Today, we will:

- Review the pdf module: **Dynamic System Response (1st-order systems)**
- Do some example problems – dynamic system response for first-order systems

Example: First-order dynamic system response

Given: A first-order low-pass filter with R = 100 kΩ and C = 0.010 μF

(***a***) **To do**: Calculate the time constant τ and the static sensitivity K of this system.

(***b***) **To do**: Discuss how time constant τ is related to the cutoff frequency of the filter.

(***c***) **To do**: For a sudden change in input voltage, how long will it take for the nondimensional output to reach 99% of its final value?

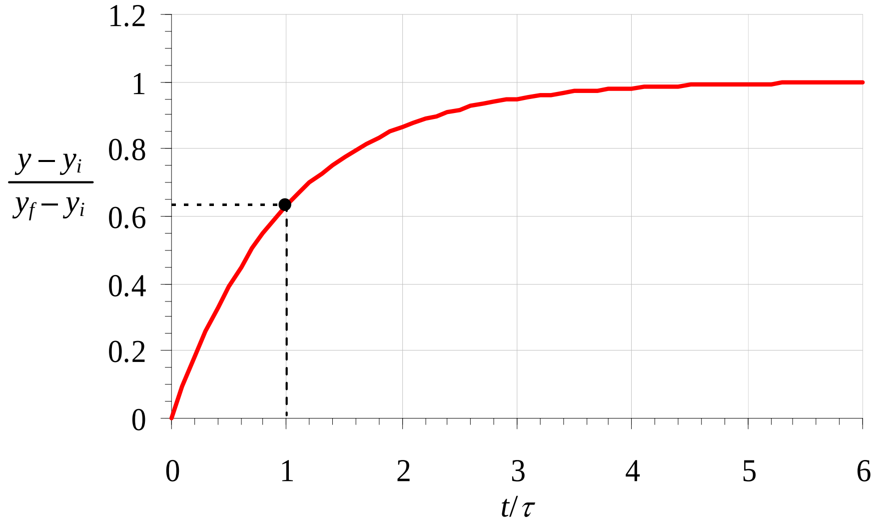
(***d***) **To do**: If y_i = 1 V and y_f = 3 V, calculate y when t = 10.0 ms.

Solution:

Example: First-order dynamic system response

Given: A thermometer behaves as a first-order dynamic system with time constant τ = 1.00 s. At t = 0, the thermometer is plunged from a tank of ice water at T = 0°C into a tank of boiling water at T = 100°C.

To do: Calculate how long (in seconds) it takes for the thermometer to read 50°C.



Solution:

$$\frac{y-y_i}{y_f-y_i}=1-e^{\frac{-t}{\tau}}$$

Example – A practical example of 1st-order dynamic system analysis

Given: A fire has occurred in a hotel room with volume $V = 85.\ \text{m}^3$. Immediately after the fire is extinguished, the mass concentration of hydrogen cyanide (HCN) is $c_i = 10{,}000\ \text{mg/m}^3$. The firemen blow "fresh" air into the room at $\dot{V} = 28.3\ \text{m}^3/\text{min}$. Since there is some smoke outside the building, the "fresh" air actually has an ambient mass concentration of HCN equal $c_a = 1.0\ \text{mg/m}^3$. The air is considered safe when the mass concentration of HCN in the room drops below $c = 5\ \text{mg/m}^3$.

To do: Calculate how long the firemen need to wait before entering the room.

Solution: Room ventilation is modeled by using a first-order ODE for mass concentration c in the room:

$$\boxed{\frac{dc}{dt} = -\frac{\dot{V}}{V}c + \frac{\dot{V}}{V}c_a}$$