Today, we will:

- Discuss **half-life** in first-order dynamic systems
- Talk about first-order dynamic systems with a **ramp function input**
- Finish reviewing the pdf module: **Dynamic System Response (2nd-order systems)**
- Do some more example problems – dynamic system response

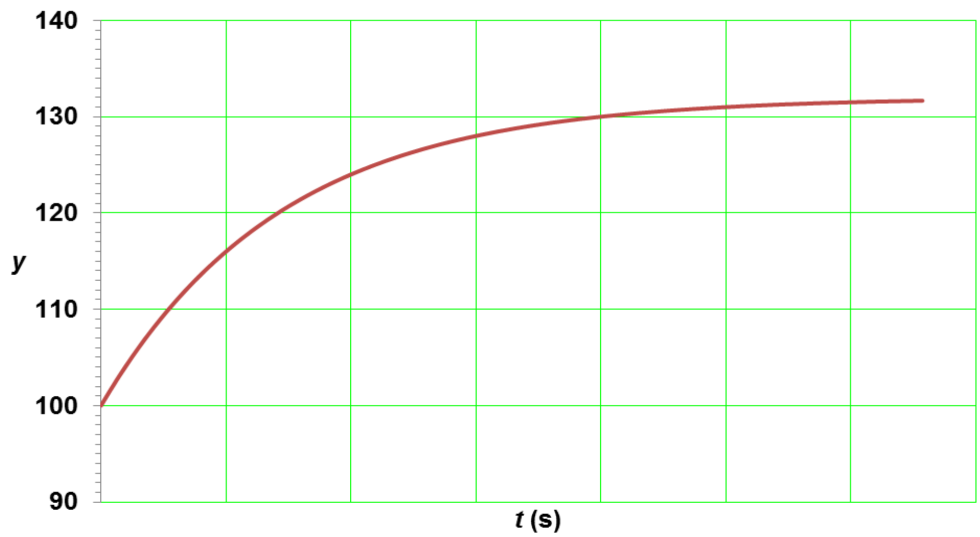
Definition of half-life: ***Half-life*** is the time required for a variable to go half-way from its present value to its final value. Half-life can also be thought of as the 50% response time.

Example: Dynamic system response (first-order and half-life)

Given: Output variable *y* responds like a first-order dynamic system when exposed to a sudden change of input variable *x*. The half-life of the system is **2.00 s**. When *x* is suddenly increased, *y* grows from an initial value of **100** to a final value of **132**.

To do: Calculate the time (in seconds) required for *y* to reach a value of **124**.

Solution:



Example: Dynamic system response (second-order)

Given: The following second-order ODE:

$$5\frac{d^2y}{dt^2}+\frac{dy}{dt}+1000y=x(t) \qquad (1)$$

The forcing function is a step function (sudden jump):

$x(t) = 0$ for $t < 0$

$x(t) = 25$ for $t > 0$

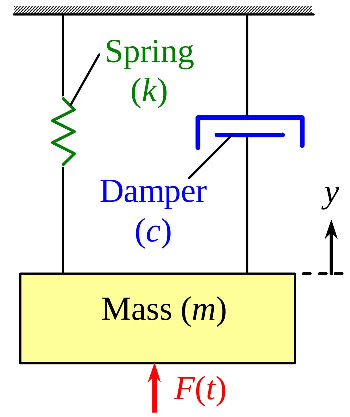
(*a*) To do: Calculate the natural frequency and damping ratio of this system.

(*b*) To do: Calculate the ***equilibrium response*** (as $t \to \infty$, what is y?).

Solution:

Example: Dynamic system response

Given: A spring-mass-damper system is set up with the following properties: mass m = 22.8 g, spring constant k = 51.6 N/cm, and damping coefficient c = 3.49 N·s/m (c is also called λ in some textbooks). The forcing function is a step function (sudden jump).



To do:

(*a*) Calculate the damping ratio of this system. Will it oscillate?

(*b*) If the system will oscillate, calculate the oscillation frequency in hertz. [*Note*: Calculate the physical frequency, not the radian frequency.] Compare the actual oscillating frequency to the undamped natural frequency of the system.

Solution: