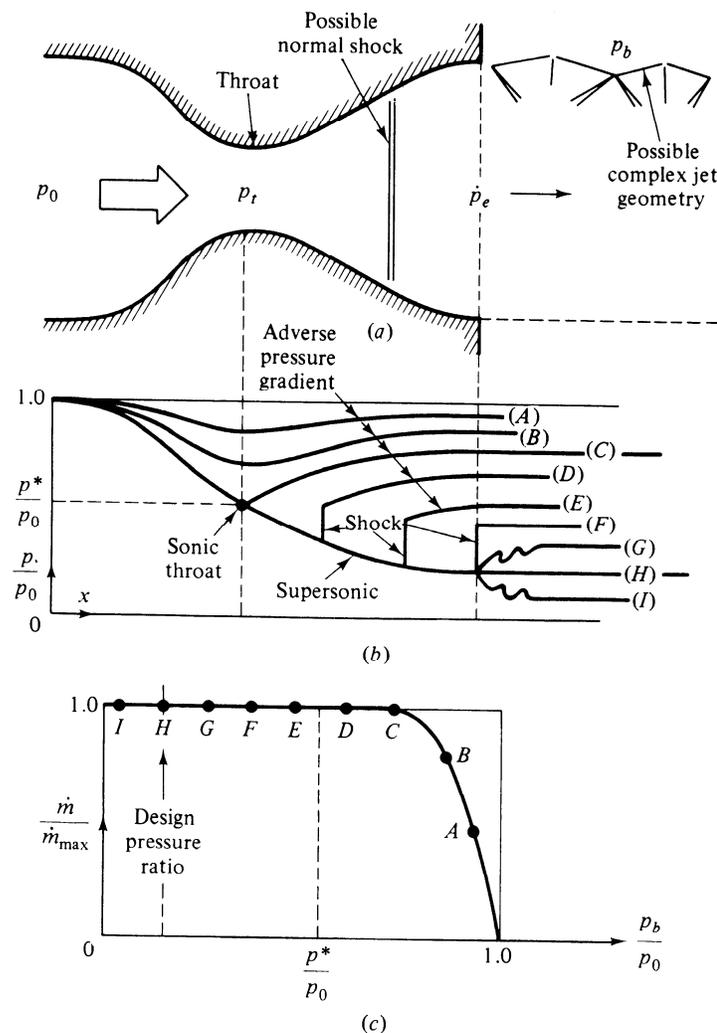


Figure 1. Operation of a converging-diverging nozzle: (a) nozzle geometry showing a possible normal shock, and possible external shock pattern; (b) pressure distribution caused by various back pressure ratios p_b/p_0 ; (c) mass flow rate as a function of back pressure ratio corresponding to each labeled pressure distribution curve A - I.

Figure 1 (taken from Reference 1) is a summary of the various possible flow regimes in a converging-diverging nozzle. Consider our experimental apparatus, where stagnation pressure P_0 (upstream of the converging-diverging nozzle) is

held constant by a pressure regulator, and back pressure P_b (downstream the nozzle) is adjustable by a valve. If the downstream valve is closed, the entire nozzle is at a pressure equal to P_0 , and there is no flow. As the valve is slowly opened, the flow is at first subsonic, and follows a pressure distribution similar to that of curve (A) in Figure 1b. Namely the pressure decrease through the converging portion of the nozzle, and then increases through the diverging nozzle. Mass flow rate \dot{m} is very small under these conditions, as seen in Figure 1c. As the valve is opened further, mass flow rate \dot{m} increases, but the overall behavior of the pressure is the same, as seen in curve (B). The flow is still subsonic, but the velocity is high, and the Mach number near the **throat** (the location of minimum area) is approaching unity.

Curve (C) represents the case where Mach number just reaches 1.0 at the throat. The Mach number increases in the converging part of the nozzle from nearly zero far upstream to 1.0 at the throat, then decreases again in the diverging portion of the nozzle, where the flow is again subsonic. Notice from Figure 1c that mass flux reaches a maximum value at this point. Why? Because now that the flow velocity is equal to the speed of sound at the throat, flow disturbances downstream of the throat, such as acoustic noise or fluctuations in back pressure, cannot travel upstream past the throat. The flow is referred to as **choked**, because further decreases in back pressure have no effect on \dot{m} . In other words, for the given upstream (supply) conditions, this is the maximum amount of mass flow that can be forced through the nozzle, regardless of the back pressure.

As P_b is reduced even further, the flow becomes *supersonic* just downstream of the throat in the diverging part of the nozzle. A **normal shock wave** forms somewhere downstream of the throat, as illustrated in curve (D). The flow jumps from supersonic to subsonic across this normal shock. Mass flux, of course, remains fixed since the flow is choked, and upstream conditions have not changed. In curve (E), the back pressure is reduced even further, causing the shock wave to move downstream. Eventually, the shock moves to the exit plane of the nozzle, as shown in curve (F). At this point, the flow in the entire diverging part of the nozzle is supersonic, but jumps to subsonic just downstream of the normal shock at the exit plane.

Further decreases in P_b no longer affect the flow in the nozzle, since the entire diverging part of the nozzle is supersonic. Again, this is most easily understood by considering a noise or pressure fluctuation downstream of the nozzle exit. In no way can such a disturbance propagate upstream and affect the nozzle, because such disturbances travel at the speed of sound, yet the flow is *faster* than the speed of sound. As sketched in Figure 2, if Fred, Sue, and Lisa are standing in a supersonic flow, Sue will hear what Lisa says, but Fred will not hear *anything* that Lisa says!

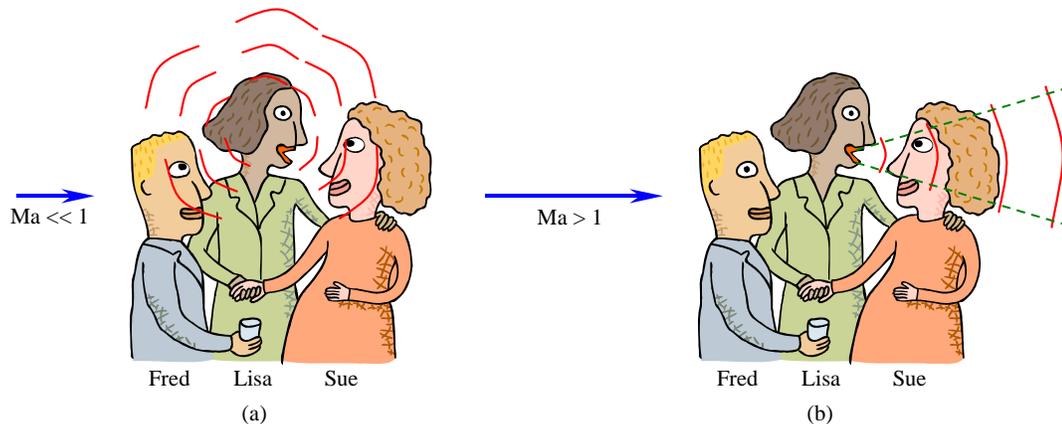


Figure 2. In subsonic, low Mach number flow (a), Both Fred and Sue can hear Lisa talking, since sound waves travel from Lisa’s mouth in all directions. However, in supersonic flow (b), sound waves (and other disturbances in the flow) travel *only* in the downstream direction; thus, while Sue can hear Lisa talking, Fred cannot. *Disturbances cannot travel upstream in a supersonic flow.*

Curve (G) represents a case where P_b is so low that the shock wave has moved *downstream* of the nozzle. In fact, a very complicated diamond-shaped pattern of shock waves forms outside of the nozzle. None of this has any effect on mass flow rate. Curve (H) represents the special case where P_b exactly matches P_e , the pressure at the exit plane. For this case, the flow exits the nozzle cleanly without any shock wave pattern outside the nozzle. This is commonly referred to in the literature as a **fully expanded** nozzle, or a nozzle running at **design conditions**. Finally, case (I) indicates that when P_b is lower than the “design” back pressure P_e , another complex flow results. In this case the flow fans out, forming a very wide exhaust plume. Such a flow can be seen when a rocket ascends to high altitudes where the pressure is very low. Even if P_b is reduced to a perfect vacuum, mass flux can not increase any further since the flow is choked, as seen in Figure 1c.

B. Shadowgraph and Schlieren Flow Visualization Techniques

Two common techniques for visualizing a compressible flow are the shadowgraph and schlieren techniques. Both take advantage of the principle that the index of refraction, n , of a gas is a strong function of gas density. If the density in the gas flow is non-uniform in the x -direction, originally parallel light rays entering the test section are bent (**refracted**), and emerge from the test section at some deflection angle α , as sketched in Figure 3.

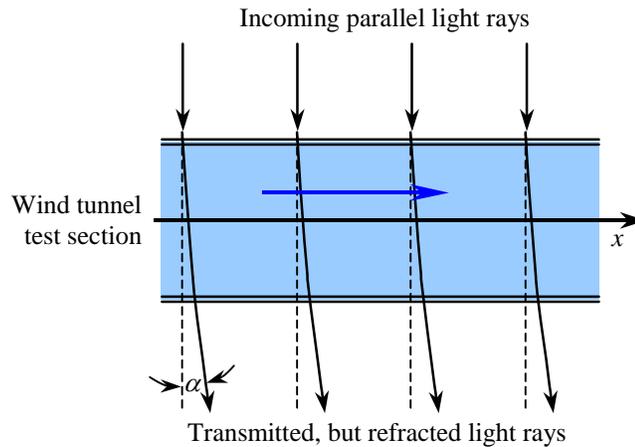


Figure 3. Principle of light ray refraction through a non-uniform density field in a flow. In this case, density increases in the x direction, causing the light rays to bend as sketched (in the direction of increasing density).

Due to the fact that light travels slower in a denser gas, the light rays always curve *in the direction of increasing density*. For uniform 1-D flow in the x direction, the deflection angle is directly proportional to the density gradient,

$$\alpha = \text{constant} \frac{\partial \rho}{\partial x} \quad (1)$$

If a viewing screen is placed on the transmitted side of the test section a **shadowgraph** image is formed on the screen. Bright spots appear where the light rays culminate together, and dark spots (shadows) appear where the rays diverge (hence the name *shadowgraph*). The convergence or divergence of the light rays is proportional to the derivative, $\partial \alpha / \partial x$, of the deflection angle. Hence, by Equation (1), the illumination depends on the *second* derivative of density. For uniform 1-D flow in the x direction,

$$\text{illumination in a shadowgraph} = \text{constant} \frac{\partial^2 \rho}{\partial x^2} \quad (2)$$

A shock wave, which is a sudden rise in density, appears as a dark line followed by a bright line in a shadowgraph.

The **schlieren technique** is somewhat more complicated than the shadowgraph method. The basic idea is that part of the refracted light is intercepted (cut off or blocked) before it reaches the viewing screen. This is accomplished by focusing the transmitted light near a knife edge (usually a razor blade), as sketched in Figure 4. The shaded region of the flow in Figure 4 represents a normal shock wave (a region of rapidly increasing density in the x direction). A light ray passing through this region is deflected, as sketched, but is blocked by the knife edge. The region on the screen corresponding to this shaded region therefore appears darker than the surrounding regions, due to the absence of striking light rays. On either side of the shock, the viewing screen appears bright. The quality (contrast and sharpness) of a schlieren image is highly dependent on the location and orientation of the knife edge. Mathematically, it turns out that the illumination of a schlieren image depends on the *first* derivative of density,

$$\text{illumination in a Schlieren image} = \text{constant} \frac{\partial \rho}{\partial x} \quad (3)$$

Thus, a one-dimensional normal shock wave appears as either a bright or dark line, depending on the actual location of the knife edge. In general, a schlieren image is a more sensitive indicator of flow disturbances than is a shadowgraph image, but the sensitivity of a schlieren optical system is highly dependent on the position of the knife edge. If too much of the light is cut off, the image gets too dark. If the knife edge is not advanced enough, the image looks like a shadowgraph.

There are many ways to adapt schlieren systems. Various light sources, source filters, and cutoffs (knife-edges) can be used to obtain more desirable results. One can even add special color filters to produce full color images. The Penn State Gas Dynamics Lab (PSGDL) is a world-leader in schlieren optical

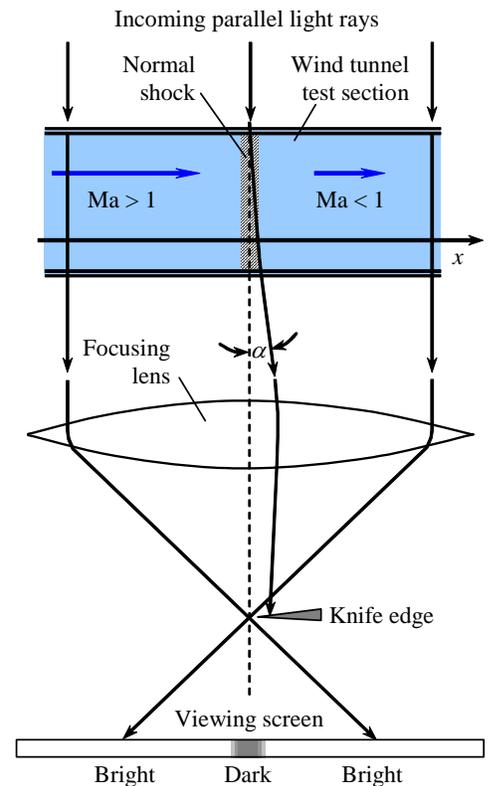


Figure 4. Schematic diagram of the operation of a schlieren optical system, used to visualize a normal shock wave. The knife edge is replaced by a color filter to enable color schlieren imaging.

technology. The PSGDL has a highly sensitive 1-meter diameter color schlieren system that is useful for studying ballistics, thermal plumes, and even the human cough (Figure 5) The PSGDL also houses the largest indoor schlieren system in the world, having a test section that measures 7.5 x 9.5 feet. This full-scale system is large enough to accommodate a wide range of applications such as leak detection, room ventilation, and explosions (Figure 6).

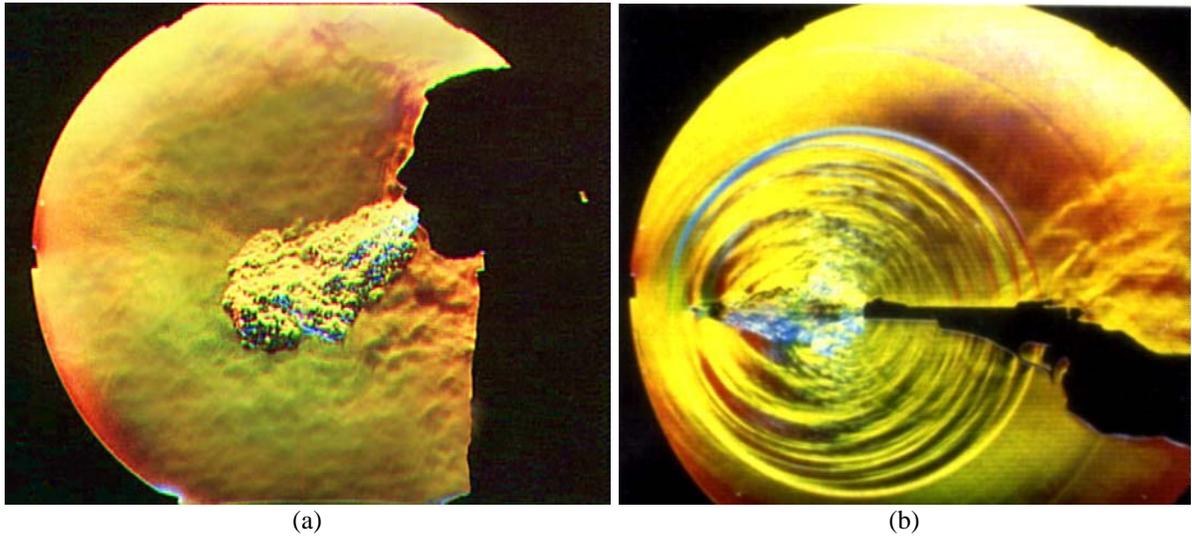


Figure 5. Color schlieren images from the 1-m system at the PSGDL: (a) the human cough, modeled by G.S. Settles, and (b) a 0.22 caliber bullet fired from a Thompson Contender marksman's pistol. The muzzle blast creates a spherical shock wave originating at the end of the barrel. The disturbances to the right of picture (b) represent heat escaping the arm of the gun holder (a thermal plume). Photos courtesy of G. S. Settles.



Figure 6. Color schlieren images from the full-scale system at the PSGDL: (a) propane gas leaking from a valve, modeled by J. D. Miller, and (b) explosion beneath a full size aircraft seat occupied by a dummy. This photo aided in the research (funded by the FAA) of aircraft hardening in response to incidents of international terrorism. Photos courtesy of G. S. Settles.

C. Description of the Facility

Our schlieren system is much smaller (70mm diameter, 500mm focal length lenses) than those of the PSGDL. In our laboratory setup, a LED is used as the light source, and two convex lenses are used in the simple dual-field-lens schlieren arrangement, as illustrated Fig. 7. The optics are set up in such a way that both schlieren and shadowgraph images can be obtained. With the knife edge in place, schlieren images are produced. If the knife edge were moved out of the way, the system would produce shadowgraph images instead. The schlieren optical system used in the undergraduate fluid flow laboratory was refurbished in the summer of 2011 by M. J. Hargather and G.S. Settles. *The optical equipment is very sensitive to misalignment, so please be very careful while moving about during this laboratory exercise.*

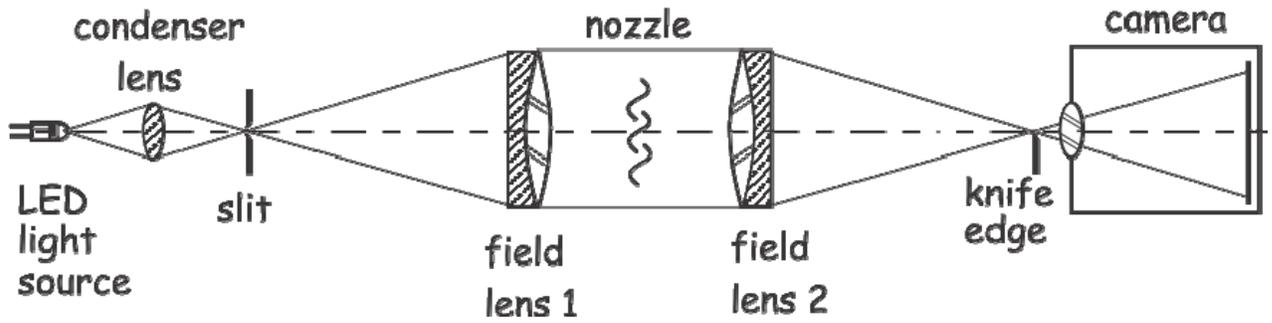


Figure 7. The simple, single-pass, dual-field-lens schlieren arrangement (from Reference 3). The knife edge is replaced by a color filter to convert from black-and-white schlieren imaging to color schlieren imaging.

A schematic diagram of the supersonic wind tunnel is shown in Figure 8, and a close-up schematic of the test section is shown in Figure 9. Pressure taps measure the static pressure at various locations in the test section, and pressure is read from the control panel. Temperatures T_1 through T_4 are also selected from the control panel, and are displayed by a digital thermometer. The converging-diverging nozzle is 6.35 mm wide, and the throat is also 6.35 mm high, giving a throat area of $A^* = 4.03 \times 10^{-5} \text{ m}^2$. As sketched in Figure 9, there are ten static pressure taps mounted in the region of the converging-diverging nozzle. Pressure tap 1 is just upstream of the nozzle, and pressure tap 2 is located at the beginning of the converging portion of the nozzle. In all calculations in this lab, it is assumed that stagnation pressure $P_0 = P_2$. Pressure taps 3-9 are in the converging-diverging portion of the nozzle, and pressure tap 10 is downstream of the nozzle. In all calculations in this lab, it is assumed that back pressure $P_b = P_{10}$. The area ratio at the exit plane is $A_e/A^* = 2.00$, but no pressure tap is present at that location.

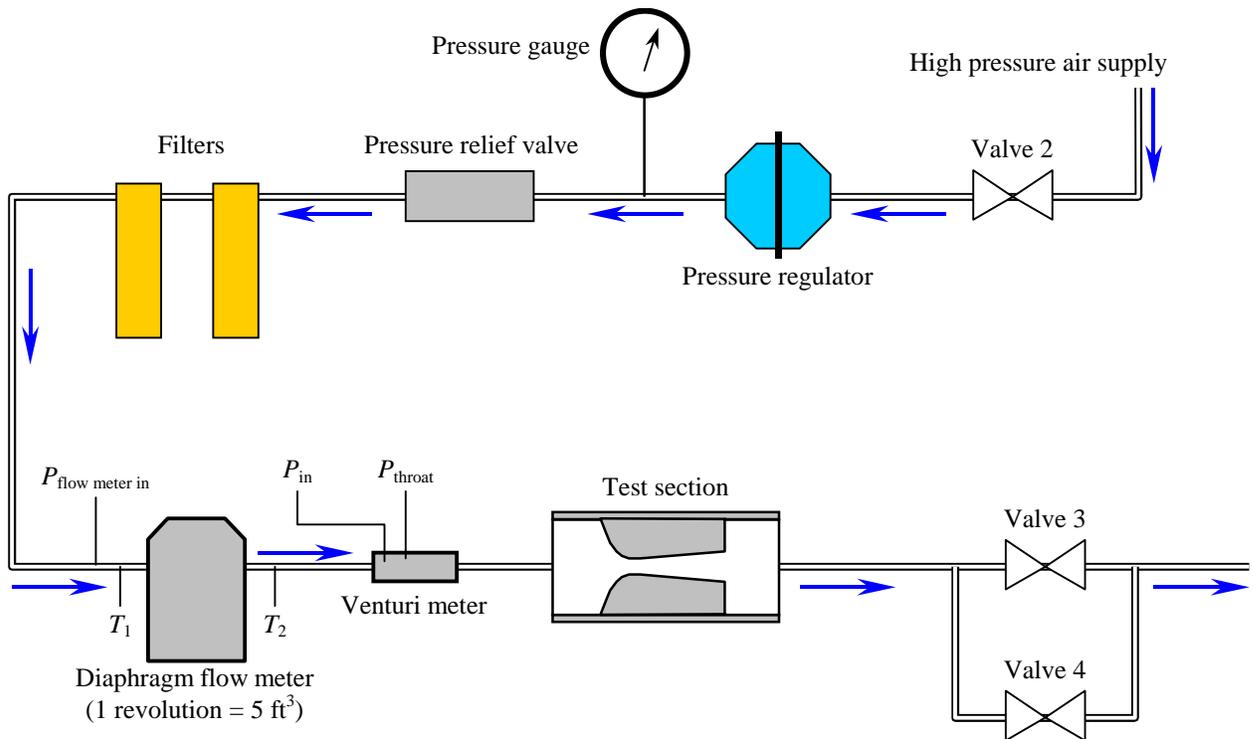


Figure 8. Schematic diagram of the supersonic wind tunnel facility. Flow is turned on by opening Valves 3 and 4, and then Valve 2. The pressure regulator maintains constant upstream pressure, even though the pressure in the high pressure air supply may vary.

The latest modification to this lab setup is color schlieren (added Spring 2012 based on a student project in Fall 2012). In the usual schlieren setup with a knife-edge (razor-blade) cutoff, rays that miss the edge form bright spots and those that hit the edge are blocked, forming dark spots that make up a monochromatic (black-and-white) schlieren image. Color schlieren, as it's done in this lab, replaces the knife edge with a composite color filter made of a central black band and two different colors of gelatin optical filters on either side. Presently the colors are a light yellow and a dark blue. The image of the LED light source through its adjustable slit should spill over the edges of the central dark band a little, thus illuminating the background of the schlieren image in a mixture of the two colors, which turns out to be greenish. Depending on which direction (left or right) the light is refracted by disturbances in the nozzle, features will appear in the schlieren image in either yellow or blue with this system. The color schlieren image is still essentially similar to the black-and-white image, but the

eye often finds it easier to interpret a color than a black-and-white image. (We can discern only about 18 different shades of gray but many thousand colors.)

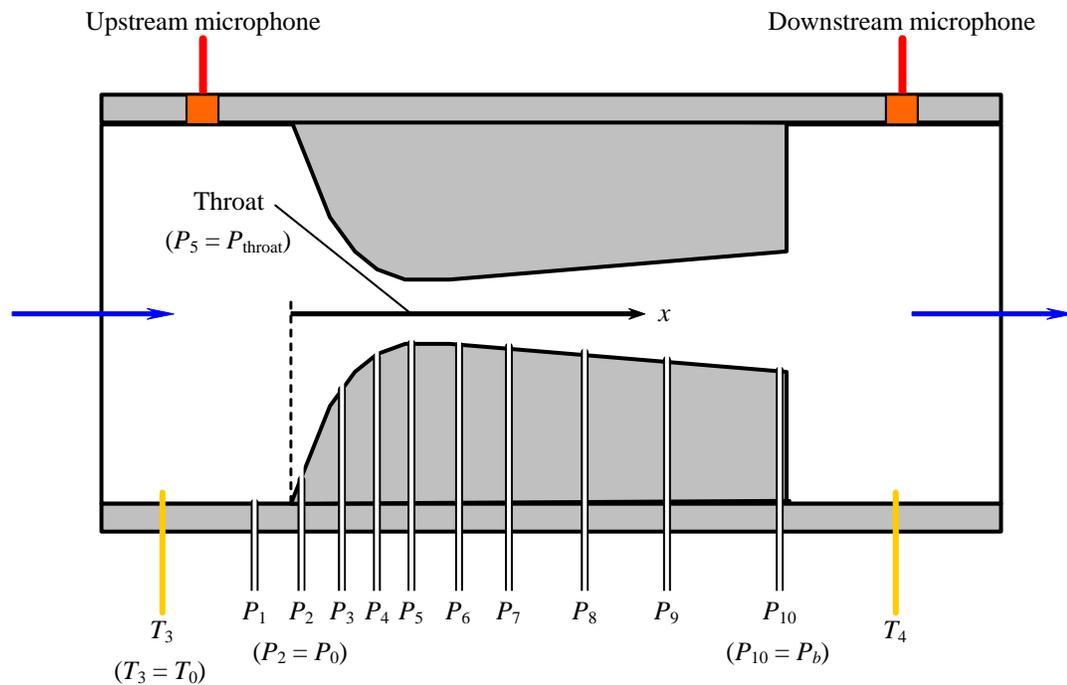


Figure 9. Close-up schematic diagram of the converging-diverging test section. Pressure taps 2 through 10 measure the static pressure at various locations in the test section. Temperatures T_3 and T_4 are upstream and downstream of the converging-diverging nozzle, respectively, and microphones are located upstream and downstream as well.

References

1. White, F. M., *Fluid Mechanics*, Ed. 5, McGraw-Hill, NY (2003), pp. 599-632.
2. Liepmann, H. W. and Roshko, A., *Elements of Gas Dynamics*, John Wiley & Sons, NY (1957), pp. 153-163.
3. Settles, G. S., *Schlieren and Shadowgraph Techniques*, Springer, NY (2001).