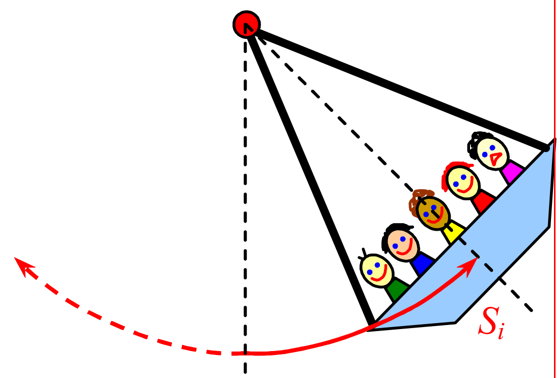
Today, we will:

- Finish the pdf module: **Dynamic System Response (response time, log-decrement)**
- Do some example problems – 2nd-order dynamic systems

Example: Dynamic system response [*explicit* solution]

Given: A pendulum-type amusement park ride behaves as a 2nd-order dynamic system with damping ratio $\zeta = 0.1$ and $f_n = 0.125$ Hz.

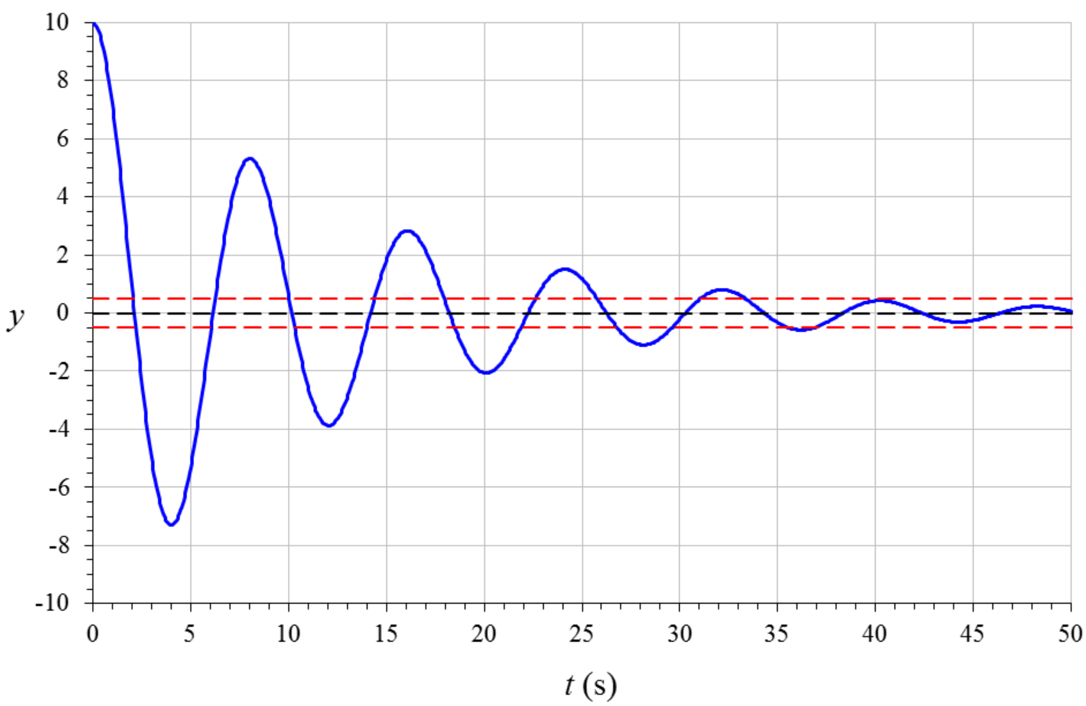


To do: For an initial displacement $S_i = 10.0$ m, calculate the damped natural frequency, the undamped natural frequency, and how long it takes for the oscillations to damp out to within 5% of S_i (the 95% response time).

Solution:

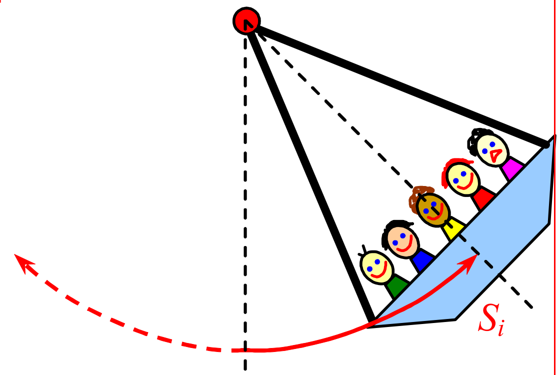
Since we know ζ and f_n, we can plot y or y_{norm} as functions of t or $\omega_n t$, using the equation for underdamped 2nd-order dynamic system response, as given in the learning module,

$$y_{\text{norm}} = \frac{y - y_i}{y_f - y_i} = 1 - e^{-\zeta\omega_n t}\left[\frac{1}{\sqrt{1-\zeta^2}}\sin\left(\omega_n t\sqrt{1-\zeta^2} + \sin^{-1}\left(\sqrt{1-\zeta^2}\right)\right)\right]$$



Example: Dynamic system response [*implicit* solution]

Given: A pendulum-type amusement park ride behaves as a 2nd-order dynamic system. We measure the *actual* (damped) period of the oscillations, T_d = 7.65 s. We also measure the two peak amplitudes at 4 periods apart, namely, S_i = 4.740 m at the first observed peak, when t = 7.65 s, and S_i = 0.239 m at a peak four periods later, when t = 38.25 s.

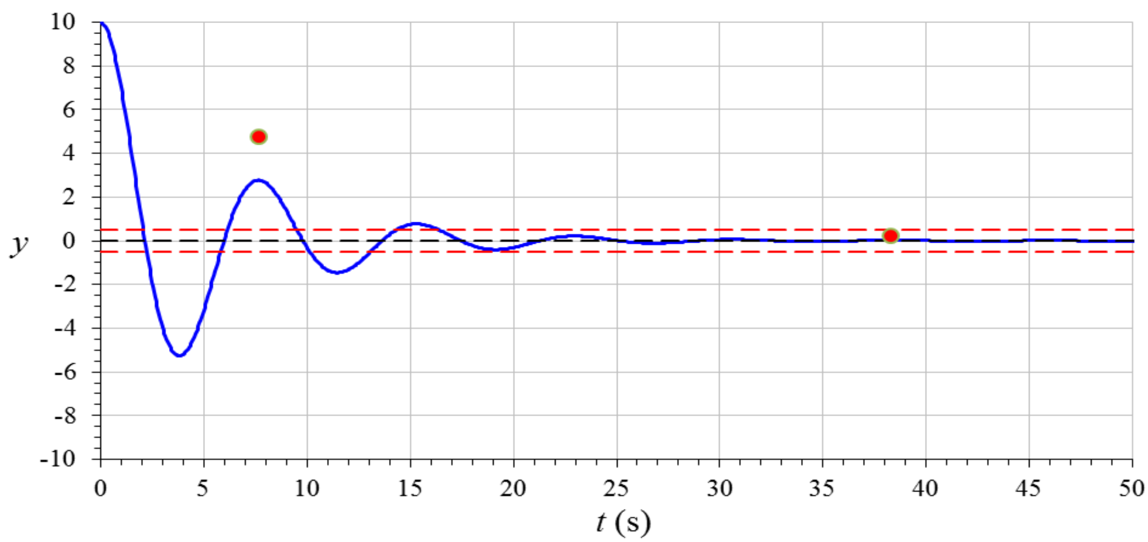


To do: Calculate the damping ratio and the undamped natural frequency.

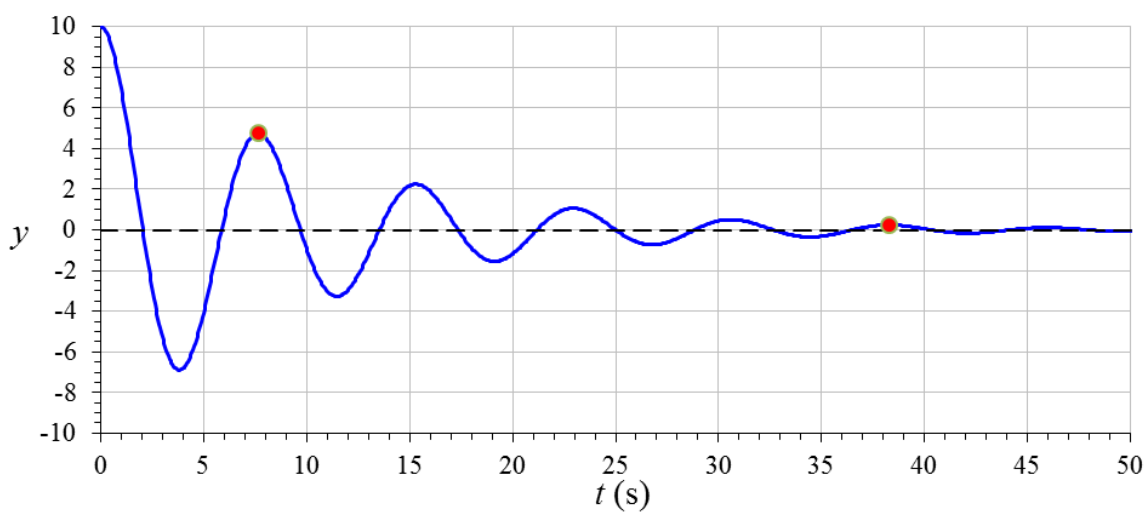
Solution:

Here we do *not* know ζ or ω_n, so this is an implicit solution, involving iteration. To solve graphically, we *guess* ζ, then plot using the equation for underdamped 2nd-order dynamic system response, as given in the learning module. We iterate until we satisfy both of the observed data points. Two cases are shown:

- Initial guess is ζ = 0.200.



- Iterate until converge on $\boldsymbol{\zeta}$ **= 0.118**.



Example: Dynamic system response (second-order, log-decrement method)

Given: Output variable y responds like a first-order dynamic system when exposed to a sudden change of input variable x. Bill measures the following (oscillatory components) at two peaks, 5 peaks apart from each other:

- $y^*_1 = 1.00$ at $t_1 = 0.00$ s.
- $y^*_6 = 0.200$ at $t_6 = 0.250$ s.

To do: Use the log-decrement method to calculate the damping ratio of this system.

Solution: Here are the log-decrement equations for your convenience:

$$\ln\left(\frac{y^*_i}{y^*_{i+n}}\right) = n\delta \quad \zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} \quad \omega_d = \frac{2\pi}{T} = 2\pi f_d \quad \omega_n = 2\pi f_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}.$$