Experimental Design

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Introduction

- When setting up an experiment, it is important to first take some time to carefully *design* the experiment.
- In this module we discuss *how to design an experiment*. A particularly helpful reference book for this material is Elmer E. Lewis, Introduction to Reliability Engineering, 2nd ed., Wiley, TA169.L47 1996, Ch. 4.

The basics of experimentation

Below are listed the basic steps for conducting an experiment. Each of these steps is critical to a successful experiment.

- ***Define the problem***
  - This may seem obvious, but it is really the most important and most critical step. Namely, the ***required output of the experiment*** must be clearly defined.
  - Examples include:
    - Determine the operating temperature range of a voltmeter such that the overall error is less than some value.
    - Determine the optimum low-pass filter for an audio system that minimizes cost while still meeting performance specifications.
- ***Design the experiment***
  - This step involves many components, including (but not limited to):
    - Conduct a ***literature survey***.
    - Perform an ***analytical analysis*** (as far as possible).
    - Select the variables to be measured. *Note*: ***Dimensional analysis*** is useful here, since it can potentially *decrease the number of required independent variables to be measured*.
    - Select the ***instruments*** to be used in the experiment.
    - Estimate the ***experimental uncertainties***.
    - Select or design a ***test matrix***. (Test matrices are discussed in detail in this module.)
    - Design the ***test rig*** and the ***experimental procedure***.
- ***Construct the experiment***
- ***Gather data***
- ***Analyze the data***
- ***Do confirmation experiments and/or follow-up experiments*** [***if necessary***].
- ***Interpret and report results/conclusions***

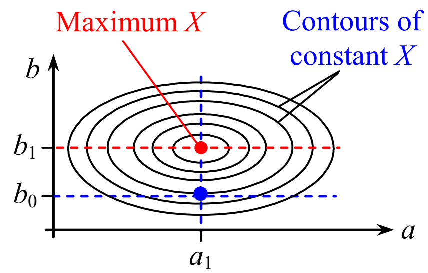
Other issues

- There are other issues that enter into consideration when conducting an experiment, such as
  - ***cost***
  - ***schedule***
  - ***personnel***
- These issues will not be addressed here.
- In this learning module, emphasis is placed on one very critical aspect of experimental design, namely *choosing a test matrix*.

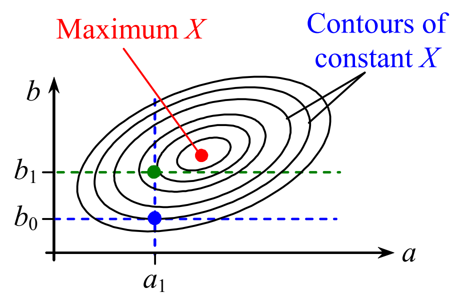
Choosing a test matrix

- By way of introduction, suppose some experiments must be conducted in order to determine an ***optimum value*** of parameter X, which is a function of parameters a, b, c, ..., i.e., $\boxed{X = X(a, b, c, ...)}$.
- The optimum value can either be a ***maximum*** (for example best efficiency, longest life, or highest strength) or a ***minimum*** (for example shortest time, minimum cost, lowest pressure drop, or smallest surface nonuniformity). In all the examples below, it is assumed that the *optimum* value of X is the *maximum* value of X. The analysis is similar if X is to be *minimized* instead.
- In this learning module, you will learn *how to choose a test matrix so that the number of necessary experimental runs (and therefore cost and time) is kept to a minimum*.
- It should be noted that parameters a, b, c, ... can be ***independent*** of each other (e.g., a does not influence the value of b or c and vice-versa), or ***dependent*** on each other (e.g., if a changes, b and/or c also change).
- The dependence or independence of the parameters on each other impacts the test matrix significantly.
- To illustrate, consider for simplicity a function X of only two parameters, a and b, i.e., $X = X(a, b)$.

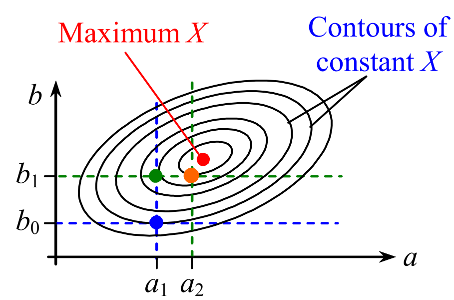
- In the simplest case, suppose that parameters *a* and *b* are *independent*. In other words, *a* has no influence whatsoever on *b*, and vice-versa.
- A ***contour plot*** (also called an ***isocontour plot***) of *X* as a function of *a* and *b* is sketched to the right.



- The ellipses represent contours of constant *X* (iso-*X* contours), increasing in value towards the center, and the red dot in the middle represents the maximum *X* value – the *optimum value*, which is desired. The goal of the experiment is to find the values of *a* and *b* (labeled a_1 and b_1 in the diagram) that yield the maximum (optimum) value of *X*.
- Note: *In an actual experiment, such a contour plot will not be available (although it can be constructed if desired). It is used here for illustrative purposes only*.
- The most simple-minded approach is called "***one parameter at a time***". As the name implies, this technique involves fixing all parameters except *one*, and varying the remaining parameter as follows:
  - Some value of *b* is chosen randomly. In the diagram, this value is b_0.
  - Several experiments are run, measuring *X* with *b* fixed at the value b_0, while varying the value of *a*. This is illustrated by the horizontal blue dashed line.
  - The maximum value of *X* is found at $a = a_1$, as indicated by the blue dot.
  - Next, *a* is kept fixed at a_1, and *b* is varied, as represented by the vertical blue dashed line.
  - The maximum value of *X* is found at $b = b_1$, as indicated by the red dot.
  - It turns out that since *a* and *b* are independent of each other, the true optimum point is at $a = a_1$ and $b = b_1$, i.e., at point (a_1, b_1), and no further testing is required.
  - The experiment is a success – the maximum value of *X* has been found.
- Now suppose instead that parameters *a* and b are *related to each other* in some way. For example, if *a* is temperature and *b* is pressure, it is quite likely that as *a* increases, *b* also increases.
- The contour plot of *X*(*a*, *b*) will not look the same as the above because of the interdependence of *a* and *b*. In fact, the contours of constant *X* may be *tilted* as sketched to the right.



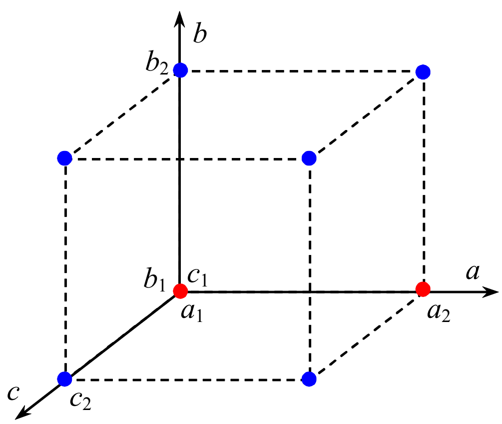
- Suppose we employ the simple-minded (one parameter at a time) experimental approach again:
  - Some value of *b* ($b = b_0$) is chosen randomly.
  - Several experiments are run, measuring *X* while keeping *b* at b_0, but varying the value of *a*. This is illustrated by the horizontal blue dashed line.
  - The maximum value of *X* from these experimental runs is found at $a = a_1$, as indicated by the blue dot.
  - Next, *a* is kept fixed at a_1, and *b* is varied, as represented by the vertical blue dashed line.
  - The maximum value of *X* from these runs is found at $b = b_1$, as indicated by the green dot.
  - Unfortunately, since *a* and b are dependent on each other, the true optimum point is *not* at point (a_1, b_1)!
  - The experiment is a failure - the maximum value of *X* has *not* been found.
- Of course, we could continue with further experiments to "zero in" on the optimum.
- For example, *b* can now be kept fixed at b_1, and parameter *a* can be varied to find the maximum *X*, as illustrated in the sketch by the horizontal green dashed line and the orange dot at (a_2, b_1):



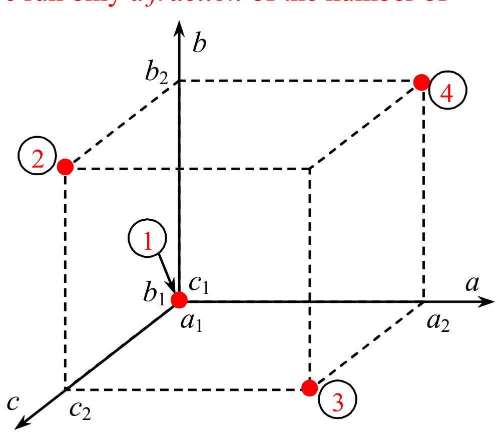
- This has gotten *closer* to the true optimum point, but it can take numerous experiments to find the optimum this way.
- The problem gets much worse when there are *several* parameters to be varied, i.e., *a*, *b*, *c*, *d*, ...
- Thus, ***the goal is to devise a test matrix that "hunts" for the optimal point most efficiently*** (i.e., with the fewest number of experimental runs).
- ***Genichi Taguchi*** is famous for devising intelligent test matrices for this very purpose. His technique is now called the ***Taguchi technique*** or the ***robust design method***. The basics of this technique are described in this learning module.

Taguchi's designed experiments

- Taguchi realized that experiments in which only one parameter at a time is varied are *inefficient*, particularly when interaction exists between parameters.
- He developed methodologies whereby ***all parameters are varied simultaneously*** to reduce the required number of experimental runs.
- These methodologies are called ***designed experiments***.
- Some of the terminology and an illustrative example are provided below.
- ***Full factorial analysis***
  - Let ***P*** equal the ***number of parameters***.
  - Let ***L*** equal the ***number of levels*** to be tested for each parameter.
  - A ***full factorial experiment*** is one in which each of the *P* parameters is varied to *L* different levels, while holding all the other parameters constant (one parameter at a time).
  - Let *N* represent the ***total number of experimental runs***. It turns out that *N* can be easily calculated from *L* and *P*, i.e., $N = L^P$ for a full factorial experiment.
  - For example, in the simple experiment above, where $X = X(a, b)$, there are *two* parameters, *a* and *b* ($P = 2$). Suppose that *four levels* of each parameter are to be tested ($L = 4$). Specifically, there are four levels of parameter *a* (a_1, a_2, a_3, and a_4), and four levels of parameter *b* (b_1, b_2, b_3, and b_4).
  - The number of required runs for a full factorial experiment with $P = 2$ and $L = 4$ is $N = L^P = 4^2 = 16$.
  - As another example, consider $X = X(a, b, c)$, and only *two levels* of each parameter are to be tested. Here, $L = 2$ and $P = 3$, thus a full factorial experiment requires $N = L^P = 2^3 = 8$ runs.

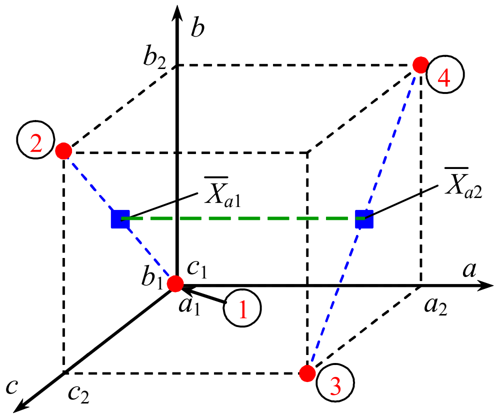


  - For this latter example, the 8 experimental runs are illustrated graphically as points on a 3-D plot. Note that here, for simplicity, the lowest level of *a*, *b*, and *c* (a_1, b_1, and c_1, respectively) are assumed to be zero so that the box can be drawn with one corner on the origin as sketched. In a real experiment, it is not necessary that the lowest level of any of the parameters be zero.
  - Notice that for *each* parameter, *two levels* of that parameter are tested while holding each of the other two parameters constant.
  - For example, consider the two red points on the plot. Parameter *a* is varied from a_1 to a_2 while *b* and *c* are fixed at b_1 and c_1 respectively.
  - Each point (6 blue ones and 2 red ones) represents an experimental run; *a total of eight runs are required for two levels and three parameters*. This is a *full factorial experiment*.
- ***Fractional factorial analysis***
  - Taguchi showed that it is not necessary to run full factorial experiments, as long as the experimental runs are chosen appropriately and intelligently.
  - A ***fractional factorial experiment*** is defined as one in which we run only a *fraction* of the number of runs necessary for a full factorial experiment.
  - Specifically, we construct an experimental test matrix in which there are still *L* levels for each parameter, and there are still *P* parameters, but some of the data points in the full factorial experiment are "skipped".
  - For the above example in which $X = X(a, b, c)$ and two levels of each parameter are tested ($P = 3$ and $L = 2$), *we can cut the number of runs in half, while still meeting the criterion of two levels for each parameter*!
  - One such fractional factorial experiment is illustrated here on the 3-D plot to the right.

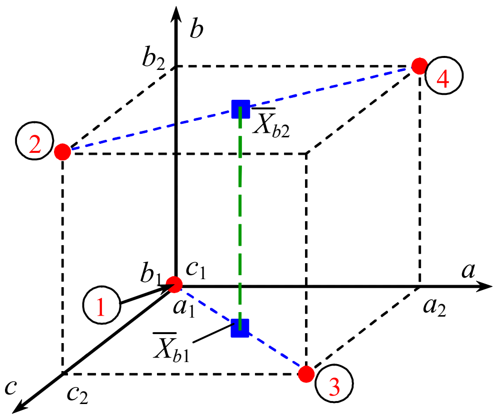


  - In this example, there are only 4 required runs (illustrated by the four red dots); the number of runs for this fractional factorial experiment has been reduced by half.
  - Run numbers 1 through 4 are labeled on the sketch for convenience. The result of the experimental run at each of these run numbers is designated by X_1, X_2, X_3, and X_4 respectively.

- Notice: There are still *two levels* tested for parameter *a* (points 1 and 2 at a_1, the lower level of *a*, and points 3 and 4 at a_2, the higher level of *a*).
- Similarly, there are still *two levels* tested for parameter *b* and for parameter *c*.
- In such an experiment, some ***averaging*** needs to be done in order to find the effect of one of the parameters on *X*.
- For example, to find the effect of parameter *a* on *X*, the average of *X* at level 1 of parameter *a* is compared to the average of *X* at level 2 of parameter *a*.
- The ***level average*** for level 1 of parameter *a* is defined as the ***average over all runs where a is at level 1***, ($a = a_1$). Here, the level average is $\overline{X}_{a1} = (X_1 + X_2)/2$.
- Similarly, the level average for level 2 of parameter *a* ($a = a_2$) is $\overline{X}_{a2} = (X_3 + X_4)/2$. This is illustrated graphically by the blue square symbols in the sketch to the right.
- Notice: When studying the effect of parameter *a* on *X*, both high and low values of *b*, and both high and low values of *c* are incorporated into the averages.



- The dashed green line shows that these two level averages indicate how *X* varies with parameter *a* alone.
- This ***averaging effect*** also helps to smooth out errors due to random noise.
- Let's do a similar analysis to find the effect of parameter *b* on *X*: The ***level average*** for level 1 of parameter *b* is defined as the ***average over all runs where b is at level 1***, ($b = b_1$). Here, the level average is $\overline{X}_{b1} = (X_1 + X_3)/2$. Similarly, the level average for level 2 of parameter *b* ($b = b_2$) is $\overline{X}_{b2} = (X_2 + X_4)/2$. This is illustrated graphically by the blue square symbols in the sketch to the right.
- You can draw similar level averages for parameter *c* on the above sketch.



- Fractional factorial methods become even *more* useful as *P* (the number of parameters) increases, because ***the number of required experimental runs can be reduced significantly***.

Taguchi design arrays

- Taguchi also developed tables called ***design arrays*** or ***test matrices*** to aid in experimental design. These arrays can be used for both full factorial and fractional factorial analyses.
- An *optimum* Taguchi design array for a fractional factorial analysis adheres to the following two rules:
  - ***Each level of each parameter appears the same number of times in the array***.
  - ***Repetitions of parameter-level combinations are minimized as much as possible***.
- **Example**: First we look at a very simple example.
  Given: $X = X(a, b, c)$ and two levels of each parameter are to be tested ($P = 3$ and $L = 2$). This is the same example illustrated graphically above.
  To do: Develop a full factorial design array and a fractional factorial Taguchi design array, and compare.
  Solution:
  - We construct a table with 5 columns, one for the run number, one for each of the three variables *a*, *b*, and *c*, and one for the result *X*.
  - For a ***full factorial test***, we require 8 runs, with both levels of each parameter tested 4 times. The design array or test matrix is shown to the right.
  - In the table, the numbers below *a*, *b*, and *c* represent the *level* of parameter *a*, *b*, or *c*, respectively. For example, the 2 in run 4, column *b* indicates level 2 of parameter *b*, i.e., b_2.

Full factorial, $P=3$, $L=2$				
Run #	***a***	***b***	***c***	***X***
1	1	1	1	X_1
2	1	1	2	X_2
3	1	2	1	X_3
4	1	2	2	X_4
5	2	1	1	X_5
6	2	1	2	X_6
7	2	2	1	X_7
8	2	2	2	X_8

- For the ***fractional factorial test*** discussed above, only *four* runs are required, as illustrated by the red dots in the above 3-D sketches (both levels of each parameter tested twice).
- We carefully construct the Taguchi array so that the run number corresponds to the circled number labeled on the red dots in the 3-D sketches above. Using these sketches as a guide, we construct the fractional factorial (Taguchi) array or test matrix shown to the right.
- The experiments are run *in order* by run number, and the result is shown in the final column. For example, run number 4 uses level 2 of parameter *a* ($a = a_2$), level 2 of parameter *b* ($b = b_2$), and level 1 of parameter *c* ($c = c_1$). The experimental result for this run is called X_4.

Taguchi, $P = 3$, $L = 2$				
Run #	***a***	***b***	***c***	***X***
1	1	1	1	X_1
2	1	2	2	X_2
3	2	1	2	X_3
4	2	2	1	X_4

Discussion: Comparing the two arrays, we see that the Taguchi fractional factorial array requires only half of the number of runs, yet still tests two levels of each of the three parameters.

- We check whether our Taguchi design array is optimum – does it satisfy the two rules given above?
  - *Does each level of each parameter appear the same number of times in the array*? Yes. For example, level 1 of parameter *a* appears twice, in runs 1 and 2. Level 2 of parameter *c* also appears twice, in runs 2 and 3, etc. This is true for any of Taguchi's design arrays.
  - *Are repetitions of parameter-level combinations minimized as much as possible*? Yes. For example, when parameter *a* is at level 1 (runs 1 and 2), parameter *b* is at level 1 in run 1 and at level 2 in run 2; there is no situation in which there are *two* runs with both *a* at level 1 and *b* at level 1. You can verify that this is the case for *any* combination of parameters.
- Since both rules are met, this is indeed an *optimum Taguchi design array* for a fractional factorial analysis.
- *Note*: In Taguchi design arrays, three levels is more useful than two levels, so that trends and maxima or minima can be discerned, as discussed later. First, an example with three levels of each parameter.

- **Example**: Now we look at a more complicated example.

Given: $X = X(a, b, c, d)$ – four parameters, and *three* levels of each parameter are to be tested ($P = 4$ and $L = 3$).

To do: Calculate how many runs would be required for a full factorial experiment. Generate a Taguchi array that still tests three levels of each parameter, but with fewer runs; compare the number of required runs.

Solution:

- A ***full factorial*** experiment would require $N = L^P = 3^4 = 81$ runs.
- A well-designed ***fractional factorial experiment*** can be created that requires only 9 runs, while still testing three levels of each of the four parameters. We choose to test each level three times: ($3 \times 3 = 9$ runs).
- The Taguchi design array for this case is shown to the right.
- We check whether our Taguchi design array satisfies the two rules given above:
  - *Does each level of each parameter appear the same number of times in the array*? Yes. For example, level 2 of parameter *b* appears three times, in runs 2, 5, and 8. Level 3 of parameter *c* appears three times, in runs 3, 5, and 7. Level 1 of parameter *d* appears three times, in runs 1, 5, and 9. This rule is satisfied for each level of each parameter.
  - *Are repetitions of parameter-level combinations minimized as much as possible*? Yes. For example, when parameter *a* is at level 2 (runs 4, 5, and 6), parameter *b* is at levels 1, 2, and 3, respectively (no repeats), parameter *c* is at levels 2, 3, and 1, respectively (no repeats), and parameter *d* is at levels 3, 1, and 2, respectively (no repeats). Similarly, when parameter *d* is at level 3 (runs 3, 4, and 8), parameter *c* is at levels 3, 2, and 1, respectively (no repeats), etc. You are welcome test other possible combinations – you will find no situation in which there are *two* runs with repeated levels of any two parameters. *This is an optimum Taguchi design array*.

Taguchi, $P = 4$, $L = 3$					
Run #	***a***	***b***	***c***	***d***	***X***
1	1	1	1	1	X_1
2	1	2	2	2	X_2
3	1	3	3	3	X_3
4	2	1	2	3	X_4
5	2	2	3	1	X_5
6	2	3	1	2	X_6
7	3	1	3	2	X_7
8	3	2	1	3	X_8
9	3	3	2	1	X_9

- The second rule is important because the level averages contain values over the entire range of each parameter, and are not falsely "weighted" at any particular level of any parameter. (Level averages are discussed in detail below.)

Discussion: The Taguchi fractional factorial design array allows us to design the experiment with only 9 runs instead of 81 runs – a great improvement! This is significant because the cost and/or time required to run the experiment are reduced by nearly an order of magnitude.

- Shown to the right is an *improper* Taguchi design array for the same fractional factorial analysis as in the above example (P = 4 and L = 3), formed by interchanging two values in column d, rows 7 & 8. At first glance, it looks okay. But why is it improper? Let's look at the rules again:
  - *Does each level of each parameter appear the same number of times in the array*? Yes. The first rule is satisfied.
  - *Are repetitions of parameter-level combinations minimized as much as possible*? No. The second rule is violated. For example, when parameter c is at level 1 (runs 1, 6, and 8), parameter d is at levels 1, 2, and 2, respectively – level 2 of parameter d is *repeated* when parameter c is at level 1. This problem is highlighted. *Can you spot any other problems with this array?*

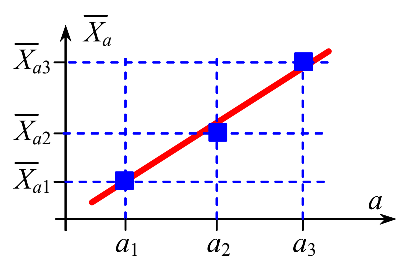
Improper array, P=4, L=3					
Run #	***a***	***b***	***c***	***d***	***X***
1	1	1	1	1	X_1
2	1	2	2	2	X_2
3	1	3	3	3	X_3
4	2	1	2	3	X_4
5	2	2	3	1	X_5
6	2	3	1	2	X_6
7	3	1	3	3	X_7
8	3	2	1	2	X_8
9	3	3	2	1	X_9

Using level averages to predict the optimum (maximum) X

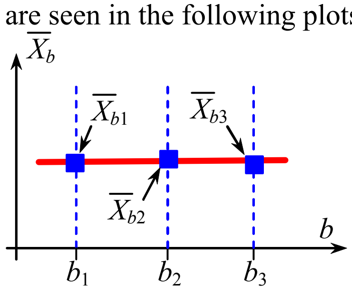
- In order to determine the effect of a parameter on X, we calculate and plot the ***level averages***, defined as ***the average value of X at each level of the parameter***.
- For example, in the *proper* Taguchi array above for the case with P = 4 and L = 3, we define the level average for level 1 of parameter a as the ***average over all runs where a is at level 1*** ($a = a_1$). Using the Taguchi array as our guide, we see that this level average is equal to $\overline{X}_{a1} = (X_1 + X_2 + X_3)/3$.
- Similarly, we define all 12 level averages (3 levels for each of the 4 parameters):

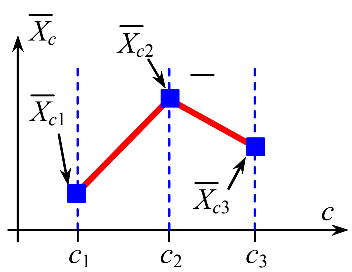
$\overline{X}_{a1} = (X_1 + X_2 + X_3)/3$	$\overline{X}_{a2} = (X_4 + X_5 + X_6)/3$	$\overline{X}_{a3} = (X_7 + X_8 + X_9)/3$
$\overline{X}_{b1} = (X_1 + X_4 + X_7)/3$	$\overline{X}_{b2} = (X_2 + X_5 + X_8)/3$	$\overline{X}_{b3} = (X_3 + X_6 + X_9)/3$
$\overline{X}_{c1} = (X_1 + X_6 + X_8)/3$	$\overline{X}_{c2} = (X_2 + X_4 + X_9)/3$	$\overline{X}_{c3} = (X_3 + X_5 + X_7)/3$
$\overline{X}_{d1} = (X_1 + X_5 + X_9)/3$	$\overline{X}_{d2} = (X_2 + X_6 + X_7)/3$	$\overline{X}_{d3} = (X_3 + X_4 + X_8)/3$

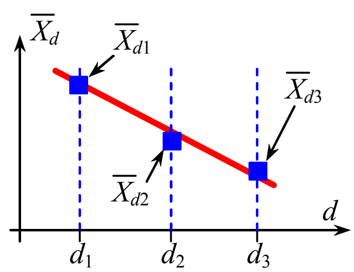
- Plots of the level averages show the effect of each parameter on result X.
- For example, to determine the effect of parameter a on X, we create a plot of $\overline{X}_a$ as a function of parameter a. Specifically, we plot level averages $\overline{X}_{a1}$, $\overline{X}_{a2}$, and $\overline{X}_{a3}$ at their corresponding values of a_1, a_2, a_3, respectively, as shown in the sketch to the right.



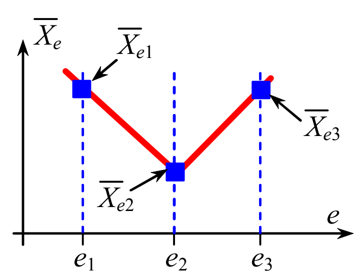
- The red line is a ***trend line***. Note that there is some scatter in the data points, and the trend line does not necessarily go through each point exactly.
- Similarly, the effects of parameters b, c, and d, along with their trends are seen in the following plots:







- With three levels, there are five possible outcomes:
  - ***No effect*** (no significant trend – a nearly horizontal line), as in the plot of $\overline{X}_b$ above.
  - ***Increasing*** (trend upward), as in the plot of $\overline{X}_a$ above.
  - ***Decreasing*** (trend downward), as in the plot of $\overline{X}_d$ above.
  - ***Maximum*** (up and then down, with maximum in between), as in the plot of $\overline{X}_c$ above.
  - ***Minimum*** (down and then up, with minimum in between), as sketched to the right for a fake parameter e. [We have only 4 parameters in our example, so we cannot illustrate all five possible outcomes; therefore, we introduce a fifth parameter e for illustrative purposes only.]



- If the levels tested in this experiment are the *only* available levels, the experiment would be complete.
- Since our goal from the outset is to maximize *X*, *we choose the level of each parameter that yields the maximum X*. In this case, we choose to run the experiment at $a = a_3$, $c = c_2$, and $d = d_1$, so as to maximize *X*.
- Since parameter *b* has no significant effect on *X*, ***parameter b can be selected strictly on the basis of cost***.
- In this particular example, the combination of parameters predicted to yield the maximum value of *X* turns out to be one of the experimental test runs, namely run number 9 in which $a = a_3$, $c = c_2$, and $d = d_1$. We are not always this fortunate!
- In many cases, the optimum case is predicted to be a particular combination of parameters that is *not* represented by any of the test runs. For such cases, a confirmation experiment may be necessary.
- In addition, if levels other than the ones tested are feasible, and funding is available, the engineer may choose to do some follow-up experiments.

Confirmation experiment and follow-up experiments

- In many cases, as discussed above, the exact combination of levels determined to be optimum may not have been one of the original test runs in the experimental design array. In such a case, it is prudent to perform at least one more test, with *each parameter set at its optimum level*, to see if the result is indeed an optimized result. Such a test is called a ***confirmation experiment***.
- Suppose, for example, that the results of the above sample experiment turned out such that levels a_3, b_1, c_2, and d_1 were found to be optimum. Unfortunately, the original experimental design array does *not* contain a test with this exact combination of the four levels. Hence, the confirmation experiment would involve testing at levels a_3, b_1, c_2, and d_1. If the Taguchi technique works properly, the maximum value of *X* should be achieved for this combination of parameters.
- We may also wish to design ***follow-up experiments*** to "zero in" on the optimum output, i.e., to determine the optimum value of *X* more precisely.
- For example, in the above sample experiment, follow-up experiments are based on the results of the first nine experimental runs:
    - Parameter *b* has negligible effect on *X*. It does not need to be included in the follow-up tests at all.
    - Parameter *a* should be tested at *higher* values, if feasible, since we found that *X* increases with *a*.
    - Parameter *d* should be tested at *lower* values, if feasible, since we found that *X* decreases with *d*.
    - Parameter *c* should be varied in smaller increments around level c_2 in order to "zero in" on the true maximum value of *X*.
- The follow-up experiments in this example are simpler to set up and less costly because there are now only three parameters to be varied – *a*, *c*, and *d* ($P = 3$ instead of 4), since *b* has negligible effect on the value of *X*.
- One possible Taguchi design array for the follow-up experiments is shown in the table to the right for three levels for each of three parameters *a*, *c*, and *d* ($P = 3$ and $L = 3$), with each level tested *twice* for each parameter: ($3 \times 2 = 6$ runs).
- Notice that only 6 experimental runs are needed for this fractional factorial experiment, compared to $N = L^P = 3^3 = 27$ runs for a full factorial experiment.
- Another possible Taguchi design array is shown to the lower right for this same example. Here, we choose to test each level of each parameter *three* times instead of two times, requiring 9 runs: (3 levels × 3 times for each level = 9 runs).
- Let's compare the two arrays (6 runs vs. 9 runs):
    - This larger array is "better" than the smaller array because it includes more combinations of levels and parameters, and the level averages are therefore more accurate and more meaningful.
    - Of course, this larger array requires a more expensive and time-consuming test.
- As with most engineering problems, there are tradeoffs between cost and quality.

Taguchi, $P = 3$, $L = 3$				
Run #	***a***	***c***	***d***	***X***
1	1	1	1	X_1
2	1	2	2	X_2
3	2	3	3	X_3
4	2	1	2	X_4
5	3	2	3	X_5
6	3	3	1	X_6

Taguchi, $P = 3$, $L = 3$				
Run #	***a***	***c***	***d***	***X***
1	1	1	1	X_1
2	1	2	2	X_2
3	1	3	3	X_3
4	2	1	2	X_4
5	2	2	3	X_5
6	2	3	1	X_6
7	3	1	3	X_7
8	3	2	1	X_8
9	3	3	2	X_9

Final comments

- There are *other* design arrays (other than Taguchi arrays) in use by engineers.
    - For example, the ***latin square test matrix*** is a popular design array.
        - For a case with 3 parameters ($P = 3$) and 5 levels each ($L = 5$), a full factorial test would require $N = L^P = 5^3 = 125$ experimental runs.
        - The latin square test matrix requires only 25 runs for $P = 3$ and $L = 5$, which represents a reduction in the number of runs by a factor of 5.

- The analyses presented here are simplified. More sophisticated analyses are available, beyond the scope of this course. They include, for example:
  - Separate treatment of ***controllable*** factors and ***uncontrollable*** (noise) factors. *In this learning module, we consider only controllable factors*.
  - ***inner arrays*** and ***outer arrays*** for testing of controllable and uncontrollable factors, respectively. *Here, we consider only inner arrays.*
  - Calculation of ***signal-to-noise ratios*** (***SNRs***) to provide quantitative analysis of the *quality* of the results.
  - Examination of ***interactions*** between controllable parameters and uncontrollable (noise) parameters. (These are called ***two-factor interactions***.)
- Nevertheless, although simple, the fractional factorial test matrices presented here are particularly useful for the initial design of an experiment, and you are encouraged to use them wherever possible.
- Taguchi also designed what are called ***orthogonal*** test matrices or ***Taguchi orthogonal arrays***. These are discussed in more detail in the next learning module.