Equation Sheet for M E 345 Quizzes and Exams

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Note: This equation sheet will be provided in electronic form as a pop-up window during the quizzes taken at the Testing Center. It is also useful for pre-quiz study purposes, so that you know what equations and tables to expect to see on the quizzes. ***Do not bring a printout of this equation sheet to the quiz***. A pop-up calculator app will also be available to help you perform simple calculations. This equation sheet has information for the entire semester ***in order of presentation***; for the earlier quizzes, ignore the later pages. This sheet is also useful for the open-book portion of the final exam.

General Equations and Constants:

Electronics: Ohm's law $\Delta V = IR$, power $\dot{W} = I\Delta V$, capacitance $I = C\dfrac{dV}{dt}$, inductance $V = L\dfrac{dI}{dt}$.

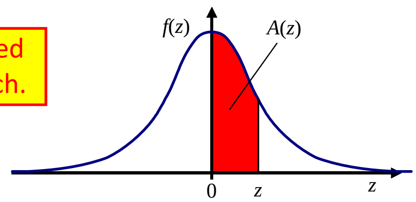
Dimensional analysis: $k = n - j$, Level of significance: $\alpha = 1 - c$.

Statistical Sampling: $f(x) = \dfrac{\text{Frequency}}{n \cdot \Delta x}$, $z = \dfrac{x-\mu}{\sigma} \approx \dfrac{x-\bar{x}}{S}$, CLT: $\sigma_{\bar{x}} \approx \dfrac{\sigma}{\sqrt{n}}$ or $S_{\bar{x}} \approx \dfrac{\sigma}{\sqrt{n}}$ or $\sigma_{\bar{x}} \approx \dfrac{S}{\sqrt{n}}$.

Gaussian PDF: The table below shows $A(z)$.

$$A(z) = \frac{1}{2}\operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \quad \text{where} \quad z = \frac{x-\mu}{\sigma} \approx \frac{x-\bar{x}}{S}$$

A(*z*) is the colored area in the sketch.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41309	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965

Student's *t* PDF: $t = \frac{\overline{x} - \mu}{S / \sqrt{n}}$, $\mu = \overline{x} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}$.

Values of $t_{\alpha/2}$ (critical values) for the student's *t* PDF

	90% confidence	95% confidence	98% confidence	99% confidence
$\alpha = \rightarrow$	0.10	0.05	0.02	0.01
df = ↓				
1	6.3138	12.7062	31.8205	63.6567
2	2.9200	4.3027	6.9646	9.9248
3	2.3534	3.1824	4.5407	5.8409
4	2.1318	2.7764	3.7469	4.6041
5	2.0150	2.5706	3.3649	4.0321
6	1.9432	2.4469	3.1427	3.7074
7	1.8946	2.3646	2.9980	3.4995
8	1.8595	2.3060	2.8965	3.3554
9	1.8331	2.2622	2.8214	3.2498
10	1.8125	2.2281	2.7638	3.1693
11	1.7959	2.2010	2.7181	3.1058
12	1.7823	2.1788	2.6810	3.0545
13	1.7709	2.1604	2.6503	3.0123
14	1.7613	2.1448	2.6245	2.9768
15	1.7531	2.1314	2.6025	2.9467
16	1.7459	2.1199	2.5835	2.9208
17	1.7396	2.1098	2.5669	2.8982
18	1.7341	2.1009	2.5524	2.8784
19	1.7291	2.0930	2.5395	2.8609
20	1.7247	2.0860	2.5280	2.8453
21	1.7207	2.0796	2.5176	2.8314
22	1.7171	2.0739	2.5083	2.8188
23	1.7139	2.0687	2.4999	2.8073
24	1.7109	2.0639	2.4922	2.7969
25	1.7081	2.0595	2.4851	2.7874
26	1.7056	2.0555	2.4786	2.7787
27	1.7033	2.0518	2.4727	2.7707
28	1.7011	2.0484	2.4671	2.7633
29	1.6991	2.0452	2.4620	2.7564
30	1.6973	2.0423	2.4573	2.7500
35	1.6896	2.0301	2.4377	2.7238
40	1.6839	2.0211	2.4233	2.7045
50	1.6759	2.0086	2.4033	2.6778
100	1.6602	1.9840	2.3642	2.6259
500	1.6479	1.9647	2.3338	2.5857
1000	1.6464	1.9623	2.3301	2.5808
1.00E+10	1.6449	1.9600	2.3263	2.5758

χ^2 PDF: $\chi^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{\sigma^2}$, $(n-1)\frac{S^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le (n-1)\frac{S^2}{\chi^2_{1-\alpha/2}}$.

Critical values for the χ^2 PDF

Critical values of $\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$ for the χ^2 distribution (both tails)								
Two tails →	90% confidence		95% confidence		98% confidence		99% confidence	
α →	0.10		0.05		0.02		0.01	
Single tail →	$1-\alpha/2$	$\alpha/2$	$1-\alpha/2$	$\alpha/2$	$1-\alpha/2$	$\alpha/2$	$1-\alpha/2$	$\alpha/2$
Probability →	0.95	0.05	0.975	0.025	0.99	0.01	0.995	0.005
df = ↓	$\chi^2_{1-\alpha/2}$	$\chi^2_{\alpha/2}$	$\chi^2_{1-\alpha/2}$	$\chi^2_{\alpha/2}$	$\chi^2_{1-\alpha/2}$	$\chi^2_{\alpha/2}$	$\chi^2_{1-\alpha/2}$	$\chi^2_{\alpha/2}$
1	0.0039	3.8415	0.0010	5.0239	0.0002	6.6349	0.0000	7.8794
2	0.1026	5.9915	0.0506	7.3778	0.0201	9.2104	0.0100	10.5965
3	0.3518	7.8147	0.2158	9.3484	0.1148	11.3449	0.0717	12.8381
4	0.7107	9.4877	0.4844	11.1433	0.2971	13.2767	0.2070	14.8602
5	1.1455	11.0705	0.8312	12.8325	0.5543	15.0863	0.4118	16.7496
6	1.6354	12.5916	1.2373	14.4494	0.8721	16.8119	0.6757	18.5475
7	2.1673	14.0671	1.6899	16.0128	1.2390	18.4753	0.9893	20.2777
8	2.7326	15.5073	2.1797	17.5345	1.6465	20.0902	1.3444	21.9549
9	3.3251	16.9190	2.7004	19.0228	2.0879	21.6660	1.7349	23.5893
10	3.9403	18.3070	3.2470	20.4832	2.5582	23.2093	2.1558	25.1881
11	4.5748	19.6752	3.8157	21.9200	3.0535	24.7250	2.6032	26.7569
12	5.2260	21.0261	4.4038	23.3367	3.5706	26.2170	3.0738	28.2997
13	5.8919	22.3620	5.0087	24.7356	4.1069	27.6882	3.5650	29.8193
14	6.5706	23.6848	5.6287	26.1189	4.6604	29.1412	4.0747	31.3194
15	7.2609	24.9958	6.2621	27.4884	5.2294	30.5780	4.6009	32.8015
16	7.9616	26.2962	6.9077	28.8453	5.8122	31.9999	5.1422	34.2671
17	8.6718	27.5871	7.5642	30.1910	6.4077	33.4087	5.6973	35.7184
18	9.3904	28.8693	8.2307	31.5264	7.0149	34.8052	6.2648	37.1564
19	10.1170	30.1435	8.9065	32.8523	7.6327	36.1908	6.8439	38.5821
20	10.8508	31.4104	9.5908	34.1696	8.2604	37.5663	7.4338	39.9969
21	11.5913	32.6706	10.2829	35.4789	8.8972	38.9322	8.0336	41.4009
22	12.3380	33.9245	10.9823	36.7807	9.5425	40.2894	8.6427	42.7957
23	13.0905	35.1725	11.6885	38.0756	10.1957	41.6383	9.2604	44.1814
24	13.8484	36.4150	12.4011	39.3641	10.8563	42.9798	9.8862	45.5584
25	14.6114	37.6525	13.1197	40.6465	11.5240	44.3140	10.5196	46.9280
26	15.3792	38.8851	13.8439	41.9231	12.1982	45.6416	11.1602	48.2898
27	16.1514	40.1133	14.5734	43.1945	12.8785	46.9628	11.8077	49.6450
28	16.9279	41.3372	15.3079	44.4608	13.5647	48.2782	12.4613	50.9936
29	17.7084	42.5569	16.0471	45.7223	14.2564	49.5878	13.1211	52.3355
30	18.4927	43.7730	16.7908	46.9792	14.9535	50.8922	13.7867	53.6719
35	22.4650	49.8018	20.5694	53.2033	18.5089	57.3420	17.1917	60.2746
40	26.5093	55.7585	24.4331	59.3417	22.1642	63.6908	20.7066	66.7660
50	34.7642	67.5048	32.3574	71.4202	29.7067	76.1538	27.9908	79.4898
100	77.9294	124.3421	74.2219	129.5613	70.0650	135.8069	67.3275	140.1697
500	449.1467	553.1269	439.9360	563.8514	429.3874	576.4931	422.3034	585.2060
1000	927.5944	1074.6794	914.2572	1089.5307	898.9124	1106.9690	888.5631	1118.9475
∞	∞	∞	∞	∞	∞	∞	∞	∞

Correlation and Trends: If $r_{xy} > r_t$, we are confident (to some confidence level) that the trend is real.

Values of r_t (critical values) for linear correlation coefficient

c = →	80%	90%	92.5%	95%	97%	98%	99%	99.5%
α = →	0.2	0.1	0.075	0.05	0.03	0.02	0.01	0.005
n = ↓								
3	0.95106	0.98769	0.99307	0.99692	0.99889	0.99951	0.99988	0.99997
4	0.80000	0.90000	0.92500	0.95000	0.97000	0.98000	0.99000	0.99500
5	0.68705	0.80538	0.83994	0.87834	0.91377	0.93433	0.95874	0.97404
6	0.60840	0.72930	0.76718	0.81140	0.85503	0.88219	0.91720	0.94170
7	0.55086	0.66944	0.70809	0.75449	0.80206	0.83287	0.87453	0.90556
8	0.50673	0.62149	0.65985	0.70673	0.75599	0.78872	0.83434	0.86974
9	0.47159	0.58221	0.61982	0.66638	0.71613	0.74978	0.79768	0.83591
10	0.44280	0.54936	0.58606	0.63190	0.68148	0.71546	0.76459	0.80461
11	0.41866	0.52140	0.55713	0.60207	0.65114	0.68510	0.73479	0.77589
12	0.39806	0.49726	0.53202	0.57598	0.62434	0.65807	0.70789	0.74961
13	0.38022	0.47616	0.50998	0.55294	0.60049	0.63386	0.68353	0.72553
14	0.36456	0.45750	0.49043	0.53241	0.57911	0.61205	0.66138	0.70344
15	0.35069	0.44086	0.47295	0.51398	0.55980	0.59227	0.64114	0.68311
16	0.33828	0.42590	0.45719	0.49731	0.54227	0.57425	0.62259	0.66434
17	0.32710	0.41236	0.44290	0.48215	0.52627	0.55774	0.60551	0.64696
18	0.31696	0.40003	0.42986	0.46828	0.51158	0.54255	0.58971	0.63083
19	0.30770	0.38873	0.41791	0.45553	0.49804	0.52852	0.57507	0.61580
20	0.29921	0.37834	0.40689	0.44376	0.48551	0.51550	0.56144	0.60176
22	0.28414	0.35983	0.38723	0.42271	0.46303	0.49209	0.53680	0.57627
24	0.27114	0.34378	0.37016	0.40439	0.44338	0.47158	0.51510	0.55370
26	0.25977	0.32970	0.35516	0.38824	0.42603	0.45341	0.49581	0.53355
28	0.24972	0.31722	0.34184	0.37389	0.41055	0.43718	0.47851	0.51542
30	0.24075	0.30606	0.32991	0.36101	0.39664	0.42257	0.46289	0.49900
32	0.23268	0.29599	0.31915	0.34937	0.38405	0.40933	0.44870	0.48404
34	0.22537	0.28686	0.30938	0.33879	0.37259	0.39725	0.43573	0.47034
36	0.21871	0.27852	0.30045	0.32911	0.36209	0.38618	0.42381	0.45773
38	0.21261	0.27086	0.29225	0.32022	0.35243	0.37598	0.41282	0.44608
40	0.20699	0.26381	0.28469	0.31201	0.34350	0.36655	0.40264	0.43527
45	0.19469	0.24833	0.26808	0.29396	0.32384	0.34575	0.38014	0.41133
50	0.18434	0.23529	0.25407	0.27871	0.30720	0.32813	0.36103	0.39093
55	0.17549	0.22411	0.24205	0.26561	0.29289	0.31295	0.34453	0.37329
60	0.16780	0.21438	0.23159	0.25420	0.28041	0.29970	0.33010	0.35783
65	0.16104	0.20582	0.22238	0.24415	0.26940	0.28799	0.31735	0.34414
70	0.15504	0.19821	0.21419	0.23520	0.25959	0.27756	0.30596	0.33191
80	0.14480	0.18522	0.20019	0.21990	0.24280	0.25970	0.28643	0.31091
90	0.13636	0.17449	0.18863	0.20725	0.22890	0.24490	0.27022	0.29345
100	0.12924	0.16543	0.17886	0.19655	0.21714	0.23236	0.25648	0.27863
150	0.10523	0.13482	0.14582	0.16033	0.17726	0.18980	0.20973	0.22807
200	0.09100	0.11664	0.12619	0.13879	0.15350	0.16441	0.18176	0.19776
300	0.07420	0.09515	0.10295	0.11327	0.12532	0.13426	0.14851	0.16167
400	0.06421	0.08236	0.08912	0.09807	0.10852	0.11629	0.12866	0.14010
500	0.05741	0.07364	0.07970	0.08770	0.09706	0.10402	0.11510	0.12535
1000	0.04056	0.05204	0.05633	0.06200	0.06863	0.07356	0.08142	0.08870

Outliers:

- For a single variable, reject if $\delta_i > \tau S$.
- For data pairs, reject if $\left|\frac{e_i}{S_{y,x}}\right| > 2$ *and* the standardized residual is inconsistent with its neighbors.

Values of the Modified Thompson τ

n	τ	n	τ	n	τ
1	-	26	4.8181	52	1.9325
2	-	27	4.9201	54	1.9335
3	1.1511	28	1.9078	56	1.9345
4	1.4250	29	1.9096	58	1.9354
5	1.5712	30	1.9114	60	1.9362
6	1.6563	31	1.9130	62	1.9370
7	1.7110	32	1.9146	64	1.9378
8	1.7491	33	1.9160	66	1.9384
9	1.7770	34	1.9174	68	1.9391
10	1.7984	35	1.9186	70	1.9397
11	1.8153	36	1.9198	75	1.9411
12	1.8290	37	1.9209	80	1.9423
13	1.8403	38	1.9220	85	1.9433
14	1.8498	39	1.9230	90	1.9443
15	1.8579	40	1.9240	95	1.9451
16	1.8649	41	1.9249	100	1.9459
17	1.8710	42	1.9257	150	1.9506
18	1.8764	43	1.9266	200	1.9530
19	1.8811	44	1.9273	250	1.9544
20	1.8853	45	1.9281	500	1.9572
21	1.8891	46	1.9288	1000	1.9586
22	1.8926	47	1.9295	2000	1.9593
23	1.8957	48	1.9301	5000	1.9597
24	1.8985	49	1.9308	10000	1.9598
25	1.9011	50	1.9314	$(\to \infty)$	1.9600

RSS Uuncertainty: $u_x = \sqrt{\sum_{i=1}^{i=K} u_i^{\,2}}$, $u_{R,\text{RSS}} = \sqrt{\sum_{i=1}^{i=N}\left(u_{x_i}\frac{\partial R}{\partial x_i}\right)^2}$.

Simple form when $R = C \cdot x_1^{a_1} \cdot x_2^{a_2} \ldots \cdot x_N^{a_N}$, $\frac{u_R}{R} = \frac{u_{R,\text{RSS}}}{R} = \sqrt{\sum_{i=1}^{i=N}\left(a_i \frac{u_{x_i}}{x_i}\right)^2}$.

Experimental Design: $N = L^P$ for *full factorial* arrays. Taguchi arrays are useful for *fractional factorial* analysis.

RSM: Coded variables $x_1 = 2\left(\frac{a - a_{\text{mid value}}}{a_{\text{range}}}\right)$, $a_{\text{mid value}} = \frac{a_{\min} + a_{\max}}{2}$, $a_{\text{range}} = a_{\max} - a_{\min}$, similar for other variables.

Direction of steepest ascent $\left(\frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2}, \frac{\partial y}{\partial x_3}\right)$ determined by regression analysis on coded variables. For known Δx_1,

$\Delta x_2 = \Delta x_1 \frac{\partial y}{\partial x_2} \Big/ \frac{\partial y}{\partial x_1}$ and $\Delta x_3 = \Delta x_1 \frac{\partial y}{\partial x_3} \Big/ \frac{\partial y}{\partial x_1}$. Transform back to physical variables, $\Delta a = \frac{a_{\text{range}}}{2} \Delta x_1$, and march.

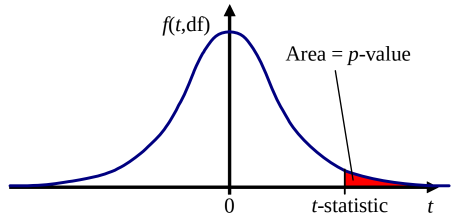
Hypothesis Testing: Single variable, $t = \dfrac{\bar{x} - \mu_0}{S / \sqrt{n}}$; see table for *p*-values at the critical *t* statistic.

Two-sample with same *n*, $t = \dfrac{\bar{\delta} - \mu_0}{S_\delta / \sqrt{n}}$; two-sample with different *n*, $t = \dfrac{\bar{x}_A - \bar{x}_B}{\sqrt{\dfrac{{S_A}^2}{n_A} + \dfrac{{S_B}^2}{n_B}}}$.

See tables below. Note that the tables here are for *n* = 10, 20 and 30 only (df = 9, 19, and 29). **Make sure you use the correct table**.

For *two tails*, multiply the value by 2, since the *t* PDF is symmetric.
The *p*-value is the colored area under the *t* PDF in the sketch.

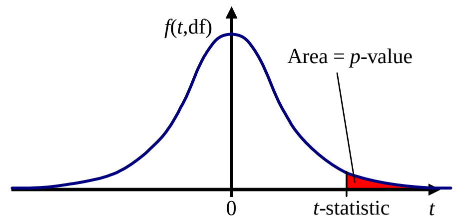
_p_-Values for the student's _t_ Distribution; one tail, _n_=10 (df=9)



Note: **This table is for df = 9** only

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.49612	0.49224	0.48836	0.48448	0.48061	0.47673	0.47286	0.46899	0.46513
0.1	0.46127	0.45741	0.45356	0.44971	0.44587	0.44204	0.43821	0.43439	0.43057	0.42676
0.2	0.42296	0.41917	0.41539	0.41162	0.40785	0.40410	0.40036	0.39662	0.39290	0.38919
0.3	0.38550	0.38181	0.37814	0.37448	0.37083	0.36719	0.36358	0.35997	0.35638	0.35280
0.4	0.34924	0.34570	0.34217	0.33865	0.33516	0.33168	0.32821	0.32477	0.32134	0.31793
0.5	0.31454	0.31116	0.30781	0.30447	0.30115	0.29785	0.29457	0.29131	0.28807	0.28485
0.6	0.28165	0.27847	0.27532	0.27218	0.26906	0.26597	0.26289	0.25984	0.25681	0.25380
0.7	0.25081	0.24784	0.24490	0.24198	0.23908	0.23620	0.23335	0.23052	0.22771	0.22492
0.8	0.22216	0.21942	0.21670	0.21400	0.21133	0.20868	0.20606	0.20345	0.20087	0.19832
0.9	0.19578	0.19327	0.19079	0.18832	0.18588	0.18346	0.18107	0.17870	0.17635	0.17402
1.0	0.17172	0.16944	0.16718	0.16495	0.16274	0.16055	0.15838	0.15624	0.15412	0.15202
1.1	0.14994	0.14789	0.14586	0.14385	0.14186	0.13989	0.13795	0.13603	0.13412	0.13224
1.2	0.13039	0.12855	0.12673	0.12494	0.12317	0.12141	0.11968	0.11797	0.11628	0.11460
1.3	0.11295	0.11132	0.10971	0.10812	0.10655	0.10499	0.10346	0.10194	0.10045	0.09897
1.4	0.09751	0.09607	0.09465	0.09325	0.09187	0.09050	0.08915	0.08782	0.08650	0.08521
1.5	0.08393	0.08266	0.08142	0.08019	0.07897	0.07778	0.07660	0.07543	0.07428	0.07315
1.6	0.07203	0.07093	0.06984	0.06877	0.06771	0.06667	0.06564	0.06463	0.06363	0.06264
1.7	0.06167	0.06072	0.05977	0.05884	0.05793	0.05702	0.05613	0.05525	0.05439	0.05354
1.8	0.05270	0.05187	0.05105	0.05025	0.04946	0.04868	0.04791	0.04715	0.04640	0.04567
1.9	0.04494	0.04423	0.04353	0.04284	0.04215	0.04148	0.04082	0.04017	0.03953	0.03890
2.0	0.03828	0.03766	0.03706	0.03647	0.03588	0.03531	0.03474	0.03418	0.03363	0.03309
2.1	0.03256	0.03203	0.03152	0.03101	0.03051	0.03002	0.02953	0.02906	0.02859	0.02813
2.2	0.02767	0.02722	0.02678	0.02635	0.02592	0.02550	0.02509	0.02468	0.02428	0.02389
2.3	0.02350	0.02312	0.02274	0.02237	0.02201	0.02165	0.02130	0.02095	0.02061	0.02028
2.4	0.01995	0.01962	0.01931	0.01899	0.01868	0.01838	0.01808	0.01779	0.01750	0.01721
2.5	0.01693	0.01666	0.01638	0.01612	0.01586	0.01560	0.01534	0.01509	0.01485	0.01461
2.6	0.01437	0.01414	0.01391	0.01368	0.01346	0.01324	0.01302	0.01281	0.01260	0.01240
2.7	0.01220	0.01200	0.01180	0.01161	0.01142	0.01124	0.01106	0.01088	0.01070	0.01053
2.8	0.01036	0.01019	0.01002	0.00986	0.00970	0.00954	0.00939	0.00924	0.00909	0.00894
2.9	0.00880	0.00866	0.00852	0.00838	0.00824	0.00811	0.00798	0.00785	0.00772	0.00760
3.0	0.00748	0.00736	0.00724	0.00712	0.00701	0.00690	0.00679	0.00668	0.00657	0.00646
3.1	0.00636	0.00626	0.00616	0.00606	0.00596	0.00587	0.00578	0.00568	0.00559	0.00550
3.2	0.00542	0.00533	0.00524	0.00516	0.00508	0.00500	0.00492	0.00484	0.00476	0.00469
3.3	0.00461	0.00454	0.00447	0.00440	0.00433	0.00426	0.00419	0.00413	0.00406	0.00400
3.4	0.00394	0.00387	0.00381	0.00375	0.00370	0.00364	0.00358	0.00352	0.00347	0.00342
3.5	0.00336	0.00331	0.00326	0.00321	0.00316	0.00311	0.00306	0.00301	0.00297	0.00292
3.6	0.00287	0.00283	0.00279	0.00274	0.00270	0.00266	0.00262	0.00258	0.00254	0.00250
3.7	0.00246	0.00242	0.00239	0.00235	0.00231	0.00228	0.00224	0.00221	0.00217	0.00214
3.8	0.00211	0.00208	0.00204	0.00201	0.00198	0.00195	0.00192	0.00189	0.00187	0.00184
3.9	0.00181	0.00178	0.00176	0.00173	0.00170	0.00168	0.00165	0.00163	0.00160	0.00158
4.0	0.00156	0.00153	0.00151	0.00149	0.00146	0.00144	0.00142	0.00140	0.00138	0.00136
4.1	0.00134	0.00132	0.00130	0.00128	0.00126	0.00124	0.00122	0.00121	0.00119	0.00117
4.2	0.00115	0.00114	0.00112	0.00110	0.00109	0.00107	0.00106	0.00104	0.00102	0.00101
4.3	0.00100	0.00098	0.00097	0.00095	0.00094	0.00093	0.00091	0.00090	0.00089	0.00087
4.4	0.00086	0.00085	0.00084	0.00082	0.00081	0.00080	0.00079	0.00078	0.00077	0.00076
4.5	0.00074	0.00073	0.00072	0.00071	0.00070	0.00069	0.00068	0.00067	0.00066	0.00065

For *two tails*, multiply the value by 2, since the *t* PDF is symmetric.
The *p*-value is the colored area under the *t* PDF in the sketch.



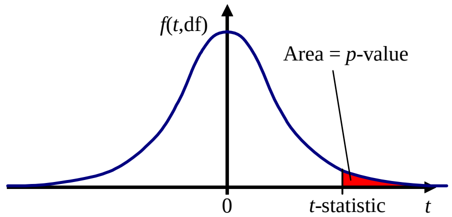
p-Values for the student's t Distribution; one tail, n=10 (df=9)

Note: **This table is for df = 19** only

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.49606	0.49213	0.48819	0.48426	0.48032	0.47639	0.47246	0.46854	0.46461
0.1	0.46070	0.45678	0.45287	0.44897	0.44507	0.44117	0.43728	0.43340	0.42953	0.42566
0.2	0.42180	0.41795	0.41411	0.41027	0.40645	0.40264	0.39883	0.39504	0.39125	0.38748
0.3	0.38372	0.37997	0.37623	0.37251	0.36879	0.36509	0.36141	0.35774	0.35408	0.35044
0.4	0.34681	0.34320	0.33960	0.33602	0.33245	0.32890	0.32537	0.32185	0.31835	0.31487
0.5	0.31141	0.30796	0.30453	0.30113	0.29774	0.29436	0.29101	0.28768	0.28436	0.28107
0.6	0.27780	0.27454	0.27131	0.26810	0.26491	0.26174	0.25859	0.25546	0.25235	0.24927
0.7	0.24621	0.24316	0.24014	0.23715	0.23417	0.23122	0.22829	0.22538	0.22250	0.21964
0.8	0.21680	0.21398	0.21119	0.20842	0.20568	0.20295	0.20026	0.19758	0.19493	0.19230
0.9	0.18969	0.18711	0.18455	0.18202	0.17951	0.17702	0.17456	0.17212	0.16970	0.16731
1.0	0.16494	0.16259	0.16027	0.15797	0.15570	0.15345	0.15122	0.14901	0.14683	0.14467
1.1	0.14254	0.14043	0.13834	0.13627	0.13423	0.13221	0.13021	0.12823	0.12628	0.12435
1.2	0.12244	0.12056	0.11870	0.11686	0.11504	0.11324	0.11146	0.10971	0.10798	0.10627
1.3	0.10458	0.10291	0.10126	0.09963	0.09803	0.09644	0.09487	0.09333	0.09181	0.09030
1.4	0.08882	0.08735	0.08590	0.08448	0.08307	0.08168	0.08031	0.07896	0.07763	0.07632
1.5	0.07502	0.07375	0.07249	0.07125	0.07002	0.06882	0.06763	0.06646	0.06531	0.06417
1.6	0.06305	0.06194	0.06086	0.05978	0.05873	0.05769	0.05666	0.05566	0.05466	0.05368
1.7	0.05272	0.05177	0.05084	0.04992	0.04902	0.04813	0.04725	0.04639	0.04554	0.04470
1.8	0.04388	0.04307	0.04228	0.04149	0.04072	0.03996	0.03922	0.03849	0.03777	0.03706
1.9	0.03636	0.03567	0.03500	0.03434	0.03368	0.03304	0.03241	0.03180	0.03119	0.03059
2.0	0.03000	0.02942	0.02886	0.02830	0.02775	0.02721	0.02668	0.02616	0.02565	0.02515
2.1	0.02466	0.02417	0.02370	0.02323	0.02277	0.02232	0.02188	0.02145	0.02102	0.02060
2.2	0.02019	0.01979	0.01939	0.01900	0.01862	0.01825	0.01788	0.01752	0.01716	0.01682
2.3	0.01648	0.01614	0.01581	0.01549	0.01518	0.01487	0.01456	0.01426	0.01397	0.01368
2.4	0.01340	0.01313	0.01286	0.01259	0.01233	0.01207	0.01182	0.01158	0.01134	0.01110
2.5	0.01087	0.01064	0.01042	0.01020	0.00999	0.00978	0.00957	0.00937	0.00918	0.00898
2.6	0.00879	0.00861	0.00842	0.00825	0.00807	0.00790	0.00773	0.00757	0.00741	0.00725
2.7	0.00709	0.00694	0.00679	0.00665	0.00651	0.00637	0.00623	0.00610	0.00597	0.00584
2.8	0.00571	0.00559	0.00547	0.00535	0.00523	0.00512	0.00501	0.00490	0.00480	0.00469
2.9	0.00459	0.00449	0.00439	0.00430	0.00420	0.00411	0.00402	0.00393	0.00385	0.00376
3.0	0.00368	0.00360	0.00352	0.00344	0.00337	0.00329	0.00322	0.00315	0.00308	0.00301
3.1	0.00295	0.00288	0.00282	0.00276	0.00270	0.00264	0.00258	0.00252	0.00247	0.00241
3.2	0.00236	0.00230	0.00225	0.00220	0.00215	0.00211	0.00206	0.00201	0.00197	0.00193
3.3	0.00188	0.00184	0.00180	0.00176	0.00172	0.00168	0.00164	0.00161	0.00157	0.00154
3.4	0.00150	0.00147	0.00144	0.00140	0.00137	0.00134	0.00131	0.00128	0.00125	0.00123
3.5	0.00120	0.00117	0.00114	0.00112	0.00109	0.00107	0.00105	0.00102	0.00100	0.00098
3.6	0.00095	0.00093	0.00091	0.00089	0.00087	0.00085	0.00083	0.00081	0.00080	0.00078
3.7	0.00076	0.00074	0.00073	0.00071	0.00069	0.00068	0.00066	0.00065	0.00063	0.00062
3.8	0.00060	0.00059	0.00058	0.00056	0.00055	0.00054	0.00053	0.00052	0.00050	0.00049
3.9	0.00048	0.00047	0.00046	0.00045	0.00044	0.00043	0.00042	0.00041	0.00040	0.00039
4.0	0.00038	0.00037	0.00037	0.00036	0.00035	0.00034	0.00033	0.00033	0.00032	0.00031
4.1	0.00030	0.00030	0.00029	0.00028	0.00028	0.00027	0.00027	0.00026	0.00025	0.00025
4.2	0.00024	0.00024	0.00023	0.00023	0.00022	0.00022	0.00021	0.00021	0.00020	0.00020
4.3	0.00019	0.00019	0.00018	0.00018	0.00018	0.00017	0.00017	0.00016	0.00016	0.00016
4.4	0.00015	0.00015	0.00015	0.00014	0.00014	0.00014	0.00013	0.00013	0.00013	0.00013
4.5	0.00012	0.00012	0.00012	0.00011	0.00011	0.00011	0.00011	0.00010	0.00010	0.00010

For *two tails*, multiply the value by 2, since the *t* PDF is symmetric.
The *p*-value is the colored area under the *t* PDF in the sketch.

_p_-Values for the student's _t_ Distribution; one tail, _n_=10 (df=9)



***Note*: This table is for df = 29 only**

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.49604	0.49209	0.48814	0.48418	0.48023	0.47628	0.47234	0.46839	0.46445
0.1	0.46052	0.45658	0.45266	0.44873	0.44481	0.44090	0.43700	0.43310	0.42920	0.42532
0.2	0.42144	0.41757	0.41371	0.40985	0.40601	0.40217	0.39835	0.39454	0.39073	0.38694
0.3	0.38316	0.37939	0.37563	0.37189	0.36815	0.36443	0.36073	0.35704	0.35336	0.34969
0.4	0.34604	0.34241	0.33879	0.33519	0.33160	0.32803	0.32447	0.32094	0.31741	0.31391
0.5	0.31042	0.30696	0.30351	0.30007	0.29666	0.29327	0.28989	0.28653	0.28320	0.27988
0.6	0.27658	0.27331	0.27005	0.26681	0.26360	0.26040	0.25723	0.25408	0.25095	0.24784
0.7	0.24475	0.24169	0.23864	0.23562	0.23262	0.22965	0.22669	0.22376	0.22086	0.21797
0.8	0.21511	0.21227	0.20945	0.20666	0.20389	0.20114	0.19842	0.19572	0.19305	0.19039
0.9	0.18777	0.18516	0.18258	0.18002	0.17749	0.17498	0.17250	0.17003	0.16760	0.16518
1.0	0.16279	0.16042	0.15808	0.15576	0.15347	0.15119	0.14895	0.14672	0.14452	0.14234
1.1	0.14019	0.13806	0.13595	0.13387	0.13181	0.12977	0.12776	0.12576	0.12379	0.12185
1.2	0.11993	0.11802	0.11615	0.11429	0.11246	0.11065	0.10886	0.10709	0.10535	0.10362
1.3	0.10192	0.10024	0.09858	0.09694	0.09533	0.09373	0.09215	0.09060	0.08907	0.08755
1.4	0.08606	0.08459	0.08313	0.08170	0.08029	0.07889	0.07752	0.07616	0.07483	0.07351
1.5	0.07221	0.07093	0.06967	0.06843	0.06720	0.06599	0.06480	0.06363	0.06248	0.06134
1.6	0.06022	0.05912	0.05803	0.05696	0.05590	0.05487	0.05385	0.05284	0.05185	0.05088
1.7	0.04992	0.04897	0.04804	0.04713	0.04623	0.04535	0.04448	0.04362	0.04278	0.04195
1.8	0.04114	0.04034	0.03955	0.03877	0.03801	0.03726	0.03653	0.03580	0.03509	0.03440
1.9	0.03371	0.03303	0.03237	0.03172	0.03108	0.03045	0.02983	0.02923	0.02863	0.02805
2.0	0.02747	0.02691	0.02635	0.02581	0.02528	0.02475	0.02424	0.02373	0.02324	0.02275
2.1	0.02227	0.02180	0.02134	0.02089	0.02045	0.02001	0.01959	0.01917	0.01876	0.01836
2.2	0.01796	0.01758	0.01720	0.01683	0.01646	0.01610	0.01575	0.01541	0.01507	0.01474
2.3	0.01442	0.01410	0.01379	0.01349	0.01319	0.01290	0.01261	0.01233	0.01205	0.01178
2.4	0.01152	0.01126	0.01101	0.01076	0.01052	0.01028	0.01005	0.00982	0.00960	0.00938
2.5	0.00916	0.00895	0.00875	0.00855	0.00835	0.00816	0.00797	0.00779	0.00761	0.00743
2.6	0.00726	0.00709	0.00692	0.00676	0.00660	0.00645	0.00630	0.00615	0.00600	0.00586
2.7	0.00573	0.00559	0.00546	0.00533	0.00520	0.00508	0.00496	0.00484	0.00472	0.00461
2.8	0.00450	0.00439	0.00429	0.00418	0.00408	0.00398	0.00389	0.00379	0.00370	0.00361
2.9	0.00352	0.00344	0.00335	0.00327	0.00319	0.00311	0.00304	0.00296	0.00289	0.00282
3.0	0.00275	0.00268	0.00262	0.00255	0.00249	0.00243	0.00237	0.00231	0.00225	0.00219
3.1	0.00214	0.00209	0.00203	0.00198	0.00193	0.00188	0.00184	0.00179	0.00175	0.00170
3.2	0.00166	0.00162	0.00158	0.00154	0.00150	0.00146	0.00142	0.00139	0.00135	0.00132
3.3	0.00128	0.00125	0.00122	0.00119	0.00116	0.00113	0.00110	0.00107	0.00104	0.00102
3.4	0.00099	0.00096	0.00094	0.00092	0.00089	0.00087	0.00085	0.00082	0.00080	0.00078
3.5	0.00076	0.00074	0.00072	0.00070	0.00069	0.00067	0.00065	0.00063	0.00062	0.00060
3.6	0.00059	0.00057	0.00056	0.00054	0.00053	0.00051	0.00050	0.00049	0.00047	0.00046
3.7	0.00045	0.00044	0.00043	0.00041	0.00040	0.00039	0.00038	0.00037	0.00036	0.00035
3.8	0.00034	0.00033	0.00033	0.00032	0.00031	0.00030	0.00029	0.00028	0.00028	0.00027
3.9	0.00026	0.00026	0.00025	0.00024	0.00024	0.00023	0.00022	0.00022	0.00021	0.00021
4.0	0.00020	0.00019	0.00019	0.00018	0.00018	0.00017	0.00017	0.00017	0.00016	0.00016
4.1	0.00015	0.00015	0.00014	0.00014	0.00014	0.00013	0.00013	0.00013	0.00012	0.00012
4.2	0.00012	0.00011	0.00011	0.00011	0.00010	0.00010	0.00010	0.00010	0.00009	0.00009
4.3	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008	0.00007	0.00007	0.00007	0.00007
4.4	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005
4.5	0.00005	0.00005	0.00005	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004

Digital Data Acquisition: $\text{quantization error} = \pm\frac{1}{2}\text{resolution} = \pm\frac{1}{2}\frac{(V_{\max} - V_{\min})}{2^N}$, $\Delta t = 1/f_s$, $\omega = 2\pi f$.

General equation for aliasing $f_{\text{perceived}} = \left| f - f_s \cdot \text{NINT}\left(\frac{f}{f_s}\right)\right|$.

Fourier transforms, DFTs and FFTs: $f_{\text{folding}} = \frac{f_s}{2} = \frac{N}{2}\Delta f$, $\Delta t = T/N$, $f_s = 1/\Delta t = N/T$, $\Delta f = 1/T$.

Filters:

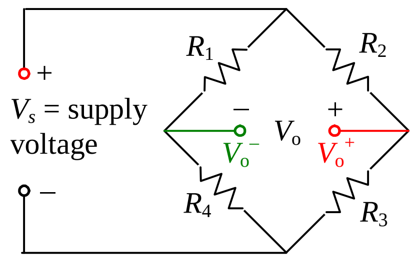
- 1st-order low-pass filter – $f_{\text{cutoff}} = \frac{\omega_{\text{cutoff}}}{2\pi} = \frac{1}{2\pi RC}$, $G = \frac{|V_{\text{out}}|}{|V_{\text{in}}|} = \frac{1}{\sqrt{1+\left(\frac{f}{f_{\text{cutoff}}}\right)^2}}$, $\phi = -\arctan\left(\frac{f}{f_{\text{cutoff}}}\right)$.
- 1st-order high-pass filter – $f_{\text{cutoff}} = \frac{\omega_{\text{cutoff}}}{2\pi} = \frac{1}{2\pi RC}$, $G = \frac{|V_{\text{out}}|}{|V_{\text{in}}|} = \frac{1}{\sqrt{1+\left(\frac{f_{\text{cutoff}}}{f}\right)^2}}$, $\phi = \arctan\left(\frac{f_{\text{cutoff}}}{f}\right)$.
- Higher-order filters – for a Butterworth low-pass filter of order n, $G = \frac{|V_{\text{out}}|}{|V_{\text{in}}|} = \frac{1}{\sqrt{1+\left(\frac{f}{f_{\text{cutoff}}}\right)^{2n}}}$.
- Gain expressed in decibels – for any filter or amplifier, $G_{\text{dB}} = 20\log_{10} G = 20\log_{10}\frac{|V_{\text{out}}|}{|V_{\text{in}}|}$.

Operational Amplifiers (op-amps):

- Open-loop gain – $V_o = g(V_p - V_n)$.
- Closed-loop configuration (with feedback loop) – $V_p \approx V_n$.
- Example circuits – buffer: $V_{\text{out}} = V_{\text{in}}$, inverting amplifier: $V_{\text{out}} = -\frac{R_2}{R_1}V_{\text{in}} = GV_{\text{in}}$, inverter: $V_{\text{out}} = -V_{\text{in}}$, inverting summer: $V_{\text{out}} = -(V_1 + V_2)$, noninverting amplifier: $V_{\text{out}} = \left(1+\frac{R_2}{R_1}\right)V_{\text{in}} = GV_{\text{in}}$.
- Other circuits – active filters, clipping circuits (high and low voltage clipping), etc.
- Common-mode rejection ratio (CMRR) – $\text{CMRR} = 20\log_{10}\frac{g}{G_{\text{CM}}}$, and G_{CM} is the common-mode gain.
- Gain-bandwidth product (GBP) – op-amp acts like low-pass filter at high frequencies:
  - *Noninverting* op-amp amplifier: $\text{GBP} = \text{GBP}_{\text{noninverting}} = f_c G_{\text{theory, noninverting}} = \text{constant}$ [GBP supplied by manufacturer's specs].
  - *Inverting* op-amp amplifier: $\text{GBP}_{\text{inverting}} = \frac{-R_2}{R_1 + R_2}\text{GBP}_{\text{noninverting}}$, $\text{GBP}_{\text{inverting}} = f_c G_{\text{theory, inverting}} = \text{constant}$.
- Loading – input and output loading can cause voltage change due to internal resistances. R_i = input resistance or input impedance.

Stress, Strain, and Strain Gages:

- Axial stress and axial strain – $\sigma_a = \dfrac{F}{A}$, $\varepsilon_a = \dfrac{\delta L}{L}$, $\sigma_a = E\varepsilon_a$.
- Transverse strain & Poisson's ratio – $\nu = \text{Poisson's ratio} = -\dfrac{\varepsilon_t}{\varepsilon_a}$, $\varepsilon_t = \dfrac{\delta w}{w} = \dfrac{\delta t}{t}$; $\varepsilon_t = \dfrac{\delta D}{D}$ for round rods.
- Stress-strain relationship on a surface; principal axes – $\varepsilon_x = \dfrac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$, $\varepsilon_y = \dfrac{1}{E}\left(\sigma_y - \nu\sigma_x\right)$.
- Wire resistance – $R = \dfrac{\rho L}{A}$, $\dfrac{\delta R}{R} = S\varepsilon_a$, where S is the strain gage factor, defined as $S = \dfrac{\delta R / R}{\varepsilon_a}$.
- Wheatstone bridge – $V_o = V_s \dfrac{R_3R_1 - R_4R_2}{\left(R_2+R_3\right)\left(R_1+R_4\right)}$, bridge is balanced when $R_3R_1 = R_4R_2$. Approximate relationship when resistances change: $\dfrac{V_o}{V_s} \approx \dfrac{R_{2,\text{initial}}R_{3,\text{initial}}}{\left(R_{2,\text{initial}}+R_{3,\text{initial}}\right)^2}\left(\dfrac{\delta R_1}{R_{1,\text{initial}}} - \dfrac{\delta R_2}{R_{2,\text{initial}}} + \dfrac{\delta R_3}{R_{3,\text{initial}}} - \dfrac{\delta R_4}{R_{4,\text{initial}}}\right)$.
- Quarter, half, and full bridge – $\varepsilon_a \approx \dfrac{4}{n}\dfrac{V_o}{V_s}\dfrac{1}{S}$ or $V_o \approx \dfrac{n}{4}\varepsilon_a S V_s$, where
  - o $n = 1$ for a quarter bridge
  - o $n = 2$ for a half bridge
  - o $n = 4$ for a full bridge



- Unbalanced bridge – $\varepsilon_a \approx \dfrac{4}{n}\dfrac{\left(V_o - V_{o,\text{reference}}\right)}{V_s}\dfrac{1}{S}$, $V_o \approx V_{o,\text{reference}} + \dfrac{n}{4}\varepsilon_a S V_s$.

Dynamic System Response:

- General governing equation (ODE). $a_n \dfrac{d^n y}{dt^n} + a_{n-1}\dfrac{d^{n-1}y}{dt^{n-1}} + \ldots + a_2\dfrac{d^2y}{dt^2} + a_1\dfrac{dy}{dt} + a_0 y = bx$.
- First-order system. $\tau\dfrac{dy}{dt} + y = Kx$, $K = b/a_0$, $\tau = a_1/a_0$.
  - o For step-function input, $\dfrac{y - y_i}{y_f - y_i} = \text{nondimensionalized output} = 1 - e^{\frac{-t}{\tau}}$, $\Gamma(t) = \dfrac{y - y_f}{y_i - y_f} = e^{\frac{-t}{\tau}}$.
  - o For ramp function input, $y_{\text{measured}} = KA\left[t - \tau\left(1 - e^{-(t/\tau)}\right)\right]$, $\text{time lag} = \tau$, $\text{error} = y_{\text{measured}} - y_{\text{ideal}} = -KA\tau$.
- Second-order system. $\dfrac{1}{\omega_n{}^2}\dfrac{d^2y}{dt^2} + \dfrac{2\zeta}{\omega_n}\dfrac{dy}{dt} + y = Kx$, $K = b/a_0$, $\omega_n = \sqrt{\dfrac{a_0}{a_2}}$, $\zeta = \dfrac{a_1}{2\sqrt{a_0 a_2}}$. For step-function input:
  - o ***Underdamped***, $\zeta < 1$, $\dfrac{y - y_i}{y_f - y_i} = 1 - e^{-\zeta\omega_n t}\left[\dfrac{1}{\sqrt{1-\zeta^2}}\sin\left(\omega_n t\sqrt{1-\zeta^2} + \phi\right)\right]$, $\phi = \sin^{-1}\left(\sqrt{1-\zeta^2}\right)$. Undamped natural frequency and ringing frequency: $f_n = \dfrac{\omega_n}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{a_0}{a_2}}$, and $\omega_d = \omega_n\sqrt{1-\zeta^2}$ or $f_d = f_n\sqrt{1-\zeta^2}$.
  - o ***Critically damped***, $\zeta = 1$, $\dfrac{y - y_i}{y_f - y_i} = 1 - e^{-\omega_n t}\left(1 + \omega_n t\right)$, and there is no phase shift.
  - o ***Overdamped***, $\zeta > 1$, $\dfrac{y - y_i}{y_f - y_i} = 1 - e^{-\zeta\omega_n t}\left[\cosh\left(\omega_n t\sqrt{\zeta^2 - 1}\right) + \dfrac{\zeta}{\sqrt{\zeta^2-1}}\sinh\left(\omega_n t\sqrt{\zeta^2 - 1}\right)\right]$.
  - o ***Log-decrement method***: $\ln\left(\dfrac{y^*{}_i}{y^*{}_{i+n}}\right) = n\delta$, $\zeta = \dfrac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$, $\omega_d = \dfrac{2\pi}{T} = 2\pi f_d$, $\omega_n = 2\pi f_n = \dfrac{\omega_d}{\sqrt{1-\zeta^2}}$.

Temperature Measurement:

- Thermocouples: $V_{1-2} = V_{1-R} - V_{2-R}$, $V_{1-3} = V_{1-2} + V_{2-3}$ for any 3 temperatures T_1, T_2, and T_3.
- **Brief thermocouple table**:

TABLE 9.2 Millivolt Output of Common Thermocouples (Reference Junction at 0°C)

Temperature (°C)	Thermocouple type					
	T	E	J	K	R	S
−250	−6.181	−9.719		−6.404		
−200	−5.603	−8.824	−7.890	−5.891		
−150	−4.648	−7.279	−6.499	−4.912		
−100	−3.378	−5.237	−4.632	−3.553		
−50	−1.819	−2.787	−2.431	−1.889		
0	0.000	0.000	0.000	0.000	0.000	0.000
20	0.789	1.192	1.019	0.798	0.111	0.113
40	1.611	2.419	2.058	1.611	0.232	0.235
60	2.467	3.683	3.115	2.436	0.363	0.365
80	3.357	4.983	4.186	3.266	0.501	0.502
100	4.277	6.317	5.268	4.095	0.647	0.645
120	5.227	7.683	6.359	4.919	0.800	0.795
140	6.204	9.078	7.457	5.733	0.959	0.950
160	7.207	10.501	8.560	6.539	1.124	1.109
180	8.235	11.949	9.667	7.338	1.294	1.273
200	9.286	13.419	10.777	8.137	1.468	1.440
220	10.360	14.909	11.887	8.938	1.647	1.611
240	11.456	16.417	12.998	9.745	1.830	1.785
260	12.572	17.942	14.108	10.560	2.017	1.962
280	13.707	19.481	15.217	11.381	2.207	2.141
300	14.860	21.033	16.325	12.207	2.400	2.323
350	17.816	24.961	19.089	14.292	2.896	2.786
400	20.869	28.943	21.846	16.395	3.407	3.260
450		32.960	24.607	18.513	3.933	3.743
500		36.999	27.388	20.640	4.471	4.234
600		45.085	33.096	24.902	5.582	5.237
700		53.110	39.130	29.218	6.741	6.274
800		61.022		33.277	7.949	7.345
900		68.873		37.325	9.203	8.448
1000		76.358		41.269	10.503	9.585
1100				45.108	11.846	10.754
1200				48.828	13.224	11.947
1300				52.398	14.624	13.155
1400					16.035	14.368
1500					17.445	15.576
1600					18.842	16.771
1700					20.215	17.942

- Thermoresistive devices – RTDs (metal, R increases with increasing T), thermistors (semiconductor, R decreases with increasing T). See tables, next page.
- Radiative devices – e.g., infrared pyrometer: $E = \varepsilon\sigma T^4$, $\sigma = 5.669\times10^{-8}\ \frac{\text{W}}{\text{m}^2\text{K}^4}$, $T_H = \left(\frac{\varepsilon_{\text{assumed}}}{\varepsilon_{\text{actual}}}\right)^{1/4} T_{\text{ind}}$.

Platinum 100-Ω RTD Table

TABLE F.3
Platinum RTD 100 Ω at 0°C, DIN curve 43760, 9–68

°C	Ohms	°C	Ohms	°C	Ohms	°C	Ohms	°C	Ohms
–200	18.53	–40	84.21	120	146.06	280	204.88	440	260.75
–195	20.65	–35	86.19	125	147.94	285	206.68	445	262.45
–190	22.78	–30	88.17	130	149.82	290	208.46	450	264.14
–185	24.92	–25	90.15	135	151.7	295	210.25	455	265.83
–180	27.05	–20	92.13	140	153.57	300	212.03	460	267.52
–175	29.17	–15	94.1	145	155.45	305	213.81	465	269.21
–170	31.28	–10	96.07	150	157.32	310	215.58	470	270.89
–165	33.38	–5	98.04	155	159.18	315	217.36	475	272.57
–160	35.48	0	100	160	161.04	320	219.13	480	274.25
–155	37.57	5	101.95	165	162.9	325	220.9	485	275.92
–150	39.65	10	103.9	170	164.76	330	222.66	490	277.6
–145	41.73	15	105.85	175	166.62	335	224.42	495	279.27
–140	43.8	20	107.79	180	168.47	340	226.18	500	280.93
–135	45.87	25	109.73	185	170.32	345	227.94	505	282.6
–130	47.93	30	111.67	190	172.16	350	229.69	510	284.26
–125	49.99	35	113.61	195	174	355	231.44	515	285.91
–120	52.04	40	115.54	200	175.84	360	233.19	520	287.57
–115	54.09	45	117.47	205	177.68	365	234.93	525	289.22
–110	56.13	50	119.4	210	179.51	370	236.67	530	290.87
–105	58.17	55	121.32	215	181.34	375	238.41	535	292.51
–100	60.2	60	123.24	220	183.17	380	240.15	540	294.16
–95	62.23	65	125.16	225	185	385	241.88	545	295.8
–90	64.25	70	127.07	230	186.82	390	243.61	550	297.43
–85	66.27	75	128.98	235	188.64	395	245.34	555	299.07
–80	68.28	80	130.89	240	190.46	400	247.06	560	300.7
–75	70.29	85	132.8	245	192.27	405	248.78	565	302.33
–70	72.29	90	134.7	250	194.08	410	250.5	570	303.95
–65	74.29	95	136.6	255	195.89	415	252.21	575	305.58
–60	76.28	100	138.5	260	197.7	420	253.93	580	307.2
–55	78.27	105	140.39	265	199.5	425	255.64	585	308.81
–50	80.25	110	142.28	270	201.3	430	257.34	590	310.43
–45	82.23	115	144.18	275	203.09	435	259.05	595	312.04

(Continued on next page)

TABLE F.3
(Continued)

°C	Ohms	°C	Ohms	°C	Ohms	°C	Ohms	°C	Ohms
600	313.65	655	331.15	710	348.3	765	365.1	820	381.55
605	315.25	660	332.72	715	349.84	770	366.61	825	383.03
610	316.86	665	334.29	720	351.38	775	368.12	830	384.5
615	318.46	670	335.86	725	352.92	780	369.62	835	385.98
620	320.05	675	337.43	730	354.45	785	371.12	840	387.45
625	321.65	680	338.99	735	355.98	790	372.62	845	388.91
630	323.24	685	340.55	740	357.51	795	374.12	850	390.38
635	324.83	690	342.1	745	359.03	800	375.61		
640	326.41	695	343.66	750	360.55	805	377.1		
645	327.99	700	345.21	755	362.07	810	378.59		
650	329.57	705	346.76	760	363.59	815	380.07		

Table taken from R. E. Fraser, *Process Measurement and Control – Introduction to Sensors, Communication, Adjustment, and Control*, Prentice-Hall, Inc., Upper Saddle River, NJ, 2001.

Resistance values for two standard thermistors

T (°C)	*T* (°F)	*R* (Ω) for type 2252	*R* (Ω) for type 5000
-10.0	14.0	12460	27670
-9.0	15.8	11810	26210
-8.0	17.6	11190	24830
-7.0	19.4	10600	23540
-6.0	21.2	10050	22320
-5.0	23.0	9534	21170
-4.0	24.8	9046	20080
-3.0	26.6	8586	19060
-2.0	28.4	8151	18100
-1.0	30.2	7741	17190
0.0	32.0	7355	16330
1.0	33.8	6989	15520
2.0	35.6	6644	14750
3.0	37.4	6319	14030
4.0	39.2	6011	13340
5.0	41.0	5719	12700
6.0	42.8	5444	12090
7.0	44.6	5183	11510
8.0	46.4	4937	10960
9.0	48.2	4703	10440
10.0	50.0	4482	9951
11.0	51.8	4273	9486
12.0	53.6	4074	9046
13.0	55.4	3886	8628
14.0	57.2	3708	8232
15.0	59.0	3539	7857
16.0	60.8	3378	7500
17.0	62.6	3226	7162
18.0	64.4	3081	6841
19.0	66.2	2944	6536
20.0	68.0	2814	6247
21.0	69.8	2690	5972
22.0	71.6	2572	5710
23.0	73.4	2460	5462
24.0	75.2	2354	5225
25.0	77.0	2252	5000
26.0	78.8	2156	4787
27.0	80.6	2064	4583
28.0	82.4	1977	4389
29.0	84.2	1894	4204
30.0	86.0	1815	4029

Measurement of Mechanical Quantities:

- Position – ultrasonic transducers: pulse-echo $x = \frac{a\Delta t}{2}$, through transmission $x = a\Delta t$;

  capacitance sensor: $C = K\varepsilon_0 A / d$, $\varepsilon_0 = 8.854\times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$.
- Angular velocity and rpm – $\left(N_{\text{rpm}} \frac{\text{rotation}}{\text{min}}\right) = \left(\omega \frac{\text{radian}}{\text{s}}\right)\left(\frac{\text{rotation}}{2\pi\ \text{radian}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right)$;

  noncontacting tachometer: $N_{\text{rpm}} = \frac{P}{n_{\text{teeth}}}$. For stroboscopic tachometers, watch out for *aliasing*.
- Torque and power – $\dot{W}_{\text{shaft}} = \omega T = \frac{2\pi}{60} N_{\text{rpm}} T$.

Pressure Measurement:

- Types of pressure – absolute, gage: $P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}}$, vacuum: $P_{\text{vac}} = P_{\text{atm}} - P_{\text{abs}}$.
- Hydrostatics – $P_{\text{below}} = P_{\text{above}} + \rho g|\Delta z|$; applications include:
  - Liquid manometers (U-tube manometers): typically, $\Delta P = (\rho_{\text{m}} - \rho) gh$ if measurement locations are at the same elevation on either side of the manometer.
  - Hydraulic jacks – the ideal mechanical advantage is $\frac{F_2}{F_1} = \frac{A_2}{A_1}$.

Velocity Measurement:

- Doppler radar velocimeter – Doppler frequency shift $\Delta f_D = \frac{2V\cos\theta}{\lambda}$.
- Displacement sensors – $V(t) = dx(t)/dt$; Acceleration sensors – $V(t) = V_0 + \int_{t_0}^{t} a(t)dt$.
- Velocity of fluids – Lagrangian methods follow fluid particle moving with the flow: $V(t) = dx(t)/dt$; Eulerian methods measure velocity field with a probe sitting in the flow.
- Laser Doppler velocimeter (LDV) – particles in fringe pattern of focal volume, $V = fs = \frac{f\lambda}{2\sin(\alpha/2)}$.
- Particle image velocimetry (PIV) – velocity vectors calculated from moving particles in the plane illuminated by a laser sheet, $V = \Delta s/\Delta t$.
- Hot-wire and hot-film anemometer – wire heated to constant temperature; King's law $E^2 = a + bV^n$.
- Pitot-static probe – Pitot formula $V = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$.

Volume Flow Rate Measurement:

- Mass flow rate vs. volume flow rate – $\dot{m} = \rho\dot{V}$. *Notation*: V = volume, V = velocity, $\dot{V}$ = volume flow rate.
- End-line measurement – $\dot{V} = V/\Delta t$ vs. in-line measurement – various types:
- Obstruction – orifice, flow nozzle, and Venturi: $\beta = d/D$, $\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$, where d = orifice diameter, D = pipe diameter, A_o = orifice area, V_1= average speed through the pipe, V_2= average speed through the orifice (or throat), $P_1 - P_2$ = measured pressure drop from upstream to downstream, C_d = discharge coefficient, and Reynolds number is based on the *pipe* not the orifice, i.e., $\text{Re} = \frac{\rho V_1 D}{\mu} = \frac{V_1 D}{\nu}$:
  - For a standard orifice flowmeter, $C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^{8.0} + 91.71\beta^{2.5}/\text{Re}^{0.75}$
  - For a standard flow nozzle flowmeter, $C_d = 0.9975 - 6.53\beta^{0.50}/\text{Re}^{0.50}$
  - For a standard Venturi flowmeter, $C_d = 0.98$