Digital Data Acquisition

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Introduction

- Instruments and data can be either ***analog*** (continuous) or ***digital*** (discrete).
- In this learning module, we discuss digital data, and how to convert between analog and digital signals.

Review - Decimal to Binary Conversion

- While we are used to working with ***decimal numbers*** (base 10), computers and many modern instruments and electronic devices use ***binary numbers*** (base 2).
- Decimal numbers contain the digits 0 through 9, while binary numbers contain only digits 0 and 1.
- As a review, here is a listing of the first eight (including zero) decimal and binary numbers:

Decimal number	Binary number
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111

- There are simple techniques to convert from decimal to binary and vice-versa:
  - Decimal to binary:
    - Successively divide by 2, as in the following example: convert the decimal number 29 into binary:

Value/2	Integer value	Remainder
29/2 = 14 + 1/2	14	1
14/2 = 7 + 0/2	7	0
7/2 = 3 + 1/2	3	1
3/2 = 1 + 1/2	1	1
1/2 = 0 + 1/2	0	1

Read this way, bottom to top.

    - The binary number is obtained from the "remainder" column, reading ***from bottom to top*** (*backwards*); the answer is 11101.
  - Binary to decimal:
    - Add up each non-zero bit, from left to right, converting the binary number 11101 into decimal:

Bit considered (underlined)	Value of the place	Value of the bit
$1110\underline{1}$	1	$1 \times 1 = 1$
$111\underline{0}1$	2	$0 \times 2 = 0$
$11\underline{1}01$	4	$1 \times 4 = 4$
$1\underline{1}101$	8	$1 \times 8 = 8$
$\underline{1}1101$	16	$1 \times 16 = 16$

Sum this column.

    - The decimal number is obtained by summing the "Value of the bit" column; the answer is 29.

- **Example**:
  Given: The decimal number $x = 4$
  To do: Convert x to a binary 4-bit number.
  Solution: We use the technique of successive division by 2, as per the following table:

Value/2	Integer value	Remainder
4/2 = 2 + 0/2	2	0
2/2 = 1 + 0/2	1	0
1/2 = 0 + 1/2	0	1

Read this way, bottom to top.

  The resulting binary number is read from the bottom to the top, $\boxed{x = 0100}$ in binary form. We have added a leading zero so that the result is a 4-bit number. If we were asked for 8 bits, we would write x = 0000 0100.
- Some calculators have built-in binary-to-decimal and decimal-to-binary converters.

Analog to Digital Conversion (A/D conversion)

- Digital data acquisition is used in digital multimeters, digital oscilloscopes, computer-controlled data acquisition systems, and many other modern instruments and electronic devices such as cell phones.
- In all these examples, the conversion of an analog signal into a digital signal is accomplished with an electronic device called an ***analog-to-digital converter***, which we abbreviate ***A/D converter***.
- The goal of an A/D converter is to change an ***analog voltage signal*** into a ***digital number*** (in binary form).
- An A/D converter is labeled as N-bit, where the ***number of bits N*** represents how many bits (ones and zeroes) are used in the digital output of the A/D converter. E.g., an 8-bit converter creates an 8-bit output.
- For simplicity, consider a 2-bit A/D converter ($N = 2$) with a ***range*** of –5 to 5 volts. The voltage range is divided into ***bins*** as follows: [*Note*: This is just one of several optional ways to assign the bins.]

Analog voltage (volts)	Bin number	Digital output (binary)	Assigned voltage, V (volts)
$-5 \le V < -2.5$	0	00	$V = V_{\min} + 0.5\Delta V = -3.75$
$-2.5 \le V < 0$	1	01	$V = V_{\min} + 1.5\Delta V = -1.25$
$0 \le V < 2.5$	2	10	$V = V_{\min} + 2.5\Delta V = 1.25$
$2.5 \le V < 5$	3	11	$V = V_{\min} + 3.5\Delta V = 3.75$

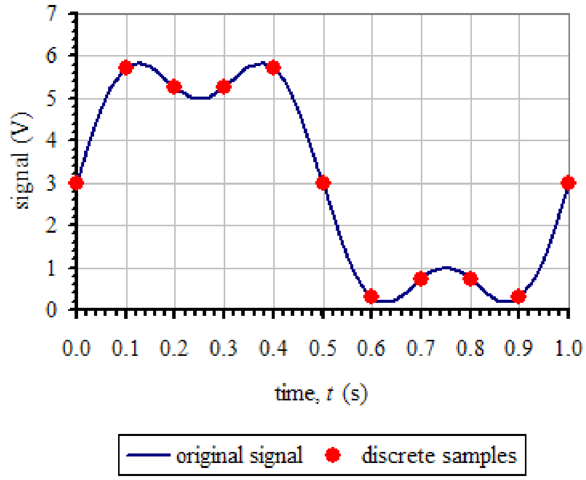
- The ***assigned voltage*** for each bin is here defined as half-way between the limits of the bin.
- The ***number of bins*** = 2^N for an N-bit A/D converter. For our example 2-bit converter, there are $2^2 = 4$ bins.
- The ***resolution*** of the A/D converter is defined as $\boxed{\text{resolution} = \Delta V = \dfrac{(V_{\max} - V_{\min})}{2^N}}$.
- In the present example (2-bit A/D converter), the resolution is $\Delta V = (5-(-5))/2^2 = 2.5\text{ V}$.
- Alternately, the resolution can be expressed as half of this value on either side, i.e., ±1.25 V. Another name for the resolution expressed this way is ***quantization error*** (also sometimes called ***quantizing error***), defined as $\boxed{\text{quantization error} = \pm\dfrac{1}{2}\text{resolution} = \pm\dfrac{1}{2}\Delta V = \pm\dfrac{1}{2}\dfrac{(V_{\max} - V_{\min})}{2^N} = \pm\dfrac{(V_{\max} - V_{\min})}{2^{N+1}}}$.
- In the present example (2-bit A/D converter), the quantization error is ±1.25 V.
- *This quantization error is too large for most practical applications*. Notice that we cannot tell the difference between an input of 2.6 V and 4.9 V – both of these input voltages fall into bin number 3, and would be assigned the output voltage of 3.75 V.
- We might express this reading as $V = 3.75$ V +/- 1.25 V to reflect the resolution of the A/D converter.
- Obviously, the bigger N, the better the resolution. Consider, for example, a 12-bit A/D converter ($N = 12$).
  - The number of bins = $2^{12} = 4096$, which is a digital output of 0 to 4095.
  - The resolution (for a converter with a range of –5 to 5 V) is $\Delta V = 0.00244141$ V.
  - The quantization error is $\pm\Delta V/2 = \pm 0.00122070$ V.
- Selected rows of the bin table for a 12-bit A/D converter with a range of –5 to 5 V are shown below: [*Note*: Again, this is just one of several optional ways to assign the bins.]

Analog voltage (volts)	Bin number	Digital output (binary)	Assigned voltage, V (volts)
$-5 \le V < -4.9976$	0	0000 0000 0000	$V = V_{\min} + 0.5\Delta V = -4.998779$
$-4.9976 \le V < -4.9951$	1	0000 0000 0001	$V = V_{\min} + 1.5\Delta V = -4.996338$
… etc. …	… etc. …	… etc. …	… etc. …
$0 \le V < 0.00244$	2048	1000 0000 0000	$V = V_{\min} + 2048.5\Delta V = 0.0012207$
… etc. …	… etc. …	… etc. …	… etc. …
$4.9951 \le V < 4.9976$	4094	1111 1111 1110	$V = V_{\min} + 4094.5\Delta V = 4.996338$
$4.9976 \le V < 5$	4095	1111 1111 1111	$V = V_{\min} + 4095.5\Delta V = 4.998779$

- Comparing the two tables above, it should be obvious that the quantization error is much less for the 12-bit A/D converter than for the 2-bit (with both converters covering the same range of –5 to 5 V.)
- Most commercially available A/D converters are 8, 12, 14, or 16-bit. [Those in our lab are 16-bit.]
- The range of most modern A/D converters is adjustable, and can be either ***monopolar*** (e.g., 0 to 1 V, 0 to 5 V, 0 to 10 V, etc.) or ***bipolar*** (e.g., –1 to 1 V, –10 to 10 V, etc.).
- ***Dynamic range***, ***DR*** = ratio of the largest voltage to the smallest *change* in voltage that can be measured, expressed in decibels (dB): monopolar: $\boxed{DR = 20\log_{10}\left(2^N\right)}$, bipolar: $\boxed{DR = 20\log_{10}\left(2^{N-1}\right)}$.
  Some authors call this the ***signal-to-quantization noise ratio***, ***SQNR*** (same equations as for DR).

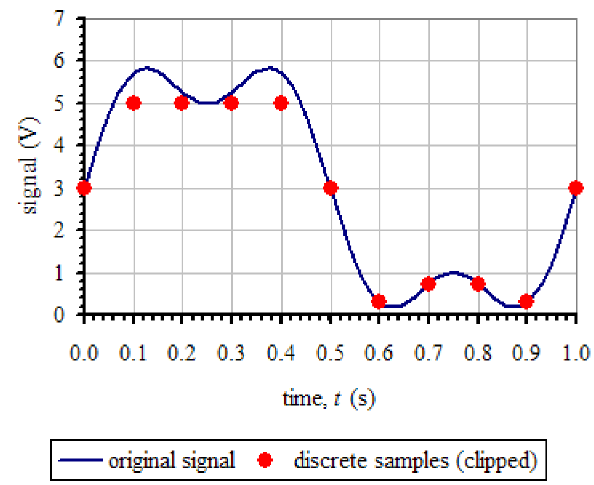
Discrete Sampling

- The main difference between analog and digital data acquisition is that digital systems sample the signal ***at discrete times only***, not continuously.
- In a digital data acquisition system, no information is recorded at times *in between* the discrete sampling times.
- Consider a continuous signal that is sampled discretely at ***sampling frequency f_s***, as sketched to the right. Here, f_s = 10 Hz.
- For a given sampling frequency f_s, the ***time period Δt*** between samples is the inverse of f_s, $\boxed{\Delta t = 1/f_s}$. Here, Δt = 1/(10 s^{-1}) = 0.1 s.
- *Warning*: If we are not careful, discrete sampling can lead to incorrect conclusions about the original signal!
- Two of the potential problems (*clipping* and *aliasing*) are discussed in detail below:



Clipping

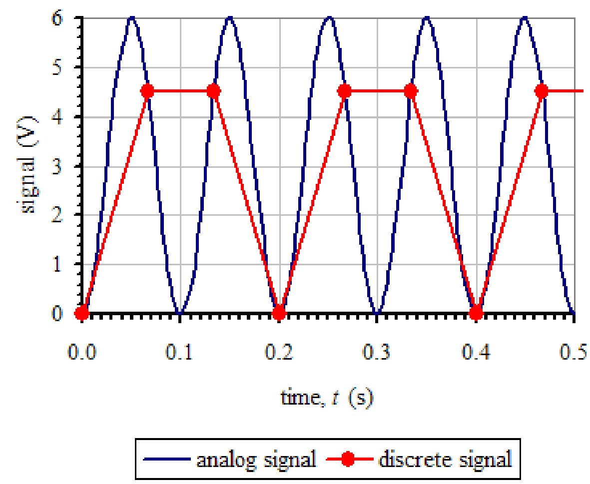
- If the original signal lies outside of the range of the A/D converter, it is ***clipped*** at the extreme value. In other words, any voltage greater than the upper limit of the A/D converter is assigned the maximum voltage of the A/D converter.
- Likewise, any voltage less than the lower limit of the A/D converter is assigned the minimum voltage of the A/D converter.
- For example, for a 12-bit A/D converter with a range of –5 to 5 V, any voltage above 5 V is assigned the maximum voltage of 4.9988 V, according to the above bin table.
- In most cases, the user does not know that the signal has been clipped, and this can lead to incorrect results, although some data acquisition systems give a warning when the signal gets clipped.
- A clipped signal (same signal as above, but clipped when using a –5 to 5 V A/D converter) is illustrated here.
- Notice that all voltages above 5 V in this example have been *clipped* to 5 V.
- Clipping would also occur if the voltage dips below -5 V.



Aliasing

- If the sampling frequency f_s is *too low*, the digital data acquisition system can actually measure an *incorrect frequency*! This is called ***aliasing***.
- Consider a pure sine wave.
- The equation for a general sine wave signal is $\boxed{g(t) = C + A\sin(2\pi ft - \phi)}$ or $\boxed{g(t) = C + A\sin(\omega t - \phi)}$, where f is the ***frequency*** (hertz), ω is the ***radian frequency*** (radians/s) and $\boxed{\omega = 2\pi f}$, A is the ***amplitude*** (volts), C is the ***DC offset*** (volts), t is the ***time*** (seconds), and ϕ is the ***phase shift*** (radians).
- Aliasing is best illustrated by example.
- Suppose the original signal is a pure 10 Hz sine wave, with an amplitude of 3 volts, a DC offset of 3 volts, and a phase shift of 90 degrees (π/2 radians):
  - The frequency of the signal is f = 10.0 Hz.
  - The ***period*** of the signal is $T = 1/f$ = 0.100 s.
  - The amplitude of the signal is A = 3.00 volts.
  - The DC offset of the signal is C = 3.00 volts.
  - The phase shift of the signal is $\phi = \pi/2$ radians.
  - For the sine wave used in this example, the equation of the signal is $\boxed{g(t) = 3 + 3\sin(2\pi(10)t - \pi/2)}$.
- This example sine wave is easily simulated in Excel or Matlab, as is the discrete sampling (and aliasing).

- Also, suppose we sample discretely (digitally) at a sampling frequency f_s = 15 Hz, 15 times per second, or one sample every $\Delta t = 1/(15\ \text{s}^{-1}) = 0.06666...$ s.

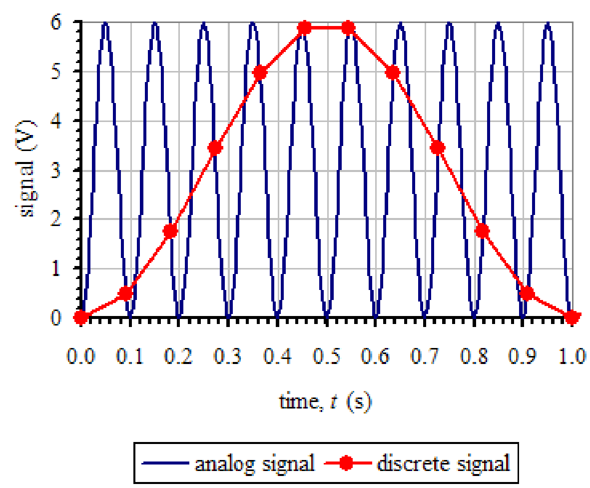


- A half second of both the analog signal and the discretely sampled data points is shown on the plot to the right.
- The aliasing is obvious – *the perceived signal looks nothing like the original*! In fact, the ***apparent frequency*** of the ***inferred*** or ***perceived signal*** (formed by connecting the discrete data points with straight line segments) is 5 Hz, and it is an odd-looking trapezoidal waveform rather than a sine wave.
- It turns out that if the sampling frequency f_s is greater than two-thirds of the actual frequency f, but less than twice the actual frequency, the ***perceived frequency*** (also called the ***aliasing frequency***, f_a) is equal to the absolute value of the difference between the sampling frequency and the actual frequency.
- In equation form, the above statement is:

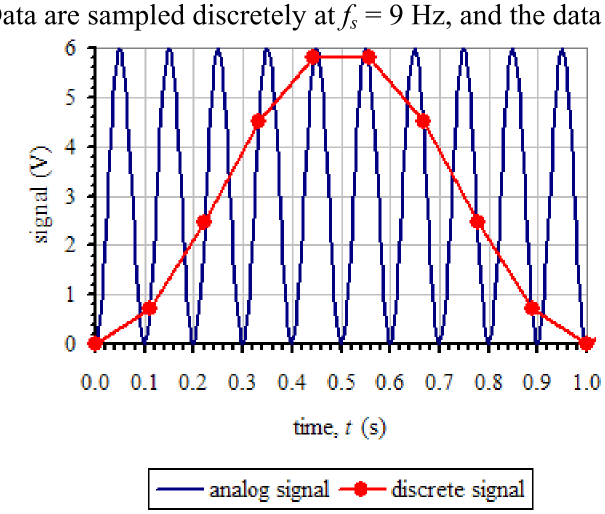
$$\boxed{\text{if } \frac{2}{3}f < f_s < 2f,\ \text{then } f_a = |f_s - f|}.$$

- In the above example, f = 10 Hz and f_s = 15 Hz, so $\frac{2}{3}f = \frac{2}{3}10 = 6.6666...$, which is less than f_s = 15 Hz.

  Therefore, the perceived frequency or aliasing frequency is $f_a = |f_s - f| = |15 - 10| = 5$ Hz. This agrees with the observed perceived frequency on the above plot (red line).



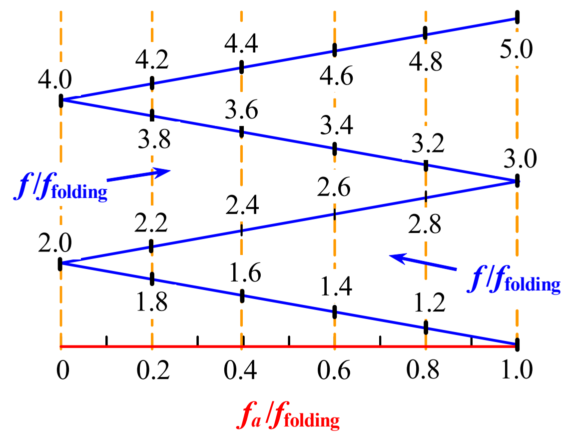
- Here is another example. For the same sine wave (f = 10 Hz), the signal is plotted to the right for 1 second. Data are sampled discretely at f_s = 11 Hz.
- Note that the perceived signal looks like a sine wave at 1 Hz! We check the predicted aliasing frequency by using the above equation: Since $\frac{2}{3}f = \frac{2}{3}10 = 6.6666...$, which is less than f_s = 11 Hz, the perceived frequency or aliasing frequency is $f_a = |f_s - f| = |11 - 10| = 1$ Hz. This agrees with the observed perceived frequency on the above plot (red line).
- One more example for the same sine wave (f = 10 Hz): Data are sampled discretely at f_s = 9 Hz, and the data are plotted to the right.



- Note that the perceived signal also looks like a sine wave at 1 Hz! We check the predicted aliasing frequency by using the above equation: Since $\frac{2}{3}f = \frac{2}{3}10 = 6.6666...$, which is less than f_s = 9 Hz, the perceived frequency or aliasing frequency is $f_a = |f_s - f| = |9 - 10| = 1$ Hz. This agrees with the observed perceived frequency on the plot to the right (red line).
- Comparing the plots for f_s = 11 Hz and f_s = 9 Hz, they look similar at first glance, since the perceived frequency is 1 Hz in both cases. However, careful inspection reveals that the sampled data points occur at different phases of the signal in the two plots. Furthermore, the 11-Hz plot contains 12 data points, while the 9-Hz plot contains only 10 data points between t = 0 and t = 1.0 s.

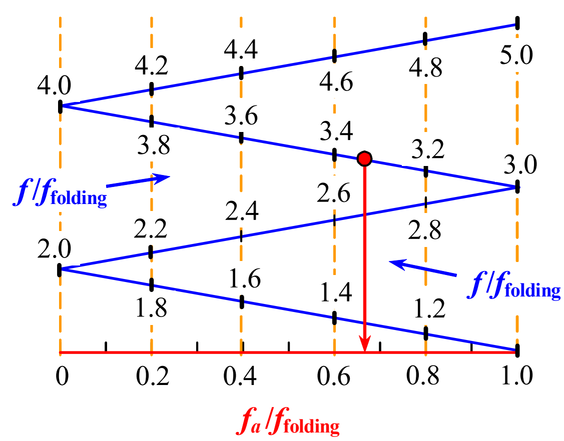
The sampling rate theorem and prediction of aliasing frequency

- The ***sampling rate theorem***, also called the ***Nyquist theorem***, helps us avoid aliasing. The sampling rate theorem is stated as follows:
  To avoid aliasing, the sampling frequency must be greater than twice the highest frequency component of the analog signal.
- The ***Nyquist criterion*** is $\boxed{f_s > 2f}$. ***The Nyquist criterion must be met in order to avoid aliasing.***
- To determine if there is aliasing, and, if so, to calculate the aliasing frequency, we apply the following rules, depending on the relative values of signal frequency f and sampling frequency f_s:
  - $\boxed{\text{If } f_s > 2f \text{, then there is no aliasing}}$. In words, if the sampling frequency is greater than twice the signal frequency, there is *no* aliasing.
  - $\boxed{\text{If } \frac{2}{3}f < f_s < 2f \text{, then } f_a = \left| f_s - f \right|}$. In words, if the sampling frequency is greater than two-thirds the signal frequency but less than twice the signal frequency, there *is* aliasing, and the aliasing frequency is equal to the absolute value of the difference between the sampling frequency and the signal frequency.
  - $\boxed{\text{If } f_s < \frac{2}{3}f \text{, then } f_a = \left(\frac{f_a}{f_{\text{folding}}} \right) f_{\text{folding}}}$, where f_{folding} is the ***folding frequency***, defined as $\boxed{f_{folding} = \frac{f_s}{2}}$, and the ratio f_a / f_{folding} is determined from the ***folding diagram***. In words, if the sampling frequency is less than two-thirds of the actual frequency, the aliasing frequency must be calculated from the folding diagram. A summary of the procedure is given below:
    - Calculate the folding frequency, $f_{\text{folding}} = f_s / 2$.
    - Locate f / f_{folding} on the folding diagram, as plotted on the right. *Note*: For values of f / f_{folding} greater than 5.0, the folding diagram can easily be extended, following the obvious pattern.
    - Read straight down from the value of f / f_{folding} to obtain the value of f_a / f_{folding} on the bottom (horizontal) axis.
    - Finally, calculate the aliasing frequency, $\boxed{f_a = \left(\frac{f_a}{f_{\text{folding}}} \right) f_{\text{folding}}}$.



  - The folding frequency is *half* of the sampling frequency because of the Nyquist criterion – you must sample at a frequency at least *twice* the signal frequency in order to avoid aliasing.
- A general equation [Shaparenko, B. and Cimbala, J. M., *Int. J. Mech. Engr Education*, Vol. 39, No. 3, pp. 195-199, 2012] is available to determine the perceived frequency of *any* signal frequency f when sampled at *any* sampling frequency f_s, whether there is aliasing or not: $\boxed{f_{\text{perceived}} = \left| f - f_s \cdot \text{NINT}\left(\frac{f}{f_s} \right) \right|}$, where
  - NINT is the "nearest integer" function.
  - In Excel, use ROUND(x,0) to round real number x to the nearest integer.

- **Example**:
  Given: A sine wave of frequency 10 Hz is sampled at a sampling frequency of 6 Hz.
  To do: Calculate the *perceived* frequency of the sampled signal.
  Solution: We follow the procedure outlined above.
  - For f = 10 Hz, $2f/3$ = 2(10 Hz)/3 = 6.66666...
  - Since f_s = 6 Hz is *less than* $2f/3$ = 6.66666..., the simple formula cannot be used. We use the folding diagram to calculate the aliasing frequency f_a.
  - The folding frequency is $f_{\text{folding}} = f_s/2$ = 6/2 = 3 Hz.



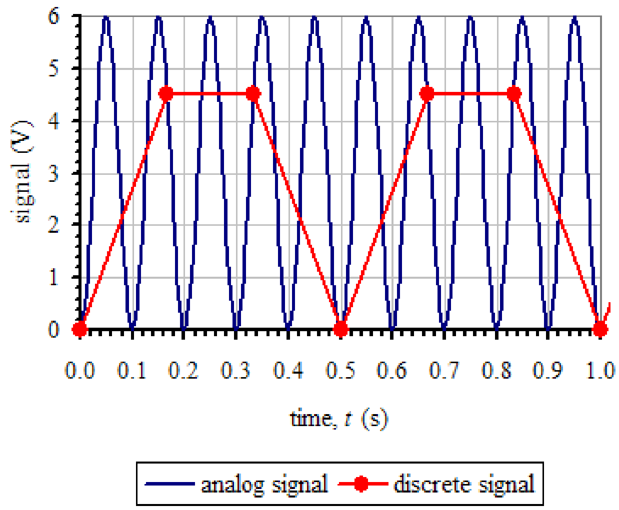
- We calculate $f/f_{\text{folding}} = 10/3 = 3.33333...$
- We locate this value of f/f_{folding} on the folding diagram (see above), and read down to the f_a/f_{folding} axis at the bottom: At $f/f_{\text{folding}} = 3.33333...$, $f_a/f_{\text{folding}} = 0.66666...$
- Finally, we calculate the aliasing frequency:
  $f_a = \left(\frac{f_a}{f_{\text{folding}}}\right) f_{\text{folding}} = (0.66666...)(3\text{ Hz}) = 2\text{ Hz}$. The perceived signal will be aliased with an aliasing frequency of $\boxed{f_a = 2\text{ Hz}}$.

Alternate Solution: We use the general equation for perceived frequency.

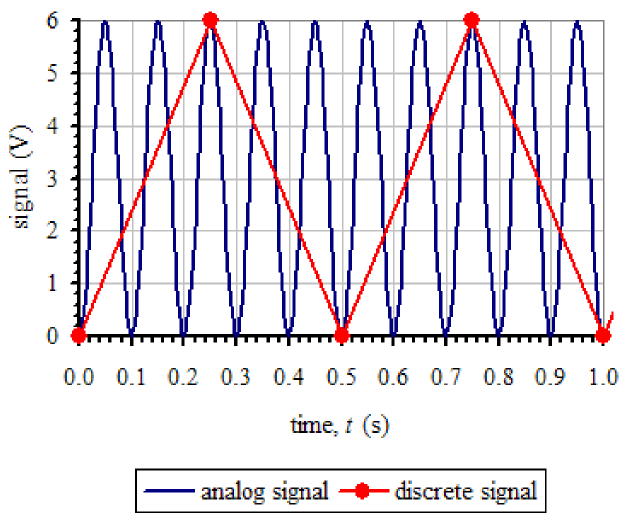
- For $f = 10$ Hz and $f_s = 6$ Hz, $f_{\text{perceived}} = \left| f - f_s \cdot \text{NINT}\left(\frac{f}{f_s}\right) \right| = \left| 10 - 6 \cdot \text{NINT}\left(\frac{10}{6}\right) \right| = \left|10 - 6 \cdot 2\right| = 2\text{ Hz}$.
- We wee that the general equation yields the correct perceived frequency, i.e., the aliasing frequency in this case, without having to calculate the folding frequency or use the folding diagram.

Discussion:

- Since f_s is *less than* $2f/3$ in this example, we cannot use the simple difference equation to calculate the aliasing frequency. In other words, $f_a \neq \left|f_s - f\right| = \left|6 - 10\right| = 4\text{ Hz}$. Rather, $f_a = 2$ Hz.
- We simulate this example using Excel, and a plot of the actual (analog) signal and the perceived (discrete digital) signal is shown to the right. The perceived signal does not even *look* like a sine wave, but it does indeed have a frequency of 2 Hz, as predicted.



- *You are encouraged to use the accompanying Excel spreadsheet to experience aliasing* – the spreadsheet is set up so that you can change the sampling frequency f_s and watch the plot change. The signal frequency is 10 Hz, but this, along with the amplitude, DC offset, and phase shift, can also be changed easily.
- For example, if f_s is changed to 4 Hz, a *triangular* wave pattern is seen as the perceived signal, with an aliasing frequency of 2 Hz, as shown below. Try to predict this aliasing frequency using the above procedure.



- Finally, we note that the folding diagram and the general equation can *always* be used, regardless of the values of f and f_s. In other words, whether or not there is aliasing, and whether or not $f_s < 2f/3$, you can still calculate the perceived frequency by following the procedure outlined above, using either the folding diagram or the general equation for perceived frequency.