AUTOMATIC CONTROL OF THE TRIGA REACTOR

(Experiment B#6)
by
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REFERENCES

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OBJECTIVE

The objective of this experiment is to design and implement an automatic controller for regulating TRIGA reactor power using a **state-of-the-art** UNIX network compatible controller integrated with a development, test, and monitoring environment based on the Mathworks MATLAB/SIMUILINK operating in a remote UNIX host computer. The controller is designed and tested with MATLAB/SIMULINK which you have used throughout the earlier labs. In this lab, the necessary real-time control software in the C computer language is automatically generated via the SIMULINK C-code generation option, directly from your SIMULINK block diagram, and then downloaded to the real-time control computer. The nonlinear aspects of nuclear reactor control will be dramatized by implementing a controller design for one power level and monitoring its diminished capacity as power is changed to a higher level.

INTRODUCTION

As you have learned in previous experiments, there is always some difference between the real-world and the mathematics we use to describe it. If we had a perfect mathematical description of a system, we could precompute the actions required to change operating conditions of the system and not even bother to measure the consequence of those actions. This would be called ***Open Loop Control***. On the other hand, ***Feedback Control*** provides a means to accommodate uncertainties between our mathematical model and the performance of the real-world system. By taking measurements of system variables we can adjust actions to that needed to accomplish performance objectives. For a reactor system, the control actions can be accomplished manually with a human operator manipulating control rod positions or with an automatic control system. A representation of an elementary automatic reactor power controller is given in Figure B6-1 where the controller computes a control rod speed demand signal $z_d(t)$ simply as a scalar gain G_c times an error signal e(t); e.g., the speed demand signal $z_d(t)=G_c e(t)$ is computed by subtracting the power measurement $n_r(t)$ from a demand signal $n_d(t)$; i.e., $e(t)=n_d(t)-n_r(t)$. The control rod mechanism dynamically responds to position the rod and thus dynamically change control rod reactivity $\rho_r(t)$ until power matches the demand signal. There are two major approaches to understanding and designing control systems: 1) ***Time Domain Analysis*** and 2) ***Frequency Domain Analysis***.

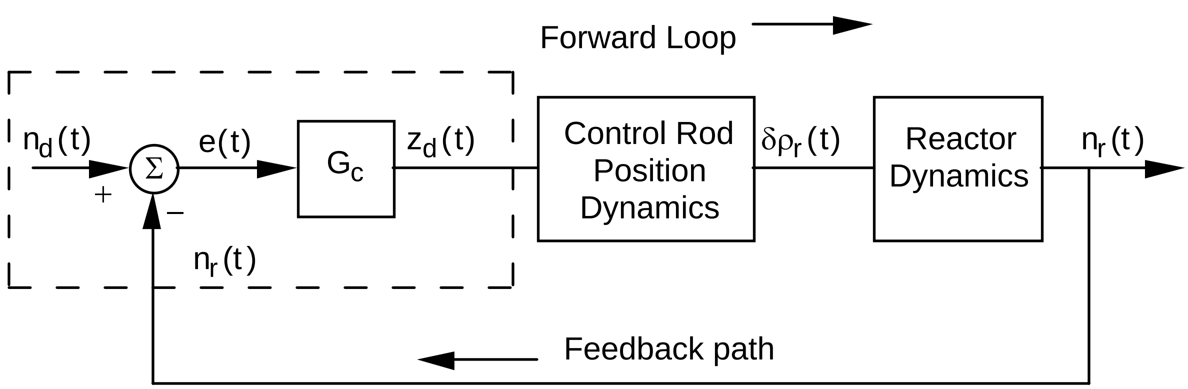

Figure B6-1 Simple Feedback Control Loop.

Time Domain Analysis

To understand how the controller operates in the time domain it is necessary to discuss the error signal e(t). As e(t) becomes very small, the power measurement must be approaching the value of the demand signal. Consider the following example. It is desired to change reactor power to a new level. To accomplish this goal, a step change in the demand signal $n_d(t)$ could be applied. A possible response is shown in Figure B6-2 where the step change is applied at t=t'. In this example the actual power of the reactor responds slower than the input demand. To reduce the response time, a modification in the controller is needed. Clearly, if the value of G_c is increased, the subsequent value of $z_d(t)$ will be magnified (initially), producing a greater velocity of the control rod. Power will respond more quickly and e(t) will ***approach*** zero faster.

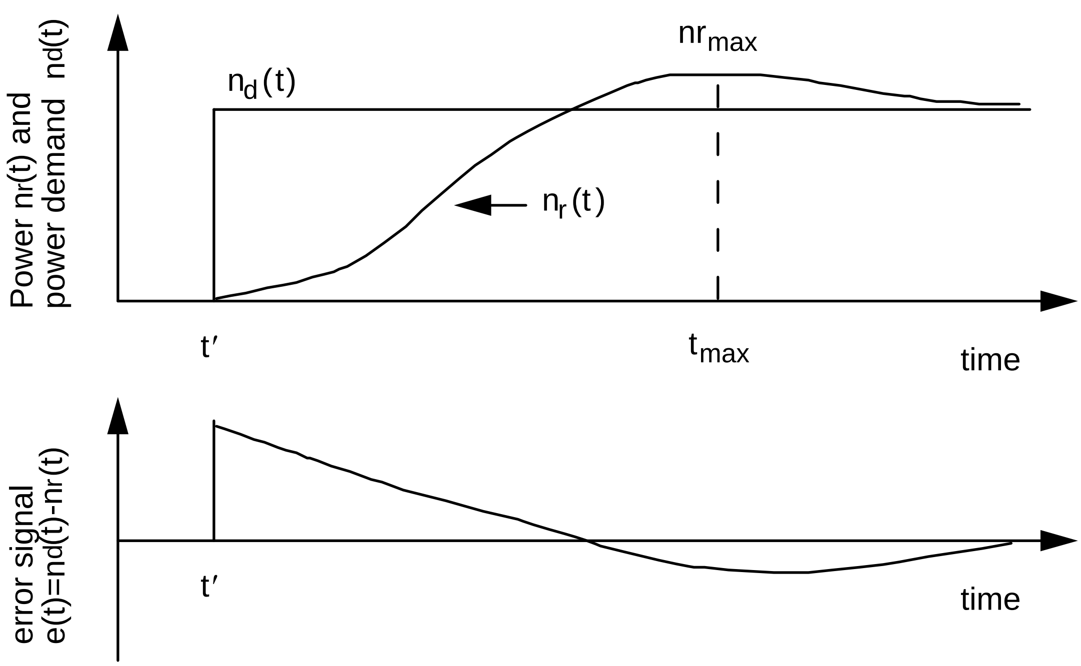

Figure B6-2 Time Domain Description of Elementary Power Controller Showing Power n_r(t) and Error Signal e(t) Response to a Step Change in Power Demand n_d(t).

The function performed by the scalar G_c is generally not limited to proportionality constants but can be a mixture of constants, integrators, and differentiators. Depending on the application, all three types of controllers may be used simultaneously, resulting in the PID controller. For this introductory control experiment, such advanced controllers will not be explored. However, you will setup, program, and operate the controller using a special Secondary Control Rod installed in the central thimble of the TRIGA core.

In addition to reducing the response time, other parameters should be considered, such as maximum overshoot and settling time. ***Maximum Overshoot*** occurs at the highest value of $n_r(t)$ as it deviates from a step change in input demand signal. ***Settling time*** is the time in which the step response settles down to about ***5 percent*** of the maximum overshoot.

Trade-offs are commonly experienced when a designer wants to optimize more than one variable. For instance, if the maximum overshoot needed to be reduced, the settling time usually becomes lengthened.

Frequency Domain Analysis

The main approach to understanding and designing a feedback control system is analysis in the frequency domain. By working in this domain a control systems engineer can more readily obtain mathematical solutions, design controllers and investigate and predict ***stability*** and ***performance*** characteristics of the system.

A very important concept in the frequency domain is the ***transfer function*** which was developed in the reactor frequency response experiment and further examined in the reactor noise experiment. Recall that the transfer function of a system is the Laplace transform of the output divided by the Laplace transform of the input (assuming zero initial conditions). For a one delayed neutron reactor, the transfer function from reactivity input $\delta\tilde{\rho}(s)$ to reactor power $\delta N_r(s)$ (B4.23) is

$$G_R(s) = \frac{\delta N_r(s)}{\delta\tilde{\rho}(s)} = \frac{\left(n_{r0}/\Lambda\right)(s+\lambda)}{s\left(s+\beta/\Lambda\right)} \qquad \text{(B6.1)}$$

(A capital letter or a ~ above a variable denotes the Laplace transform of the corresponding lower case time domain variable.)

By obtaining a transfer function for each component in Figure B6-1, they can be readily combined to design a feedback controller. By representing the control rod drive dynamics with $G_r(s)$, the dynamics of the system in the frequency domain can be summarized in block diagram form as shown in Figure B6-3.

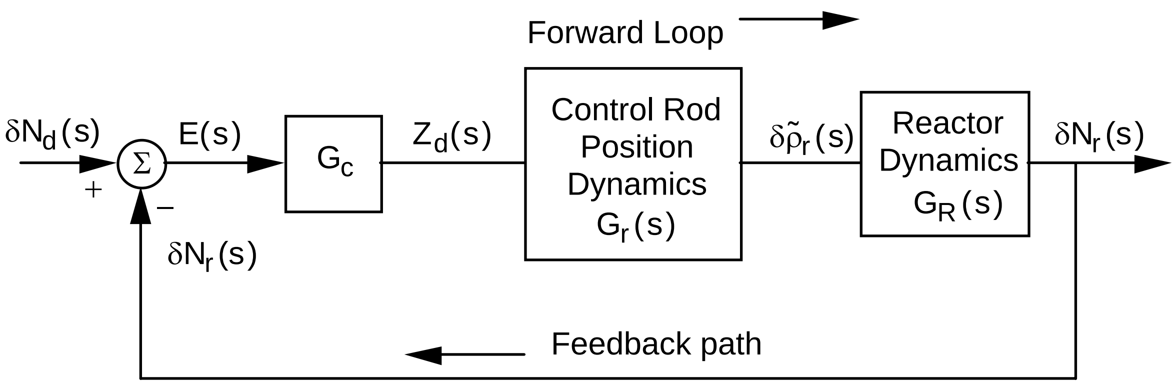

Figure B6-3 Transfer Function Block Diagram of Reactor Control.

The characteristics of a simple closed loop feedback controller is governed by the ***closed loop identity relationship***. This identity relates the transfer function of the closed loop system to the individual transfer functions in the forward and feedback path of the system. The transfer functions along a path are simply combined by multiplication; i.e., the forward path of Figure B6-3 represented by G(s) is equal to $G_c G_r(s) G_R(s)$. Figure B6-4 shows a general feedback controlled system where dynamical elements H(s) are also contained in the feedback path.

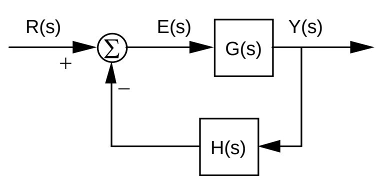

Figure B6-4 General Form of Feedback Control: R(s) is the Reference (demand) Signal; Y(s) is the System Output Signal.

The closed loop identity, which relates output response Y(s) to the demand or reference function R(s) is

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \qquad \text{(B6.2)}$$

B-6-6

In the reactor controller of Figures B6-1 and B6-3 there are no dynamical elements in the feedback path so that H(s) is simply unity.

By initially assuming ideal signal conditioning and rod drive dynamics which instantaneously provides a speed that matches the speed demand signal, control rod position dynamics can be represented as a pure integral function which has a Laplace transform equal to 1/s. The control rod transfer function $G_r(s)$ can be simply represented as W/s where W is the total reactivity worth of the rod and rod speed z_r is computed in units of fraction of core length per second (positive in the direction of removal from the reactor). The closed loop transfer function of the elementary controlled system in Figures B6-1 and B6-3 for a one-delayed neutron group reactor without temperature feedback is thus

$$\frac{Y(s)}{R(s)} = \frac{\delta N_r(s)}{\delta N_d(s)} = \frac{\dfrac{G_c\,(W/s)(n_{r0}/\Lambda)\,(s+\lambda)}{s\,(s+\beta/\Lambda)}}{1+\dfrac{G_c\,(W/s)(n_{r0}/\Lambda)\,(s+\lambda)}{s\,(s+\beta/\Lambda)}} \qquad \text{(B6.3)}$$

which can be algebraically manipulated into the ratio of two polynomials

$$\frac{\delta N_r(s)}{\delta N_d(s)} = \frac{(G_c W n_{r0})(s+\lambda)}{\Lambda \cdot s^3 + \beta \cdot s^2 + (G_c W n_{r0})\cdot s + (G_c W n_{r0})\lambda} \qquad \text{(B6.4)}$$

The denominator polynomial is the characteristic equation. The 3 roots of this equation are the exponents in the exponential functions of a time domain response, similar to that studied in the digital simulation laboratory (B1) and the rod calibration experiment (B2). As indicated, the controller gain G_c is located in two terms of the characteristic equation. By adjusting the gain, the roots of the characteristic equation are changed which results in a different dynamical system response.

Unlike all the reactor analysis and experiments conducted in this laboratory sequence, the roots of the characteristic (B6.4) will invariably include complex numbers; i.e., $r_i=\sigma_i \pm j\omega_i$, i=1,2,3 and $j=\sqrt{-1}$. The roots of the open loop reactor kinetics characteristic equation (inhour equation A1.6 and B2.4) are always real. Generally, this is not true for higher order dynamical systems and in particular for feedback controlled systems. When complex roots occur, they always occur in complex conjugate pairs, $r_i=\sigma_i+j\omega_i$ and $r_{i+1}=\sigma_i-j\omega_i$. The combination of exponential functions with these complex exponents will produce a damped sinusoidal response (assuming that the real part σ_i is negative, $Ae^{\sigma_i t}\sin(\omega t)$. If σ_i is 0, a self-excited oscillatory response at the frequency ω_i will be observed, A $\sin(\omega t)$. If σ_i ***is positive, the system is unstable*** and the system output (reactor power) will grow until the system is destroyed, a safety system intervenes, or the automatic controller reaches the limit of control capability that it has available (runs out of rod positioning capability).

Most any linear systems analysis computer software package will include a polynomial root solving subroutine. (The SIMULINK simulation language includes such functions.) A standard presentation of a controller analysis is a complex plane plot of characteristic equation root locations versus controller gain. Such a plot is called a ***root locus plot.*** Figure B6-5 shows a root locus of (B6.4) as the gain G_c is varied from 0.0 to 2.8 with parameters n_{r0}=1, β=0.007, λ=0.1, Λ=0.000035, and W=0.0024. At a gain G_c of 0, the system has two roots at the origin, σ_i=0, ω_i=0, and a third root at σ=-200. As gain is increased ever so slightly above 0, the two roots at the origin form a complex conjugate pair which trace out a circle until the gain exceeds about 1.0.

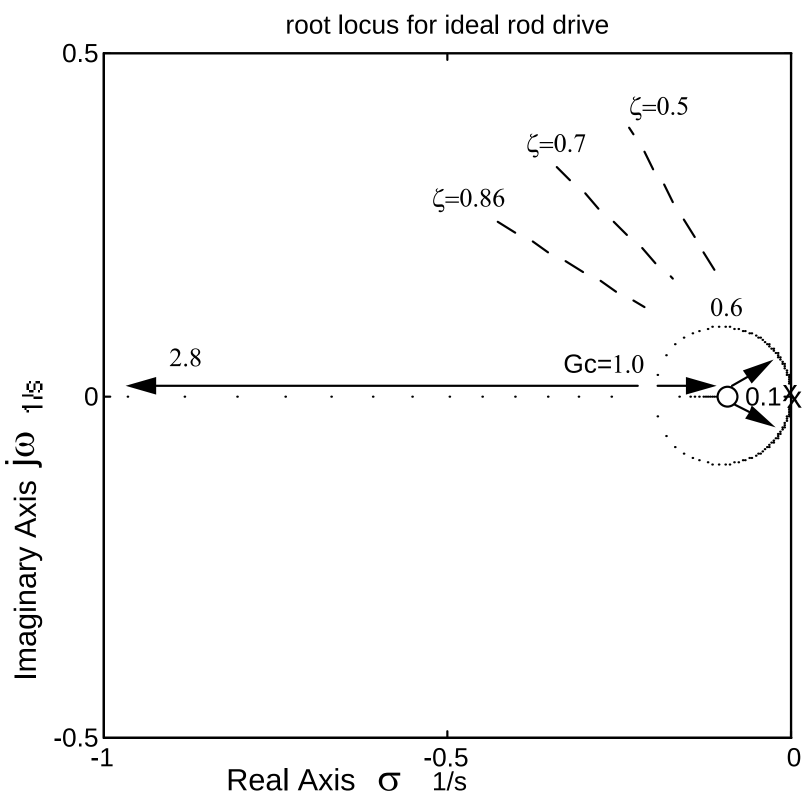

Figure B6-5 Root Locus With Ideal Control Rod Drive.

As gain is increased further, the roots again become real with one approaching σ=-1 and the other becoming increasingly negative. For this ideal closed loop system, it can be shown that it is stable for all positive gains. However, implementation of a controller with a large gain would be susceptible to realistic reactor noise and other uncertainties and would not be practical.

A measure of controller performance is the damping ratio ζ of the dominant roots. The ***damping ratio*** is defined as the ratio of the real part of the root to the magnitude of the root; i.e., $\zeta = |\sigma_i| / \sqrt{\sigma_i^2 + \omega_i^2}$. The usefulness of the damping ratio is that it readily identifies the expected time response of the system due to that root. Figure B6-6 shows example step responses of a 2nd order system with several damping ratios.

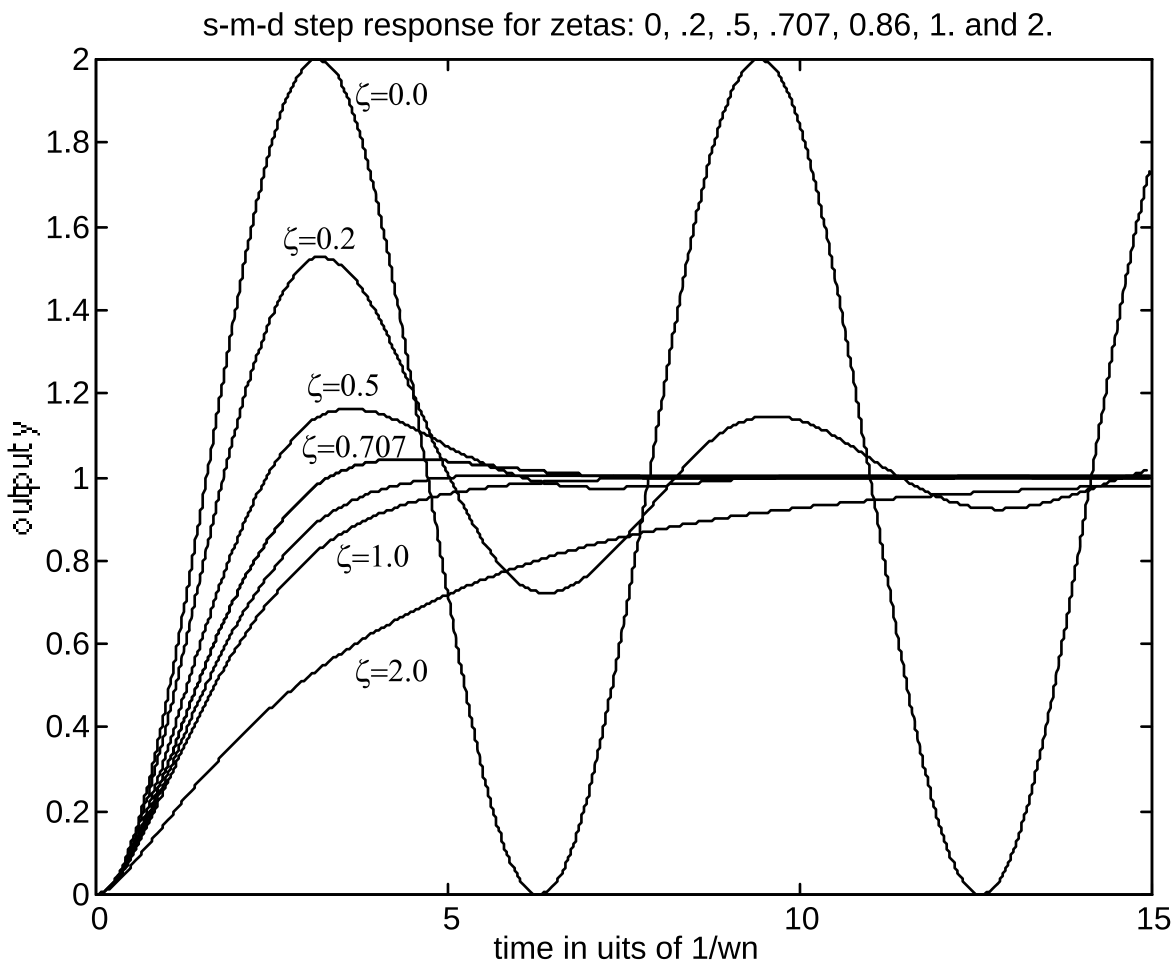

Figure B6-6 Second Order System Response to Step Inputs.

The response at a damping ratio of 0.707 appears attractive in that it is a fast response with very little overshoot. On the root locus plot, a diagonal line at 45 degrees indicates the 0.707 damping ratio condition. A typical goal of a control system design is to ***seek a damping ratio in the range of 0.5 to 0.9***. Based on the root locus of Figure B6-5, a controller gain of 0.56 provides a damping ratio of 0.707. (The units of this gain are *fraction of core length per second per fractional power error;* If the error between the demand and measured relative powers is 0.1, the resulting control rod speed demand signal would be 0.056 fraction of core length per second with this gain; in-other-words, if there is a 10% power error the speed demand would be 5.6% of core length per second.)
The root locus analysis presented in Figure B6-5 is for a mythical control rod drive mechanism which can instantaneously change control rod speed to match a demand signal. A more realistic assumption would be to assume that the control rod speed responds as a pure exponential with a time constant of τ. This is also called a first order lag response. The resulting control rod drive transfer function is then

$$G_r = \frac{\delta\tilde{\rho}_r(s)}{\delta Z_d(s)} = \frac{W}{s\,(\tau s + 1)} \qquad \text{(B6.5)}$$

and the corresponding closed loop transfer function is

$$\frac{\delta N_r(s)}{\delta N_d(s)} = \frac{(G_c W n_{r0})(s+\lambda)}{\Lambda\tau\cdot s^4 + (1+\beta\tau)\cdot s^3 + \beta s^2 + (G_c W n_{r0})\cdot s + (G_c W n_{r0})\lambda} \qquad \text{(B6.6)}$$

This new denominator polynomial again has a dependency on the controller gain but now also depends on the control rod dynamics given the time constant τ. Unlike the controller with an idealized rod drive mechanism, the stability of the system with the more realistic controller is dependent on the controller gain selection. A critical value for G_C can be determined which will cause the roots of the characteristic equation to have positive real parts and lie to the right of the origin in the root locus plot. Figure B6-7 shows a series of root locus analysis for the rod time constants of 1, 2, and 4 seconds. The root locus with the 1 second time constant appears very similar to the ideal case for gains less than 0.9. However, with a time constant of 2 or 4 seconds, a response with a damping ratio of 0.707 cannot be obtained and the system will become unstable at a lower gain value.

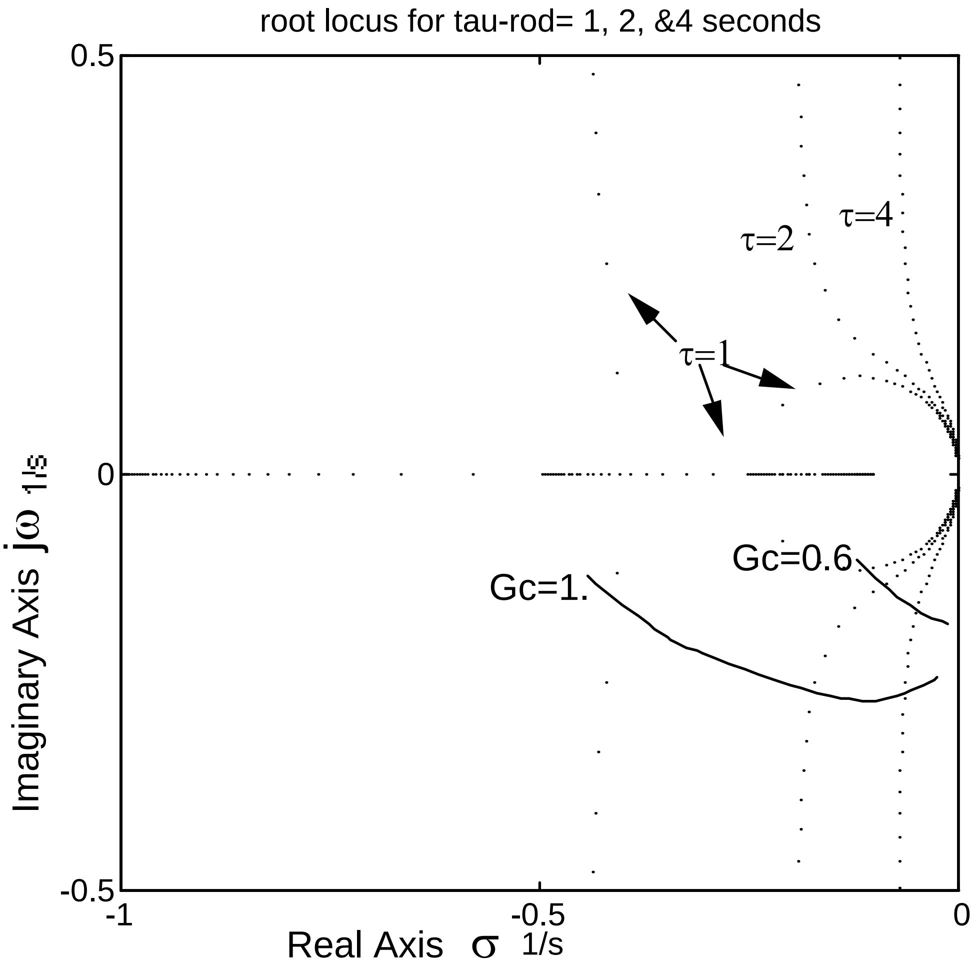

Figure B6-7 Root Locus With Control Rod Dynamics

EQUIPMENT

The equipment used in this laboratory is a Gateway computer with National Instruments data acquisition card. The software is the real-time workshop option of the Mathworks MATLAB/Simulink. The real time target option for a windows-based system is used.

Figure B6-1 is physically realized in Figure B6-8, excluding the sensor and driver sections. Inside the computer a new value of power is obtained at discrete sampling intervals T. Using k to denote an integer constant, the discrete nature of computer time is thus represented as t=kT. An output signal will be setup by subtracting the feedback signal $n_r(kT)$ from a setpoint $n_d(kT)$and multiplied by the controller gain G_c. The SIMULINK block diagram representing this controller is presented at the top Figure B6-8, cc_simrt.m

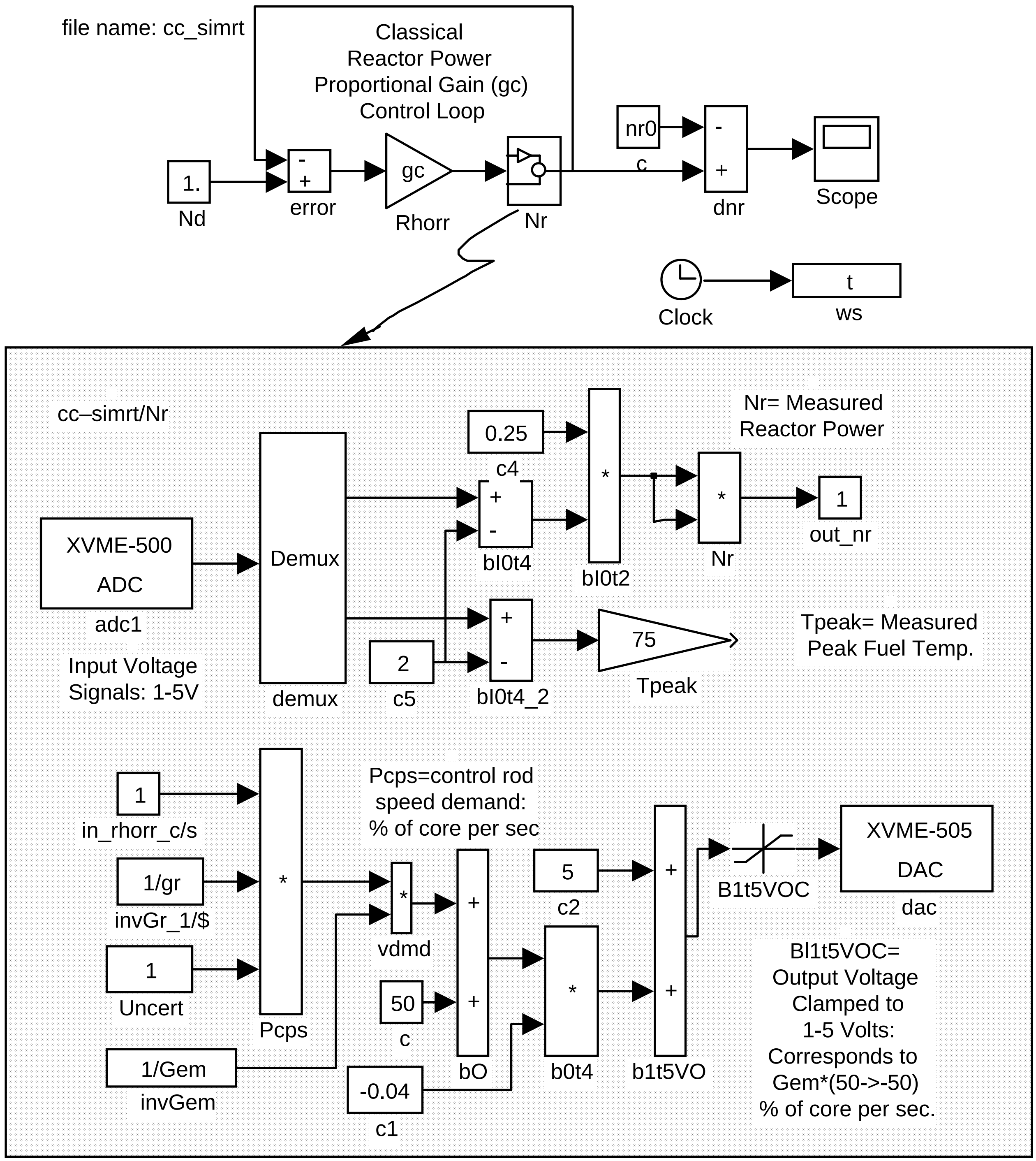

Figure B6-8 SIMULINK Controller Specification.

The block labeled Nr at the top of Figure B6-8 exemplifies a key feature of SIMULINK for organizing large and complex simulations in to logical and manageable chunks. The computations and processing represented by the Nr block are shown in the shaded region of the Figure. On the top level SIMULINK block diagram shown at the top of the Figure, one would simply double click on the Nr block to reveal its processing definitions in a secondary window. As shown, the Nr block receives a reactivity rate demand signal from the output of the controller, gc, and converts it to an appropriate voltage for a Digital to Analog Converter (DAC) to convert and apply to the rod drive mechanism. The Nr block also defines the digitization of the voltage signal representing the reactor power measurement to engineering units.

EXPERIMENT OVERVIEW

The experiment is designed to demonstrate a key element of reactor dynamics in feedback control. Even though you may never be a control engineer, it is your responsibility as a Nuclear Engineer to inform control specialists, who may not be familiar with reactor dynamics, about this characteristic. *The result of the nonlinear characteristic of reactor dynamics in a feedback control system is that a change in power level is equivalent to changing the gain in the controller.* To demonstrate this effect in an experiment, we will start at a reactor power level 100 watts which we will define as a relative power of 1.0, n_{r0}=1.0, and select a controller gain which provides a well-damped response. As discussed in class, a closed loop controller with an ideal signal conditioning and control rod drive mechanism, which could instantaneously change speed to match a demand signal, would be stable at all controller gains. We, of course, do not have an ideal rod or signal conditioning, and there is a gain which will cause the controlled system to be unstable. Instead of experimentally increasing the gain in search of this unstable condition, we are instead going to approach the unstable condition by increasing the reactor power (in small steps). The purpose of this exercise is to dramatize that changing reactor power level is like changing the controller gain in a closed loop reactor power controller. We will increase reactor power to a maximum of 1 Kilowatt, n_{r0}=10, a gain increase of a factor of 10. We will do this in small steps not to exceed 25% of the current power level or 100 watts whichever is less. At each power increase, we will record the overshoot and settling time and thus estimate the frequency and damping ratio of the dominant root which can then be plotted on a root locus plot. When things have equilibrated, reactor noise analysis will also be conducted to obtain the power spectrum before deciding on the next power increase.

EXPERIMENT PROCEDURES

In order to become familiar and comfortable with the equipment setup, a rod position control experiment will be first conducted with the reactor in a standby condition. The position control experiment is conducted with the reactor in a standby condition (no power) and we can afford to push the controlled system to an unstable response (undamped sinusoid) with a certain value of gain. The fact that we can make the position control experiment unstable proves that there are additional dynamics beyond a simple time constant. (In class we showed, that if the control electronics and rod dynamics were strictly a first order system described with a simple time constant τ, it would be stable at all controller gains.)