Signal Reconstruction – the Cardinal Series

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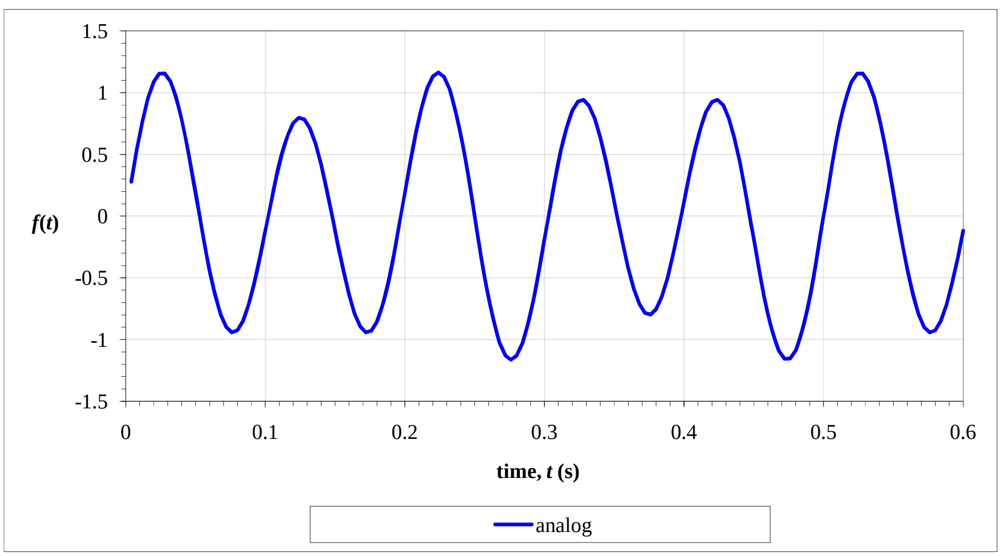
Introduction

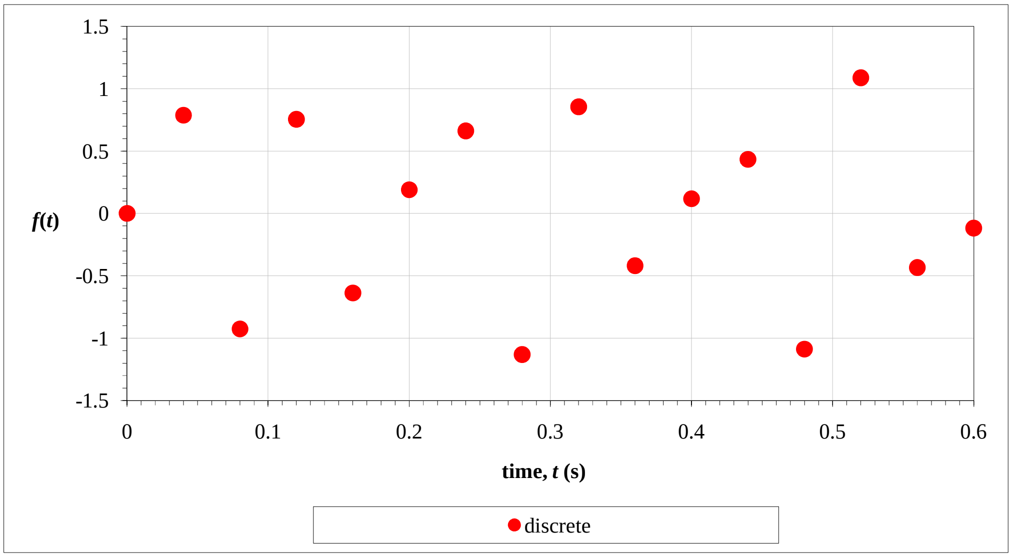
- In a previous learning module, we discuss how to convert an analog signal into a digital signal.
- In this learning module, we discuss how to *reconstruct* a digital signal back into an analog signal.

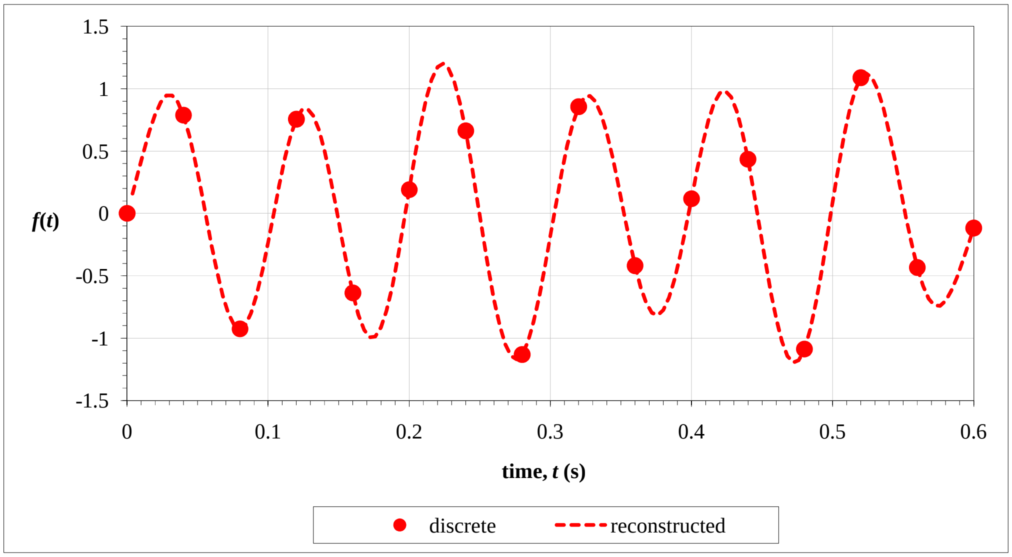
Signal Reconstruction

- As long as the Nyquist criterion is met (sampling frequency f_s is at least twice the maximum signal frequency), we can theoretically reconstruct the original analog waveform from a set of discrete samples.
- One method of reconstruction is the ***cardinal series***. It is the only reconstruction technique we discuss here.
- For N discrete data points, the cardinal series is $\boxed{f(t)=\frac{1}{\pi}\sum_{n=0}^{n=N-1} f_d(n\Delta t)\frac{\sin\left[\pi\left(\frac{t}{\Delta t}-n\right)\right]}{\frac{t}{\Delta t}-n}}$, where t is time (we assume that the signal starts at $t = 0$), n is the data point number (the summation is over all N discrete data points, from $n = 0$ to $n = N - 1$), $f(t)$ is the reconstructed waveform, N is the total number of discrete data points available, $\boxed{\Delta t = 1/f_s}$ is the time period between discrete data points, and $f_d(n\Delta t)$ is the discretely sampled value for the n^{th} discretely sampled data point.
- For an infinite number of discrete data points, the cardinal series reconstructs the waveform *exactly*.
- In practice, however, with only a *finite* number of discrete samples available, the cardinal series still does a very good job of reconstructing the original signal.
- ***Example*** **-** The function $\boxed{f_a(t)=(1.0)\sin(2\pi\cdot 10t)+(0.20)\sin(2\pi\cdot 6t)}$ represents a superposition of two sine waves, the primary one at 10 Hz, and the secondary one at 6 Hz. The 6 Hz wave has an amplitude of 0.20, while the 10 Hz wave has an amplitude of 1.0. The analog function (blue curve) is plotted below:

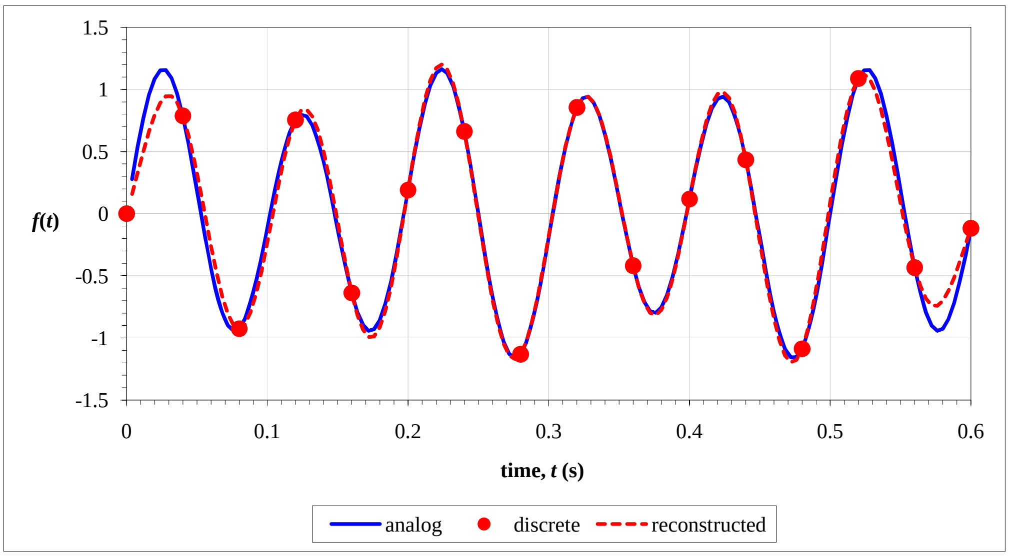


- The Nyquist criterion for this signal is $f_s > 20$ Hz, since 10 Hz is the maximum frequency of the signal.
- We acquire discrete (digital) data at $f_s = 25$ Hz, which exceeds the Nyquist criterion. So, theoretically, we should be able to reconstruct the analog signal nicely from a set of discretely sampled data points.
- Data points are acquired for 0.6 seconds at $f_s = 25$ Hz (a total of $N = 16$ discrete data points). A plot of the discrete data points $f_d(n\Delta t)$ (red circles) is shown below:
- Taken alone, the discrete values do not appear to contain enough information to reconstruct the original signal, but the cardinal series reconstruction is excellent
- A plot of the reconstructed waveform $f(t)$ (dashed red curve) is also shown below; it deviates from the original signal significantly only near the end points of the time span.





- A summary plot of the actual (analog) signal $f_a(t)$ (blue curve), the discrete data points $f_d\left(n\Delta t\right)$ (red circles), and the reconstructed waveform $f(t)$ (dashed red curve) is shown below:



- ***Caution***: The cardinal series does not work properly if the Nyquist criterion is not satisfied in the sampling of the original data.