Introduction to Mechanical Engineering Measurements

Author: John M. Cimbala, Penn State University
Latest revision, 19 August 2011

Two Main Purposes of Measurements

- ***Engineering experimentation*** **-** This is where we seek *new information*, and is generally done when developing a *new product*. Some example questions which may be asked by the engineer are: How hot does it get? How fast does it go?
- ***Operational systems*** **-** This is where we monitor and control processes, generally on *existing equipment* rather than in the design of new products. For example, consider the heating and/or air conditioning control system in a room. The system measures the temperature, and then controls the heating or cooling equipment.

Dimensions and Units

- Primary (or Basic) Dimensions
  There are seven ***primary dimensions*** (also called ***basic dimensions***). All other dimensions can be formed by combinations of these. The primary dimensions are:

  mass, length, time, temperature, current, amount of light, and amount of matter.
- Unit Systems
  Unit systems were invented so that *numbers* could be assigned to the dimensions.
  - There are three primary unit systems in use today:
    - the ***International System of Units*** (SI units, from *Le Systeme International d'Unites*, more commonly simply called the ***metric system of units***)
    - the ***English Engineering System of Units*** (commonly called ***English system of units***)
    - the ***British Gravitational System of Units*** (BG)
  - The latter two are similar, except for the choice of primary mass unit and use of the degree symbol.
- The two dominant unit systems in use in the world today are the metric system (SI) and the English system. [The BG system is no longer popular, and I do not recommend that you use it.]
- The table below shows each of the primary dimensions, along with their symbols and units in the SI, English, and BG unit systems:

Primary dimension	**Symbol**	**SI unit**	**English unit**	**BG unit**
mass	m	kg (kilogram)	lbm (pound-mass)	slug
length	L	m (meter)	ft (foot)	ft (foot)
time	t	s (second)	s (second)	s (second)
thermodynamic temperature	T	K (kelvin)	R (rankine)	^{o}R (^{o}Rankine)
current	I (or i)	A (ampere)	A (ampere)	A (ampere)
amount of light (luminous intensity)	C (or I)	cd (candela)	cd (candela)	cd (candela)
amount of matter	N	mol (mole)	mol (mole)	mol (mole)

- All other dimensions and units can be derived as combinations of these seven. These are called ***secondary dimensions***, with their corresponding ***secondary units***. A few examples are given in the table below:

Secondary dimension	**Symbol**	**SI unit**	**English unit**	**BG unit**
force	F	N (newton = kg·m/s^2)	lbf (pound-force)	lbf or lb (pound)*
acceleration	a	m/s^2	ft/s^2	ft/s^2
pressure	P (sometimes p)	Pa (pascal = N/m^2)	lbf/in^2 (psi)	lbf/ft^2 (psf)
energy	E	J (joule = N·m)	ft·lbf (foot-pound)	ft·lbf (foot-pound)
power	$\dot{W}$ (sometimes P)	W (watt = J/s)	ft·lbf/s	ft·lbf/s

*Some users of the BG system use lb (pound) and do not distinguish between lbf and lbm. In this course we will never use lb, but will always use either lbf (pound force) or lbm (pound mass) to avoid confusion.

- *Note*: Some authors substitute force for mass in the list of primary dimensions. This is an alternative way to consider primary dimensions. We will not use this alternative method in this course, but you should be aware that it is sometimes used. The system using mass as a primary dimension is the more popular of the two.

- Example
  Given: An engineer is measuring surface tension.
  To do: Express the dimensions of surface tension in terms of *primary* dimensions only.
  Solution:
  - *Note*: In this course, the notation "{something}" means "the dimensions of something".
  - Surface tension has dimensions of force per unit length, or {F/L}.
  - Force has dimensions of mass times acceleration, or $\{mL/t^2\}$.
  - Hence, surface tension has dimensions of $\{(mL/t^2)/L\}$, or $\{m/t^2\}$.
  - The final result is thus $\boxed{\{\textbf{surface tension}\} = \{\mathbf{m/t^2}\}}$
- There are many other units, both metric and English, in use today. For example, power is often expressed in units of Btu/hr, Btu/s, cal/s, ergs/s, or horsepower, in addition to the standard units of watt and ft·lbf/s.
- There are ***conversion factors*** listed in handbooks to enable conversion from any of these units to any other.
- Even though the English system is much more difficult to use than the metric system, it is still widely in use in industry today. Therefore, we still have to learn both systems, and as engineers we must be comfortable using both systems.
- Quirks - There are quirks in both English and metric systems:
  - English **-** There are two standard units for mass: lbm and slug. Students are often confused by these. A lbm represents the mass that weighs one pound-force (lbf) on earth. It is not proper to say that a lbm is equal to a lbf since the former is a unit of mass and the latter is a unit of force. However, it *is* proper to say that a lbm *weighs* one lbf (on earth). A slug is much bigger than a lbm. In fact, a slug is 32.174 lbm. A slug, then, weighs 32.174 lbf (on earth).
  - Metric **-** The standard unit for mass is the kilogram (kg), and the standard unit for force is the newton (N). Unfortunately, most people use kg as a measure of *weight*, which is technically incorrect. Note that one kg *weighs* 9.807 N (on earth). When you buy a box of cereal, the printing may say "net weight 1 pound (454 grams)." Technically, this means that the cereal inside the box weighs 1 lbf, and has a mass of 454 gm (0.454 kg). The actual *weight* of the cereal in the metric system is $W = mg = (0.454\ \text{kg})(9.807\ \text{m/s}^2)(\text{N}\cdot\text{s}^2/\text{kg}\cdot\text{m}) = 4.45\ \text{N}$, but the manufacturer gives the weight in kg – a quirk of the metric system.
- Unit system conversion is a common source of error, and has even led to catastrophic failures. Shown here is a newspaper article from 1999 showing how unit conversion errors led to the destruction of a NASA space probe:

6 — The Daily Collegian Friday, Oct. 1, 1999

Error destroys space probe

By MATTHEW FORDAHL
Associated Press Writer

LOS ANGELES — A mixup over metric and English measurements likely caused the loss of the $125 million Mars Climate Orbiter as it started to circle the planet last week, officials said yesterday.

The error caused the probe to fly too close to the red planet, causing the spacecraft to break up or burn up in the Martian atmosphere that it had been designed to study, mission controllers at NASA's Jet Propulsion Laboratory said.

Two teams — one at JPL and another at Lockheed Martin in Colorado where the spacecraft was built — used different measurement systems, and quality control failed to notice the discrepancy, said Edward Weiler, NASA's associate administrator for space science.

"People sometimes make errors," he said. "The problem here was not the error, it was the failure of ... the checks and balances in our processes to detect the error. That's why we lost the spacecraft."

The mistake was made as commands were being sent to the probe to place it in proper orbit around Mars. It was not announced who used English measurements — like feet and pounds — and who used metric measurements — like meters and grams.

Metric measurements are typically used in navigating spacecraft as well as most scientific studies.

"Our inability to recognize and correct this simple error has had a major implications," said JPL director Edward Stone. "We have under way a thorough investigation to understand the issue."

Two separate review committees are investigating the incident at JPL; a third board will be formed shortly by NASA.

The spacecraft, built in about four years on what for space exploration was a shoestring budget, was to have been a shining example of NASA's policy of faster, better and cheaper solar system missions.

Last week, NASA Administrator Daniel Goldin said the investigators will not be casting blame but making sure that whatever caused the loss does not happen again.

The orbiter's sibling spacecraft, Mars Polar Lander, is set to arrive Dec. 3.

"Our clear short-term goal is to maximize the likelihood of a successful landing of the Mars Polar Lander," Weiler said. "The lessons from these reviews will be applied across the board in the future."

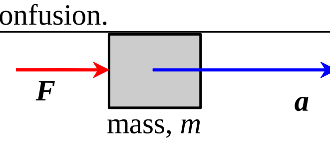
The orbiter was to have acted as a relay for the lander.

Comment about the Gravitational Conversion Constant g_c

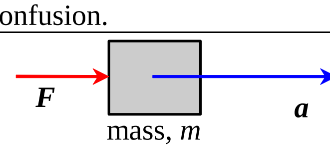
- Some authors define the ***gravitational conversion constant*** g_c, which is inserted into Newton's second law of motion. Instead of $\boldsymbol{F} = m\boldsymbol{a}$, they write $\boldsymbol{F} = m\boldsymbol{a}/g_c$, where g_c is defined in the English Engineering System of Units as $g_c = 32.174 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}$ and in SI units as $g_c = 1 \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2}$.
- *I strongly discourage use of the gravitational conversion constant g_c, since it leads to much confusion*. Instead, Newton's law should remain in the fundamental form in which it was created, without an artificial constant thrown into the equation simply for unit's sake.
- There has been much confusion (and error!) because of the differences between lbf, lbm, and slug. The use of g_c has complicated and further confused the issue, in my opinion, and should never have been invented.

Relationship between Force and Mass units

- The relationship between force, mass, and acceleration can be clearly understood by applying Newton's second law. The following table is provided to avoid confusion, especially with English units.
- **SI units**:

Relationship	Newton's second law, $\boldsymbol{F} = m\boldsymbol{a}$. [*Note*: Bold notation indicates a vector.] By definition of the fundamental units, this yields 1 N = 1 kg·m/s².
Conversion	$\left(\frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}\right)$ This is a unity conversion factor [equal to 1 and dimensionless].
Discussion	This unity conversion factor simplifies the units and avoids confusion.
Example	How much force (in Newtons) is required to accelerate a mass of 13.3 kg at a constant acceleration of 1.20 m/s² to the right?  **Solution**: $\boxed{F_x = ma_x = (13.3\ \text{kg})\left(1.20 \frac{\text{m}}{\text{s}^2}\right)\left(\frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}\right) = 16.0\ \text{N}}$ to the right, since F_x is the x-component of vector $\boldsymbol{F}$, and a_x is the x-component of acceleration vector $\boldsymbol{a}$.
Terminology	It is *not* proper to say that 1.00 kg *equals* 9.81 N, but it *is* proper to say that 1.00 kg *weighs* 9.81 N (on earth). This is obtained by utilizing Newton's second law with gravitational acceleration: $W = mg = (1.00\ \text{kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)\left(\frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}\right) = 9.81\ \text{N}$.

- **English units**:

Relationship	Newton's second law, $\boldsymbol{F} = m\boldsymbol{a}$. [*Note*: Bold notation indicates a vector.] By definition of the fundamental units, this yields 1 lbf = 1 slug·ft/s² or 1 lbf = 32.174 lbm·ft/s².
Conversion	$\left(\frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}\right)$ *or* $\left(\frac{\text{lbf} \cdot \text{s}^2}{32.174\ \text{lbm} \cdot \text{ft}}\right)$ *or* $\left(\frac{\text{slug}}{32.174\ \text{lbm}}\right)$ All are unity conversion factors.
Discussion	This unity conversion factor simplifies the units and avoids confusion.
Example	How much force (in pounds-force) is required to accelerate a mass of 13.3 lbm at a constant acceleration of 1.20 ft/s² to the right?  **Solution**: $\boxed{F_x = ma_x = (13.3\ \text{lbm})\left(1.20 \frac{\text{ft}}{\text{s}^2}\right)\left(\frac{\text{lbf} \cdot \text{s}^2}{32.174\ \text{lbm} \cdot \text{ft}}\right) = 0.496\ \text{lbf}}$ to the right, since F_x is the x-component of vector $\boldsymbol{F}$, and a_x is the x-component of acceleration vector $\boldsymbol{a}$.
Terminology	It is *not* proper to say that 1.00 lbm *equals* 1.00 lbf, but it *is* proper to say that 1.00 lbm *weighs* 1.00 lbf (on earth). This is obtained by utilizing Newton's second law with gravitational acceleration: $W = mg = (1.00\ \text{lbm})\left(32.174 \frac{\text{ft}}{\text{s}^2}\right)\left(\frac{\text{lbf} \cdot \text{s}^2}{32.174\ \text{lbm} \cdot \text{ft}}\right) = 1.00\ \text{lbf}$.

Significant Digits

- Engineering measurements are generally accurate to at most only a few digits. Three (sometimes four) digits of accuracy are considered “standard” for engineering analysis.
- The number of ***significant digits*** is defined as *the number of relevant or useful digits in a measurement*.
- The best way to illustrate is to write the number in standard exponential (scientific) notation instead of common real number (engineering) notation, and then count the number of digits.
- Here are some examples:

Common notation	Underlined notation	Exponential notation	# significant digits	Comments
134.2	134.<u>2</u>	1.342 x 10^2	4	just count the number of digits
0.0056	0.005<u>6</u>	5.6 x 10^{-3}	2	the leading zeroes are not significant
0.00506	0.0050<u>6</u>	5.06 x 10^{-3}	3	the leading zeroes are not significant, but any zeroes between two numbers *are* significant
0.00560	0.0056<u>0</u>	5.60 x 10^{-3}	3	the leading zeroes are not significant, but the trailing zeroes *are* significant
400	400	4 x 10^2	infinite	integer values have an infinite number of significant digits
400.	40<u>0</u>	4.00 x 10^2	3	a decimal point (or underline) indicates that all digits to the left of the decimal point are significant, and that this is *not* an integer value
400.0	400.<u>0</u>	4.000 x 10^2	4	the zero to the right of the decimal point *is* significant
40,300.	40,30<u>0</u>	4.0300 x 10^4	5	a decimal point (or underline) indicates that all digits to the left of the decimal point are significant, and that this is *not* an integer value
40,300	40,300	403 x 10^2	infinite	integer values have an infinite number of significant digits; do not use a decimal point when writing an integer in exponential notation
400 (to 2 significant digits)	4<u>0</u>0	4.0 x 10^2	2	words in parenthesis are necessary to indicate a smaller number of significant digits in common notation whenever trailing zeroes are present

- Things get a little tricky and ambiguous when dealing with large numbers. For example, suppose someone reports the population of a large city as 3,485,000, and says nothing about significant digits. Is it rounded to the nearest *thousand* (3.485×10^6; 4 significant digits)? Is it rounded to the nearest *hundred* (3.4850×10^6; 5 significant digits)? It is impossible to know. We suspect that it the population is not *exactly* 3,485,000 (7 significant digits), although that is a remote possibility.
- One way around this ambiguity is to <u>underline</u> the least significant digit. In our population example, if the population were rounded to the nearest thousand, we would write 3,48<u>5</u>,000 since the first zero is *not* significant. If the population were rounded to the nearest 100, we would write 3,485,<u>0</u>00 since the first zero *is* significant, and so on.
- *When performing multiplication or division calculations, the answer has the same number of significant digits as the component with the <u>least</u> number of significant digits*.
- <u>Example</u>
  Given: A force of 4.210 lbf is measured, and it is applied to a mass of 2.23 lbm so as to accelerate this mass.
  To do: Calculate the acceleration.
  Solution: Use Newton's second law, i.e. $F = ma$, and solve for the acceleration:

$$a = \frac{F}{m} = \frac{4.210\ \cancel{\text{lbf}}}{2.23\ \cancel{\text{lbm}}}\left(\frac{32.174\ \cancel{\text{lbm}}\cdot\text{ft/s}^2}{\cancel{\text{lbf}}}\right) = 60.7410\ \frac{\text{ft}}{\text{s}^2}$$

. The answer we report is $\boxed{a = 60.7\ \frac{\text{ft}}{\text{s}^2}}$.

- Note some important points in this simple example:
  - The final answer is reported to only three significant digits, since m is precise to only 3 significant digits, and $3 < 4$, where F is precise to 4 significant digits. The answer is *not* 60.74104933, even though that is what the calculator shows! All the digits following the first three are meaningless.
  - However, it is good to write down the answer to several additional significant digits, as above, if this value is to be used in subsequent calculations. Failure to do so can result in round-off error.

Unity Conversion Factors

- The quantity in parentheses in the above equation is simply a conversion factor. When written as a ratio of units as above, a conversion factor has *no units,* and has a value of *unity*: $\boxed{\left(\dfrac{32.174\ \text{lbm}\cdot\text{ft/s}^2}{\text{lbf}}\right)=1}$.
- We call such conversion factors, when written this way, ***unity conversion factors***.
- Here are some other examples of unity conversion factors:

$$\boxed{\left(\frac{\text{kg}\cdot\text{m/s}^2}{\text{N}}\right)}\quad\boxed{\left(\frac{12\ \text{in}}{1\ \text{ft}}\right)}\quad\boxed{\left(\frac{\text{J/s}}{\text{W}}\right)}\quad\boxed{\left(\frac{737.56\ \text{lbf}\cdot\text{ft/s}}{1\ \text{kW}}\right)}\quad\boxed{\left(\frac{1\ \text{m}^3\text{/s}}{15{,}580\ \text{gal/min}}\right)}\quad\boxed{\left(\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right)}\quad\boxed{\left(\frac{\text{Pa}\cdot\text{m}^2}{\text{N}}\right)}$$

- Always do your unit conversions in this way, using unity conversion factors – this will avoid errors.

Rounding Off

- There are standard rules for rounding off values to a desired number of significant digits. First, the number is truncated to its desired length. Then, the *excess* (leftover) digits are examined as if they were a decimal fraction:
  - If the decimal fraction is *less than* 0.5, *truncate* the excess digits.
  - If the decimal fraction is *greater than* 0.5, *round up* the least significant digit in the number by one.
  - If the decimal fraction is *exactly* 0.5, the convention is to *round up* if the least significant digit is *odd,* and to *truncate* (round down) if the least significant digit is *even*. [*Note*: The digit zero is *even*.]
  - Here is a little trick to remember this, courtesy of Steve Galamba: Either way, you get an even digit!
- Examples:
  - Round 548,392 to three significant digits. Answer: 548,000 [round down].
  - Round 548,592 to three significant digits. Answer: 549,000 [round up].
  - Round 548,500 to three significant digits. Answer: 548,000 [round down since 8 is even].
  - Round 547,500 to three significant digits. Answer: 548,000 [round up since 7 is odd].
- Things get a little tricky when *adding* or *subtracting* numbers. For example, suppose the population of a large city is 3,485,000, rounded to the nearest thousand (3.485×10^6 or 3,485,000; 4 significant digits). If 12 people move into the city, what is the new population? We are tempted to say 3,485,000 + 12 = 3,485,012, but this implies 7 significant digits of precision. Actually, since we cannot have fractions of people, it implies *infinite* precision, i.e., an integer number. The correct answer is 3,485,000, since the 12 extra people do not change the population to the nearest thousand – we round off.
- Suppose 1,862 people move to the city. The new population is 3,485,000 + 1,862 = 3,486,862 rounded off to the nearest thousand to maintain the original number of significant digits, i.e., 3,487,000.
- *When performing addition or subtraction, the number of significant digits is determined by the leftmost decimal column that contains a least significant digit.* The best way to add or subtract numbers is to align the decimal point, and highlight the leftmost significant digit. For example, here is how to add 13.68 + 0.08672:

$$\begin{array}{r} 13.6\boxed{8} \\ +\ 0.0\boxed{8}67\underline{2} \\ \hline 13.7\boxed{6}672 \end{array}$$

  We highlight the fourth column from the left, since it contains the *leftmost* least significant digit. After rounding up, our final answer is therefore **13.77**, precise to four significant digits.
- When adding or subtracting numbers, it is possible for the result to have a greater number of significant digits than any of the component numbers. For example, 5.86 + 7.21 = 13.07 [result precise to four significant digits]. This is useful when calculating mean (average) values of a measurement sample.
- Example: Consider the following twelve numbers: 7.53, 8.76, 7.42, 8.15, 7.79, 7.88, 7.91, 8.24, 8.13, 7.74, 7.80, and 8.06. The average is calculated by adding up all the numbers and dividing by 12. The sum is 95.41 (four significant digits). Calculation of the average yields 95.41/12 = 7.950833333. However, we must round to a maximum of four significant digits because of the division. Here, although each data point is precise to only 3 significant digits, our final answer is **7.951** (rounded to four significant digits). However, as mentioned previously, if we need to use this average in further calculations, we should carry along a few more digits to avoid potential round-off errors.
- *Note*: Some authors argue that an average cannot have more significant digits than its components. They would say that the correct final answer to the above example is **7.95** (rounded to three significant digits). There is some support for this alternative answer here since there is so much scatter in the original data.