Name: ____

Date of lab: ____________________ Section number: M E 345.______

Lab 3

Precalculations – Individual Portion
Correlation and Regression: Statistical Analysis of Trends

Precalculations Score (for instructor or TA use only):	_____ / 20

1. **(4)** Imagine an electrical circuit in which the resistance is constant, but the voltage is varied. Both voltage and current are measured as data pairs. Do you expect current to be correlated with voltage? (No Yes) Explain.

2. **(4)** What is the definition of the ***spring constant*** of a spring, and what are its dimensions and typical SI units?

3. **(4)** Think about the pressure inside a balloon. As the balloon is inflated (as it gets bigger and bigger), do you expect the pressure inside the balloon to go down, stay the same, or go up? Explain. Do you expect pressure to be linearly correlated with balloon diameter? Why or why not?

4. **(8)** In this lab you will be asked to design an experiment where you will adjust the inclination angle of an accelerometer and measure the voltage output of the instrument. What instrument will you use to measure the voltage? If the accelerometer has a linear voltage output with respect to acceleration, do you expect the voltage output with respect to inclination angle to be linear? (Keep in mind that these accelerometers can measure the static acceleration from gravity, even when sitting still.) Explain.

Cover Page for

Lab 3

Lab Report – Group Portion

Correlation and Regression: Statistical Analysis of Trends

Name 1: ____ Section M E 345.______

Name 2: ____ Section M E 345.______

Name 3: ____ Section M E 345.______

[Name 4: ____ Section M E 345.______]

Date when the lab was performed: ____________________

Group Lab Report Score (For instructor or TA use only):

Lab experiment and results, plots, tables, discussion questions, etc.	_____ / 80
TOTAL	_____ **/ 80**

Lab Participation Grade and Deductions – The instructor or TA reserves the right to deduct points for any of the following, either for all group members or for individual students:

- Arriving late to lab or leaving before your lab group is finished.
- Not participating in the work of your lab group (freeloading).
- Causing distractions, arguing, or not paying attention during lab.
- Not following the rules about formatting plots and tables.
- Grammatical errors in your lab report.
- Sloppy or illegible writing or plots (lack of neatness) in your lab report.
- Other (at the discretion of the instructor or TA).

Name	Reason for deduction	Points deducted	Total grade (out of 80)

Comments (for instructor or TA use only):

Correlation and Regression: Statistical Analysis of Trends

Author: John M. Cimbala; also edited by Peter Ingram, Michael Robinson, and Savas Yavuzkurt, Penn State University
Latest revision: 11 September 2013

Introduction and Background (*Note*: To save paper, you do *not* need to print this section for your lab report.)
Consider a set of n measurements of some variable y as a function of another variable x. Typically, y is some measured output as a function of some known input, x, and such data are often taken for the purpose of ***static calibration*** – that is, ***calibration in which time is not relevant in the measurement***. For example, the voltage output of a pressure transducer is calibrated as a function of pressure, which is in turn measured with a reliable, trusted instrument. In general, in such a set of measurements, there may be:

- some ***scatter*** (***precision error*** or ***random error***)
- a ***trend*** – in spite of the scatter, ***y may show an overall increase or perhaps an overall decrease with x***

Correlation analysis is used to calculate a ***linear correlation coefficient***. The linear correlation coefficient r_{xy} is used to determine if there is a trend. If there *is* a trend, ***regression analysis*** is used to find an equation for y as a function of x that provides the best fit to the data. The best fit is determined by minimizing the square of the error between the fit and the data, and is therefore also called a ***least squares fit***.

In this lab, you will take several sets of data (x and y data pairs), and examine whether or not the data are correlated. Regression analysis will be performed on three data sets – one from a hanging spring, one from the pressure inside an inflated balloon, and one from an accelerometer.

Objectives

1. Become more familiar with correlation and regression analysis.
2. Examine how a spring stretches as a function of applied load.
3. Examine how pressure inside a balloon varies with balloon diameter.
4. Examine how accelerometer voltage changes with inclination angle.
5. Improve Microsoft Excel skills, particularly its statistical analysis features, such as regression analysis.

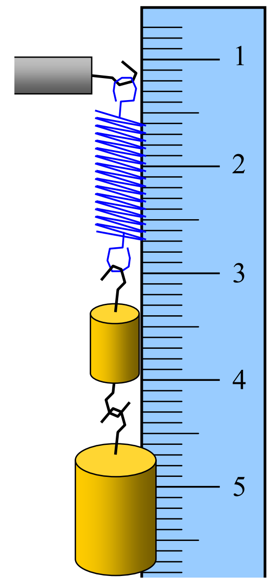
Equipment

- test stand with triangular base, various hooks, rods, and clamps
- spring
- ruler and tape measure
- mass set (10 g to 1000 g) with hooks
- balloon, typically 9-inch or 12-inch (get from instructor or TA)
- pressure gage (either 0 to 20 or 0 to 40 inches of water range)
- powered breadboard and box of jumper wires for breadboarding
- Inclinometer
- ADXL335 Accelerometer

Procedure

Testing the Linearity of a Spring

1. Construct a test stand such that the spring hangs vertically downward from a hook, and the centimeter ruler is aligned vertically, parallel to the axis of the spring, as in the sketch. (Use whatever clamps, hooks, and rods that you need to construct the test stand.)



2. **(1)** Record the location of the bottom of the spring when *no weight* is hanging from the spring. (*Note*: You can use the bottom of the coils or the top of the bottom loop or the bottom of the bottom loop – whatever you desire, but be *consistent* in all your measurements.

   Vertical location with no weight, $x_{\text{zero-load}}$ = ___________ cm

3. Prepare a table in Excel with *at minimum* the following four (labeled) columns, and enter the data from your first set of measurements (m = 0):
   - applied mass m (grams, g)
   - measured distance x (centimeters, cm)
   - applied force F (newtons, N)
   - displacement $x - x_0$ (centimeters, cm)

   where displacement is the measured distance minus the zero value, and represents the amount of spring displacement from the ***zero-load*** case.
4. Hook a 50 g mass to the bottom loop of the spring, and enter the data into your spreadsheet. Repeat in intervals of 50 g (50, 100, 150, 200, ...). *Caution: Do not exceed 15 cm of displacement, or you may overstretch and permanently damage the spring.*
5. Repeat *backwards* down to zero (..., 200, 150, 100, 50, 0). When finished you should have two data points at each mass level.
6. Perform a regression analysis on displacement as a function of applied force, *using all the data points* (Data Analysis-Regression). Place the results within the same Excel worksheet. Highlight the linear correlation coefficient, r_{xy} on the spreadsheet. Do not print out the spreadsheet yet – this will be done in Step 10.
7. Prepare a plot of displacement as a function of applied force. Use symbols for these data points (no line, just symbols). Be sure to include axes labels. Put your plot on the same spreadsheet as the previous step.
8. On the *same plot*, plot a line based on the results of the regression analysis, i.e., plot the linear least-squares curve fit of the experimental data. Use a line for these data points (no symbols, just a line).
9. **(5)** Attach a printout of your spreadsheet showing the table, the regression analysis and results, and the plot.

   See attached spreadsheet printout.

10. **(5)** Repeat the regression analysis, but *ignore some of the data, as appropriate* (it should be obvious which data points to ignore – an official outlier analysis is *not* necessary). Attach a printout of your spreadsheet, including the plot and regression analysis. Is the correlation better now? Discuss.

    See attached spreadsheet printout.

11. Tear down your test stand so that the next group has to start over from scratch.

Discussion Questions for the Spring Displacement Experiment

1. **(5)** Is there any ***hysteresis*** in your measurements? Discuss why or why not.

2. **(5)** From your spring displacement results, does spring displacement increase linearly with applied load, as predicted by theory? Why or why not? Discuss.

3. **(5)** Calculate the spring constant, *k*, for the spring, including units, showing all your work here.

Testing the Correlation of Internal Balloon Pressure to Balloon Diameter

1. Get a *new* balloon from the TA. For health reasons, one person in the group should be designated as the "**Official Balloon Blower**."
2. **(3)** Blow up the balloon to nearly maximum size, but try not to pop the balloon. Twist the stem a couple times to seal the air from leaking, and attach its opening to the pressure gage. Untwist the stem. Measure the diameter of the balloon. Record the pressure and the balloon diameter as the first data pair.

   Balloon pressure, P = ___________ inches of water column

   Balloon diameter (including *units*), D = ___________ ____________

3. Prepare a table in Excel with *at minimum* the following four (labeled) columns, and enter the data from your first set of measurements: *Note*: You will need to convert to the desired units.
   - balloon diameter D (centimeters, cm)
   - balloon internal pressure P (inches of water column, in. H_2O)
   - balloon internal pressure P (pounds per square inch, psi)
   - balloon internal pressure P (pascals, Pa, or N/m^2)
4. Let *a small amount* of air out of the balloon *without disconnecting it from the pressure gage*, and generate a second data pair (pressure and balloon diameter). Repeat for at least 15 balloon diameters, and enter the data into your spreadsheet. *Be careful to deflate the balloon slowly so that you obtain enough points*; if you re-inflate the balloon after the skin has already been stretched, the data will not be consistent. *Please see the TA if you have any questions about performing this experiment.*
5. Plot the data pairs (pressure in pascals as a function of diameter in centimeters) using symbols only (no line).
6. **(5)** Perform a linear regression analysis of balloon pressure (y) as a function of balloon diameter (x). Calculate r_{xy} and plot the best-fit straight line through the data (line – no symbols) *on the same plot*.

   Linear correlation coefficient, r_{xy} = ___________

7. **(8)** Attach a printout of your spreadsheet showing the table, the regression analysis and results, and the plot.

   See attached spreadsheet printout.

8. When all done, you may pop the balloon (everybody likes to pop balloons!) and dispose of it.

Discussion Questions for the Balloon Experiment

1. **(5)** You were asked to perform a linear regression analysis of balloon pressure (y) as a function of balloon diameter (x), to calculate the linear correlation coefficient r_{xy}, and to plot the data points and the best-fit straight line on the same plot. Is the correlation nearly linear? Why or why not?

2. **(5)** Compare your linear correlation coefficient r_{xy} to the critical value for your number of data pairs (interpolate from the table if necessary). Can you officially say to 95% confidence that there is a trend of balloon pressure vs. balloon diameter? Discuss.

3. **(5)** If the behavior is *not* linear, try to find a different curve fit that fits the data better. You are welcome to "play" with either pressure or diameter as the dependent (y) or independent (x) variable – any way necessary to generate a curve fit that goes through the data points reasonably well. Attach your plot and discuss.

Testing the Correlation of Accelerometer Output to Inclination Angle

1. **(8)** You will need to design an experiment to relate the inclination angle of an accelerometer to the output voltage of the accelerometer. Read the tutorial "Using_the_ADXL335_Accelerometer.pdf ", which is posted on the class website to help you get started. We also have **magnetic-base inclinometers** in the lab that attach well to the breadboard. The inclinometer is a simple mechanical device with a dial that indicates the angle of the inclinometer relative to the horizon. Many smart phones also have inclination measurement apps that you could use. Create a table with many (you will need to decide how many) values of inclination angle and accelerometer voltage on the Xout channel. Take measurements over the range between when the accelerometer is parallel to the table and when it is perpendicular to the table.

See attached Table ________________

2. **(5)** Generate a linear calibration equation of the accelerometer that converts the voltage output of the accelerometer to an acceleration in *g*'s. Hint: When the X axis of the accelerometer is perpendicular to the direction of gravity, it experiences roughly zero *g*'s of acceleration. When the X axis of the accelerometer is parallel to the direction of gravity it experiences roughly one *g* of acceleration. Think about how you could change your data from inclination angle vs. voltage output to acceleration vs. voltage output. Show all units.

Calibration Equation:

Discussion Questions for the Accelerometer Experiment

1. **(5)** Were the voltage measurements you obtained linear with respect to inclination angle? Does this match your expectations? Explain.

2. **(5)** *Quantitatively*, how well do your collected data match your linear equation? *Hint*: Think about what statistical parameters describe how well an equation fits a set of data. Discuss.