# **Taguchi Orthogonal Arrays**

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### Introduction

- There are options for creating Taguchi arrays for the design of experiments, depending on how many times you choose to test each level of each parameter.
- For example, consider an experiment with 3 parameters and 3 levels of each parameter (P = 3 and L = 3), as discussed in a previous learning module. We showed two Taguchi arrays for this case:
  - o a 6-run array for testing each level of each parameter *twice*
  - o a 9-run array for testing each level of each parameter *three times*
- The 9-run array is more desirable (if cost and time permit) because for each level of any one parameter, *all* three levels of the other parameters are tested. Of course, either array here costs less to run than a full-factorial analysis, since the number of required runs for a full factorial analysis is  $N = L^P = 3^3 = 27$ .
- We call a Taguchi array an *orthogonal array* (some authors call it a *full orthogonal array*) when *for each level of a particular parameter, all L levels of each of the (P-1) other parameters are tested at least once.*
- Sometimes, as *P* increases, it is necessary to test all levels of all parameters more than once in order to meet the rules for Taguchi arrays, as discussed previously. In such cases, there should be no *unnecessary* repeats.
- For example, consider the P = 4, L = 3 case on the next page. When parameter *a* is at level 1, parameter *b* is tested at levels 1, 2, and 3 (all levels). Similarly, parameters *c* and *d* are tested at levels 1, 2, and 3 (all *L* levels). The same thing holds when parameter *a* is at level 2 or level 3. The same thing holds for *all* of the parameters. Hence, we see that the definition of an orthogonal array holds for this case for each level of a particular parameter, all *L* levels of each of the (*P*-1) other parameters are tested at least once only once in this particular case. The required number of runs is therefore 3 (L = 3 levels) × 3 (each level of each parameter tested 3 times) = 9 required runs for this orthogonal array.
- Consider another example, the P = 5, L = 3 case on the next page. When parameter *a* is at level 1, parameter *b* is tested at level 1 twice, level 2 twice, and level 3 twice (twice for all 3 levels). Similarly, parameters *c* and *d* are tested twice at levels 1, 2, and 3 (all *L* levels). The same thing holds when parameter *a* is at level 2 or level 3. The same thing holds for *all* of the parameters. Hence, our definition of an orthogonal array also holds for this case for each level of a particular parameter, all *L* levels of each of the (*P*-1) other parameters are tested at least once actually *twice* in this particular case. The required number of runs is therefore 3 (L = 3 levels) × 6 (each level of each parameter tested 6 times) = 18 required runs for this orthogonal array.
- Orthogonal arrays are the "best" and most common type of Taguchi array, and *you are encouraged to use orthogonal arrays whenever time and cost permit*. A table of Taguchi orthogonal arrays is provided below for values of *P* (number of parameters) ranging from 2 to 5, and *L* (number of levels) ranging from 2 to 5.

$\begin{array}{c} P = \rightarrow \\ L = \downarrow \end{array}$				3						4				5									
										Tagu	ıchi,	<i>P</i> =	= 4, j	L =	2	Taguchi, $P = 5, L = 2$							
											Run #	a	b	с	đ	X	Run #	a	b	с	đ	e	X
	Taguch	i, <i>P</i> =	=2, 1	L=2	Taguel	ni, P	9 = 3	, L	= 2		1	1	1	1	1	$X_1$	1	1	1	1	1	1	$X_1$
	Run #	a	b	X	Run #	a	b	с	X		2	1	1	1	2	$X_2$	2	1	1	1	2	2	$X_2$
	1	1	1	$X_1$	1	1	1	1	$X_1$		3	1	2	2	1	$X_3$	3	1	2	2	1	1	$X_3$
2	2	1	2	$X_2$	2	1	2	2	$X_2$		4	1	2	2	2	$X_4$	4	1	2	2	2	2	$X_4$
	3	2	1	$X_3$	3	2	1	2	$X_3$	-	5	2	1	2	1	$X_5$	5	2	1	2	1	2	$X_5$
	4	2	2	$X_4$	4	2	2	1	$X_4$		6	2	1	2	2	$X_6$	6	2	1	2	2	1	$X_6$
	-	1									7	2	2	1	1	$X_7$	7	2	2	1	1	2	$X_7$
											8	2	2	1	2	$X_8$	8	2	2	1	2	1	$X_8$

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$L = \downarrow$		2			3				4				5						
~ *												Ta	gucł	ni, P	= 5	, L =	= 3		
												Run #	a	b	с	d	e X		
												1	1	1	1	1	1 X <sub>1</sub>		
												2	1	2	2	2	2 X <sub>2</sub>		
	Taguah	; D-	-2 1 -3	Tama	h; D	- 3 T -	2	Tamio		- 1	I = 3	3	1	3	3	3	3 X <sub>3</sub>		
	Run #	a 1, 1	$\frac{-2, L-3}{b}$	Run #	ш, <i>г</i>	$\frac{-3, L}{b}$	$\frac{3}{X}$	Run #	a b	с <del>с</del>	$\frac{L-3}{d X}$	4	2	1	1	2	2 X <sub>4</sub>		
	1	1	1 X <sub>1</sub>	1	1	1 1 .	$X_1$	1	1 1	1	$1 X_1$	5	2	2	2	3	3 X <sub>5</sub>		
	2	1	2 X2	2	1	2 2 .	$X_2$	2	1 2	2	$2 X_2$	6	2	3	3	1	1 X <sub>6</sub>		
	3	1	3 X3	3	1	3 3 .	X 3	3	1 3	3	3 X3	7	3	1	2	1	3 X <sub>7</sub>		
3	4	2	1 X4	4	2	1 2 -	$X_4$	4	2 1	2	3 X4	8	3	2	3	2	1 X <sub>8</sub>		
	5	2	2 X5	5	2	2 3 -	$X_5$	5	2 2	3	1 X <sub>5</sub>	9	3	3	1	3	$2 X_9$		
	6	2	3 X <sub>6</sub>	6	2	3 1 -	$X_6$	6	2 3	1	2 X <sub>6</sub>	10	1	1	3	3	2 X <sub>10</sub>		
	7	3	1 X7	7	3	1 3 -	X7	7	3 1	3	2 X7	11	1	2	1	1	3 X <sub>11</sub>		
	8	3	2 X8	8	3	2 1 -	$X_8$	8	3 2	1	3 X8	12	1	3	2	2	1 X <sub>12</sub>		
	9	3	3 X9	9	3	3 2 -	X9	9	3 3	2	1 X9	13	2	1	2	3	1 X <sub>13</sub>		
				<u> </u>				++			+ + +	14	2	2	3	1	2 X 14		
												15	2	1	2	2	3 X 15		
												10	2	2	3	2	5 A 16		
												18	2	2	2	1	$\frac{1}{2} \frac{X_{17}}{X_{17}}$		
	<b>T</b> 1						_					10	,	5	2	1	2 2 18		
	Laguch		A T 4						·				1	· .	-	-			
1	Run #	1, P=	=2, L=4	Tague	hi, P	=3, L =	4 V	Pup #	ni, P =	= 4, 1	L = 4	Ta Run #	guch	i, P	= 5	, L =	= 4		
	<b>Run</b> #	1, P= a	$\begin{array}{c c} =2, L=4 \\ \hline b & X \\ \hline 1 & X_1 \end{array}$	Tague	hi, P a 1	= 3, L = <b>b</b> c 1 1	$\frac{4}{X}$	1 aguel Run #	ni, P = a b 1 1	= 4, 1 c	$L = 4$ $\frac{d X}{1 X_1}$	Ta <b>Run</b> #	iguch a 1	i, P b	= 5 c	, L = <u>d</u> 1	$= 4$ $e  X$ $1  X_1$		
	<b>Run</b> #	1, P = a 1	$ \begin{array}{c c} =2, L=4 \\ \hline b & X \\ \hline 1 & X_1 \\ \hline 2 & X_2 \end{array} $	Tague           Run #           1           2	hi, P a 1 1	= 3, L = $b c$ $1 1 c$ $2 2 c$	$\frac{4}{X}$ $\frac{1}{X_1}$	I ague!           Run #           1           2	ni, P = a b 1 1 1 2	= 4, 1 c 1 2	$L = 4$ $\frac{d  X}{1  X_1}$ $2  X_2$	Ta <b>Run #</b> 1 2	iguch a 1	i, P b 1 2	= 5 c 1 2	, L = <u>d</u> 1 2	$ \begin{array}{c c}                                    $		
	Run # 1 2 3	1, P = a 1 1 1 1	$ \begin{array}{c c} =2, L=4 \\ \hline b & X \\ \hline 1 & X_1 \\ \hline 2 & X_2 \\ \hline 3 & X_3 \end{array} $	Taguel           Run #           1           2           3	hi, P a 1 1	= 3, L = $b c$ $1 1 c$ $2 2 c$ $3 3 c$	$\frac{4}{X_1}$ $\frac{1}{X_2}$ $\frac{1}{X_2}$	I aguel           Run #           1           2           3	ni, P = <u>a</u> <u>b</u> 1 1 1 2 1 3	= 4, 1 c 1 2 3	$L = 4$ $\frac{d  X}{1  X_1}$ $2  X_2$ $3  X_3$	Ta <b>Run</b> # 1 2 3	aguch a 1 1	i, P b 1 2 3	= 5 c 1 2 3	, L = <u>d</u> 1 2 3			
	Run # 1 2 3 4	1, P = a 1 1 1 1 1 1	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Taguel           Run #           1           2           3           4	hi, P a 1 1 1 1	$ \begin{array}{c} = 3, L = \\ \hline b c \\ \hline 1 1 \\ 2 2 \\ \hline 3 3 \\ 4 4 \\ \end{array} $	$\frac{4}{X}$ $\frac{1}{X_1}$ $\frac{1}{X_2}$ $\frac{1}{X_3}$ $\frac{1}{X_4}$	I aguel           Run #           1           2           3           4	$\begin{array}{c c} \text{ni, } P = \\ \hline a & b \\ \hline 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{array}$	= 4, 1 c 1 2 3 4	$L = 4$ $\frac{d  X}{1  X_1}$ $2  X_2$ $3  X_3$ $4  X_4$	Ta Run # 1 2 3 4	aguch a 1 1 1 1 1	i, P b 1 2 3 4	= 5 c 1 2 3 4	, L = <u>d</u> 1 2 3 4	= 4 e X 1 X <sub>1</sub> 2 X <sub>2</sub> 3 X <sub>3</sub> 4 X <sub>4</sub>		
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	Run #           1           2           3           4           5           6	$     \begin{array}{c}                                     $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Taguel           Run #           1           2           3           4           5           6	hi, P a 1 1 1 1 1 2 2 2	$\begin{array}{c c} = 3, L = \\ \hline b & c \\ \hline 1 & 1 & - \\ 2 & 2 & - \\ 3 & 3 & - \\ 4 & 4 & - \\ \hline 1 & 2 & - \\ 2 & 1 & - \\ \end{array}$	$ \frac{4}{X} \\ \frac{X}{X_1} \\ \frac{X_2}{X_3} \\ \frac{X_4}{X_5} \\ \frac{X_6}{X_6} $	I aguel           Run #           1           2           3           4           5           6	$\begin{array}{c} \text{ni, } P = \\ \hline a & b \\ \hline 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ \hline 2 & 1 \\ 2 & 2 \end{array}$	= 4, 1 c 1 2 3 4 2 1	$L = 4$ $d X$ $1 X_1$ $2 X_2$ $3 X_3$ $4 X_4$ $3 X_5$ $4 X_6$	Ta           Run #           1           2           3           4           5           6	a 1 1 1 1 1 2 2	i, P b 1 2 3 4 1 2	= 5 c 1 2 3 4 2 1	$L = \frac{d}{1}$ 2 3 4 3 4 4	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		
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4	Run #           1           2           3           4           5           6           7           8           9           10           11	$ \begin{array}{c}     a \\     \hline     a \\     a \\     \hline     a \\     a \\ $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Taguel           Run #           1           2           3           4           5           6           7           8           9           10           11	hi, P a 1 1 1 1 2 2 2 2 3 3 3 3	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \frac{4}{X} \\ \frac{1}{X_{1}} \\ \frac{1}{X_{2}} \\ \frac{1}{X_{3}} \\ \frac{1}{X_{4}} \\ \frac{1}{X_{5}} \\ \frac{1}{X_{6}} \\ \frac{1}{X_{7}} \\ \frac{1}{X_{8}} \\ \frac{1}{X_{9}} \\ \frac{1}{X_{10}} \\ \frac{1}{X_{11}} \\ \frac{1}{X_{10}} \\ $	I aguel           Run #           1           2           3           4           5           6           7           8           9           10           11	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	= 4, 1 c 1 2 3 4 2 1 4 3 3 3 4 1 1	$     \begin{array}{r}       L = 4 \\       \hline       d & X \\       1 & X_1 \\       2 & X_2 \\       3 & X_3 \\       4 & X_4 \\       3 & X_5 \\       4 & X_6 \\       1 & X_7 \\       2 & X_8 \\       4 & X_9 \\       3 & X_{10} \\       2 & X_{11} \\     \end{array} $	Ta           Run #           1           2           3           4           5           6           7           8           9           10           11	guch a 1 1 1 1 2 2 2 2 3 3 3 3 3	i, P b 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 3	= 5 c 1 2 3 4 2 1 4 3 3 4 1 1 4 3 4 1 1 1 2 3 4 1 2 3 4 2 1 1 2 3 4 4 2 1 1 1 1 1 1 1 1 1 1 1 1 1	$, L = \frac{d}{1}$ $1$ $2$ $3$ $4$ $3$ $4$ $1$ $2$ $4$ $3$ $4$ $3$ $2$	$= 4$ $= 4$ $= X$ $1 X_1$ $2 X_2$ $3 X_3$ $4 X_4$ $4 X_5$ $3 X_6$ $2 X_7$ $1 X_8$ $2 X_9$ $1 X_{10}$ $4 X_{11}$		
4	Run #           1           2           3           4           5           6           7           8           9           10           11           12	$ \begin{array}{c}     a \\     a \\     1 \\     1 \\     1 \\     1 \\     1 \\     1 \\     1 \\     1 \\     2 \\     2 \\     2 \\     2 \\     2 \\     3 $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Taguel           Run #           1           2           3           4           5           6           7           8           9           10           11           12	hi, P a 1 1 1 1 2 2 2 2 2 3 3 3 3 3 3	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \frac{4}{X} \\ \frac{1}{X_{1}} \\ \frac{1}{X_{2}} \\ \frac{1}{X_{2}} \\ \frac{1}{X_{3}} \\ \frac{1}{X_{4}} \\ \frac{1}{X_{5}} \\ \frac{1}{X_{6}} \\ \frac{1}{X_{7}} \\ \frac{1}{X_{8}} \\ \frac{1}{X_{9}} \\ \frac{1}{X_{10}} \\ \frac{1}{X_{11}} \\ \frac{1}{X_{12}} \\ \frac{1}{X_{10}} \\ 1$	I aguel       Run #       1       2       3       4       5       6       7       8       9       10       11       12	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	c         c           1         2           3         4           2         1           4         3           3         4           1         2	$     \begin{array}{c}       L = 4 \\       \hline       d & X \\       1 & X_1 \\       2 & X_2 \\       3 & X_3 \\       4 & X_4 \\       3 & X_5 \\       4 & X_4 \\       3 & X_5 \\       4 & X_6 \\       1 & X_7 \\       2 & X_8 \\       4 & X_9 \\       3 & X_{10} \\       2 & X_{11} \\       1 & X_{12}   \end{array} $	Ta           Run #           1           2           3           4           5           6           7           8           9           10           11           12	guch a 1 1 1 1 2 2 2 2 3 3 3 3 3 3	i, P b 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 4	= 5 c 1 2 3 4 2 1 4 3 3 4 1 4 3 4 1 2 1 1 2 1 1 2 1 1 2 3 4 2 1 1 2 3 4 2 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1	$, L = \frac{d}{1}$ $1$ $2$ $3$ $4$ $3$ $4$ $1$ $2$ $4$ $3$ $2$ $1$ $1$	= 4           e         X           1         X1           2         X2           3         X3           4         X4           4         X5           3         X6           2         X7           1         X8           2         X9           1         X10           4         X11           3         X12		
4	Run #           1           2           3           4           5           6           7           8           9           10           11           12           13	$ \begin{array}{c}     a \\     a \\     1 \\     1 \\     1 \\     1 \\     1 \\     1 \\     1 \\     1 \\     2 \\     2 \\     2 \\     2 \\     2 \\     3 \\     3 \\     3 \\     3 \\     3 \\     4 \\   \end{array} $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Taguel           Run #           1           2           3           4           5           6           7           8           9           10           11           12           13	hi, P a 1 1 1 1 2 2 2 2 2 3 3 3 3 4	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \frac{4}{X} \\ \frac{1}{X_{1}} \\ \frac{1}{X_{2}} \\ \frac{1}{X_{2}} \\ \frac{1}{X_{3}} \\ \frac{1}{X_{4}} \\ \frac{1}{X_{5}} \\ 1$	I aguel       Run #       1       2       3       4       5       6       7       8       9       10       11       12       13	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	c         c           1         2           3         4           2         1           4         3           3         4           1         2           2         4	$L = 4$ $d X$ $1 X_1$ $2 X_2$ $3 X_3$ $4 X_4$ $3 X_5$ $4 X_6$ $1 X_7$ $2 X_8$ $4 X_9$ $3 X_{10}$ $2 X_{11}$ $1 X_{12}$ $2 X_{13}$	Ta           Run #           1           2           3           4           5           6           7           8           9           10           11           12           13	guch a 1 1 1 1 2 2 2 2 2 3 3 3 3 4	i, P b 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 1 2 3 4 1	= 5 c 1 2 3 4 2 1 4 3 3 4 1 2 4 1 2 4 3 3 4 1 2 4 3 3 4 4 3 3 4 4 3 3 4 4 5 6 6 6 6 6 6 6 6 6 6 6 6 6	$, L = \frac{d}{1}$ $1$ $2$ $3$ $4$ $3$ $4$ $1$ $2$ $4$ $3$ $2$ $1$ $2$ $1$ $2$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $		
4	Run #           1           2           3           4           5           6           7           8           9           10           11           12           13           14	$ \begin{array}{c}     a \\     \hline     a \\     a \\     \hline     a \\     a \\     \hline     a \\     a \\   $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Taguel         Run #         1         2         3         4         5         6         7         8         9         10         11         12         13         14	hi, P a 1 1 1 1 2 2 2 2 2 3 3 3 3 4 4	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \frac{4}{X} \\ \frac{1}{X_1} \\ \frac{1}{X_2} \\ \frac{1}{X_2} \\ \frac{1}{X_3} \\ \frac{1}{X_4} \\ \frac{1}{X_5} \\ \frac{1}{X_6} \\ \frac{1}{X_7} \\ \frac{1}{X_8} \\ \frac{1}{X_9} \\ \frac{1}{X_9} \\ \frac{1}{X_1} \\ \frac{1}{$	I aguel       Run #       1       2       3       4       5       6       7       8       9       10       11       12       13       14	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	c         c           1         2           3         4           2         1           4         3           3         4           1         2           4         3           3         4           1         2           4         3           3         3           4         3           3         3           4         3           3         3           4         3	$L = 4$ $d X$ $1 X_1$ $2 X_2$ $3 X_3$ $4 X_4$ $3 X_5$ $4 X_6$ $1 X_7$ $2 X_8$ $4 X_9$ $3 X_{10}$ $2 X_{11}$ $1 X_{12}$ $2 X_{13}$ $1 X_{14}$	Ta           Run #           1           2           3           4           5           6           7           8           9           10           11           12           13           14	guch a 1 1 1 1 2 2 2 2 2 3 3 3 3 4 4	i, P b 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 2 3 4 1 2 2 3 4 1 2 3 3 4 1 2 3 3 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 4 4 4 1 2 3 3 4 4 4 1 2 3 3 4 4 2 3 3 4 4 4 1 2 3 3 4 4 1 2 3 3 4 4 1 2 3 3 3 4 4 1 2 3 3 4 4 3 3 3 3 4 4 1 2 3 3 3 4 4 1 2 3 3 3 3 4 4 1 2 3 3 4 4 3 3 3 4 4 3 3 3 3 3 3 4 4 3 3 4 4 2 3 3 3 4 4 4 1 2 3 3 3 4 4 1 2 3 3 3 3 3 4 4 3 4 1 2 3 3 4 4 3 3 1 2 3 3 1 2 3 3 3 4 4 1 2 3 3 3 3 4 4 1 2 3 3 3 3 3 3 4 4 1 2 3 3 3 3 4 4 1 2 3 3 4 4 1 2 3 3 3 3 4 4 1 2 3 3 3 3 4 4 1 2 3 3 3 3 3 4 4 1 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	= 5 c 1 2 3 4 2 1 4 3 3 4 1 2 4 3 4 1 2 4 3 4 4 3 3 4 4 3 3 4 4 3 3 4 4 5 6 6 6 6 6 6 6 6 6 6 6 6 6	, L = $d$ $1$ $1$ $2$ $3$ $4$ $3$ $4$ $1$ $2$ $4$ $3$ $2$ $1$ $2$ $1$ $2$ $1$	$= 4$ $= X$ $1 X_1$ $2 X_2$ $3 X_3$ $4 X_4$ $4 X_5$ $3 X_6$ $2 X_7$ $1 X_8$ $2 X_9$ $1 X_{10}$ $4 X_{11}$ $3 X_{12}$ $3 X_{13}$ $4 X_{14}$		
4	Run #           1           2           3           4           5           6           7           8           9           10           11           12           13           14           15	$ \begin{array}{c}     a \\     a \\     1 \\     1 \\     1 \\     1 \\     1 \\     1 \\     1 \\     1 \\     1 \\     2 \\     2 \\     2 \\     2 \\     2 \\     2 \\     3 \\     3 \\     3 \\     3 \\     3 \\     4 \\     4 \\     4 \\   \end{array} $	$\begin{array}{c c} =2, L=4\\ \hline b & X\\ 1 & X_1\\ 2 & X_2\\ 3 & X_3\\ 4 & X_4\\ 1 & X_5\\ 2 & X_6\\ 3 & X_7\\ 4 & X_8\\ 1 & X_9\\ 2 & X_{10}\\ 3 & X_{11}\\ 4 & X_{12}\\ 1 & X_{13}\\ 2 & X_{14}\\ 3 & X_{15} \end{array}$	Taguel           Run #           1           2           3           4           5           6           7           8           9           10           11           12           13           14           15	hi, P a 1 1 1 1 2 2 2 2 2 2 3 3 3 3 4 4 4 4	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		I aguel       Run #       1       2       3       4       5       6       7       8       9       10       11       12       13       14       15	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	c         c           c         1           2         3           4         2           1         4           3         3           4         1           2         4           3         2	$L = 4$ $d X$ $1 X_1$ $2 X_2$ $3 X_3$ $4 X_4$ $3 X_5$ $4 X_6$ $1 X_7$ $2 X_8$ $4 X_9$ $3 X_{10}$ $2 X_{11}$ $1 X_{12}$ $2 X_{13}$ $1 X_{14}$ $4 X_{15}$	Ta Run # $ $	guch a 1 1 1 1 2 2 2 2 2 3 3 3 3 4 4 4 4	i, P b 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 3 4 1 2 3 3	= 5 c 1 2 3 4 2 1 4 3 3 4 1 2 4 3 4 1 2 4 3 2 4 3 2 4 3 3 4 4 2 1 4 3 3 4 4 5 6 6 6 6 6 6 6 6 6 6 6 6 6	$, L = \frac{d}{1}$ 1 2 3 4 3 4 1 2 4 3 2 4 3 2 1 2 1 4 4 4	= 4           e         X           1         X1           2         X2           3         X3           4         X4           4         X5           3         X6           2         X7           1         X8           2         X9           1         X10           4         X11           3         X12           3         X13           4         X14           1         X15		

Faguahi	Orthogonal	Ammoria	Daga 2
ragueni	Onnogonal	Allays.	Page 5

$\begin{array}{c} P = \rightarrow \\ L = \downarrow \end{array}$			•	3					4		5											
	Taguchi, $P=2, L=5$				Taguchi, $P = 3, L = 5$					Tagu	5	Taguchi, $P = 5, L = 5$										
	Run #	a	b	X	Run #	a	b	с	X	Run #	a	b	с	đ	$\boldsymbol{X}$		Run #	a	b	с	đ	<i>e X</i>
	1	1	1	$X_1$	1	1	1	1	$X_1$	1	1	1	1	1	$X_1$		1	1	1	1	1	$1 X_1$
	2	1	2	$X_2$	2	1	2	2	$X_2$	2	1	2	2	2	$X_2$		2	1	2	2	2	$2 X_2$
	3	1	3	$X_3$	3	1	3	3	<i>X</i> <sub>3</sub>	3	1	3	3	3	$X_3$		3	1	3	3	3	3 X3
	4	1	4	$X_4$	4	1	4	4	$X_4$	4	1	4	4	4	$X_4$	_	4	1	4	4	4	4 X <sub>4</sub>
	5	1	5	$X_5$	5	1	5	5	$X_5$	5	1	5	5	5	$X_5$		5	1	5	5	5	5 X <sub>5</sub>
	6	2	1	$X_6$	6	2	1	2	$X_6$	6	2	1	2	3	$X_6$		6	2	1	2	3	4 X <sub>6</sub>
	7	2	2	$X_7$	7	2	2	3	$X_7$	7	2	2	3	4	$X_7$		7	2	2	3	4	5 X7
	8	2	3	$X_8$	8	2	3	4	$X_8$	8	2	3	4	5	$X_8$		8	2	3	4	5	1 X <sub>8</sub>
	9	2	4	$X_9$	9	2	4	5	X٩	9	2	4	5	1	$X_9$		9	2	4	5	1	2 X9
	10	2	5	X10	10	2	5	1	X <sub>10</sub>	10	2	5	1	2	$X_{10}$		10	2	5	1	2	3 X <sub>10</sub>
	11	3	1	X <sub>11</sub>	11	3	1	3	<i>X</i> <sub>11</sub>	11	3	1	3	5	X11		11	3	1	3	5	$2 X_{11}$
5	12	3	2	<i>X</i> <sub>12</sub>	12	3	2	4	<i>X</i> <sub>12</sub>	12	3	2	4	1	$X_{12}$		12	3	2	4	1	3 X <sub>12</sub>
	13	3	3	X <sub>13</sub>	13	3	3	5	X <sub>13</sub>	13	3	3	5	2	X <sub>13</sub>		13	3	3	5	2	4 X <sub>13</sub>
	14	3	4	$X_{14}$	14	3	4	1	X <sub>14</sub>	14	3	4	1	3	$X_{14}$		14	3	4	1	3	5 X <sub>14</sub>
	15	3	5	X <sub>15</sub>	15	3	5	2	X <sub>15</sub>	15	3	5	2	4	X <sub>15</sub>	_	15	3	5	2	4	1 X <sub>15</sub>
	16	4	1	X16	16	4	1	4	X16	16	4	1	4	2	X16	_	16	4	1	4	2	5 X <sub>16</sub>
	17	4	2	X <sub>17</sub>	17	4	2	5	<i>X</i> <sub>17</sub>	17	4	2	5	3	X <sub>17</sub>	_	17	4	2	5	3	1 X <sub>17</sub>
	18	4	3	X <sub>18</sub>	18	4	3	1	X <sub>18</sub>	18	4	3	1	4	X <sub>18</sub>	_	18	4	3	1	4	2 X <sub>18</sub>
	19	4	4	X 19	19	4	4	2	X 19	19	4	4	2	5	X19	_	19	4	4	2	5	3 X <sub>19</sub>
	20	4	5	X <sub>20</sub>	20	4	5	3	X <sub>20</sub>	20	4	5	3	1	$X_{20}$	_	20	4	5	3	1	4 X <sub>20</sub>
	21	5	1	X <sub>21</sub>	21	5	1	5	X <sub>21</sub>	21	5	1	5	4	X <sub>21</sub>	_	21	5	1	5	4	3 X <sub>21</sub>
	22	5	2	X <sub>22</sub>	22	5	2	1	X <sub>22</sub>	22	5	2	1	5	X <sub>22</sub>		22	5	2	1	5	4 X <sub>22</sub>
	23	5	3	X <sub>23</sub>	23	5	3	2	X <sub>23</sub>	23	5	3	2	1	X <sub>23</sub>		23	5	3	2	1	5 X <sub>23</sub>
	24	5	4	X <sub>24</sub>	24	5	4	3	X <sub>24</sub>	24	5	4	3	2	X <sub>24</sub>		24	5	4	3	2	1 X <sub>24</sub>
	25	5	5	X <sub>25</sub>	25	5	5	4	$X_{25}$	25	5	5	4	3	$X_{25}$		25	5	5	4	3	$2 X_{25}$

• **Bottom line**: Experimental test arrays are usually chosen based on a compromise between the *cost* of the experiments (cost includes the *time* required to run the experiments) and required *accuracy* of the results. Below is a hierarchy of how you should choose a test array:

# Full Factorial Array:

Increasing accuracy

If cost is not a big issue (in other words, you have enough time, and the runs are inexpensive and don't take too long), or if the accuracy of the results is critical, use a full factorial array.

# Taguchi Orthogonal Array:

If the cost (including time) of a full factorial array analysis is high, and the accuracy of the results is not so critical, use an orthogonal Taguchi array.

## Taguchi Non-Orthogonal Array:

If the cost is prohibitive (runs are extremely expensive or time consuming), and you can accept limited accuracy, use a non-orthogonal Taguchi array (but be sure to optimize it using the two rules given in the previous learning module for fractional factorial analysis).

Decreasing cost