Today, we will:

- Review: Do some more example problems – significant digits
- Review the pdf module: **Dimensional Analysis** and do some example problems
- Review the pdf module: **Review of Basic Electronics**

Example: Significant digits

Given: Three quantities are measured: $a = 7.55$, $b = 6.044$, and $c = 10.451$.

To do:

(*a*) Calculate $a - b$, giving your answer to the appropriate precision and number of significant digits.

Solution: Align the decimal places:

$$\begin{array}{r} 7.55 \\ -\ 6.04\underline{4} \\ \hline 1.5\underline{0}6 \end{array}$$

The left-most column that has a least significant digit carries into the answer

So, the answer is $\boxed{1.51}$ (to 3 sig. digits)

(*b*) Given the same three quantities: $a = 7.55$, $b = 6.044$, and $c = 10.451$. Calculate $a + b + c$, giving your answer to the appropriate precision and number of significant digits.

Solution: Align the decimal places and underline the least significant digit, as previously

$$\begin{array}{r} 7.5\underline{5} \\ +\ 6.04\underline{4} \\ +\ 10.45\underline{1} \\ \hline 24.0\underline{4}5 \end{array}$$

Round down since 4 is even.

ANSWER: $\boxed{24.04}$

Note: use all digits for any follow-on calculations, such as Part (c)

(*c*) Given the same three quantities: $a = 7.55$, $b = 6.044$, and $c = 10.451$. Calculate the average of a, b, and c to the appropriate precision and number of significant digits.

Solution:

DO NOT USE 24.04 IN THIS CALCULATION

$$\text{Average} = \frac{\text{Sum}}{3} = \frac{24.0\underline{4}5}{3} = 8.01\underline{5}$$

Answer: $\boxed{8.01\underline{5}}$

Since the numerator has 4 sig. digits & the denominator has ∞ sig. digits (it is an integer), The answer is limited to 4 sig. digits

Example: Primary dimensions – shear stress, force per unit length, and power

(***a***) **Given**: In fluid mechanics, shear stress τ is expressed in units of N/m^2. = Force/area

To do: Express the primary dimensions of τ, i.e., write an expression for $\{\tau\}$.

Solution:

$$\{\tau\} = \left\{\frac{\text{Force}}{\text{Area}}\right\} = \left\{\frac{mL/t^2}{L^2}\right\} = \left\{\frac{m}{Lt^2}\right\}$$

Answer: $\boxed{\left\{\frac{m}{Lt^2}\right\} \text{ or } \left\{m^1 L^{-1} t^{-2}\right\}}$

(***b***) **Given**: Ray is conducting an experiment in which quantity a has dimensions of force per unit length.

To do: Express the primary dimensions of a, i.e., write an expression for $\{a\}$.

Solution:

$$\left\{\frac{F}{L}\right\} = \left\{\frac{mL/t^2}{L}\right\} = \left\{\frac{m}{t^2}\right\}$$

Answer: $\boxed{\left\{\frac{m}{t^2}\right\} \text{ or } \left\{m^1 t^{-2}\right\}}$

(***c***) **Given**: Power $\dot{W}$ has the dimensions of energy per unit time.

To do: Write the dimensions of power in terms of primary dimensions.

Solution:

Energy = Force × Distance (same as work)

Power = Energy / time

So... $$\{\dot{W}\} = \left\{\frac{\text{Force}\cdot L}{t}\right\} = \left\{\frac{mL}{t^2}\frac{L}{t}\right\} = \left\{\frac{mL^2}{t^3}\right\}$$

Answer: $\boxed{\{\dot{W}\} = \left\{\frac{mL^2}{t^3}\right\} = \left\{m^1 L^2 t^{-3}\right\}}$

Example: Dimensional analysis – shaft power

Given: The output power $\dot{W}$ of a spinning shaft is a function of torque T and angular velocity ω.

[Torque = force × moment arm = Force × length]

To do: Express the relationship between $\dot{W}$, T, and ω in dimensionless form.

Solution:

Step 1: (List the variables & count) $\dot{W} = \text{fnc}(T, \omega)$ $\boxed{n=3}$

Step 2: (List the dimensions)

(from previous problem) $\left\{\frac{mL^2}{t^3}\right\}$ $\left\{\frac{mL^2}{t^2}\right\}$ $\left\{\frac{1}{t}\right\}$

[Note: ω = radians per second, but radian is a dimensionless quantity, so $\{\omega\} = 1$]

Step 3: (Pick reduction, j) → TRY $j = 3$ = # primary dimensions in the problem (m, L, t)

So $k = n - j = 3 - 3 = 0$ ✗ — can't work, so let $j = j - 1 = 3 - 1 = 2$ $\boxed{j=2}$

Step 4: (Pick j repeating variables)

We have no choice but to pick the two independent variables, T & ω

Note: $k = n - j = 3 - 2 = 1$, So we expect k Π's. Here we expect only one Π

$\boxed{\text{Repeating variables} = T \ \& \ \omega}$

Step 5: (Calculate the Π's)

$$\Pi_1 = \dot{W} \underbrace{T^a \omega^b} = \text{dimensionless}$$

(we force the Π to be dimensionless)

This is the dependent Π_1 since it was the dependent variable

The repeating variables are given exponents which we must calculate

Notice that we force the Π to be dimensionless ($m^0 = 1$, $L^0 = 1$, $t^0 = 1$)

So, $\Pi_1 = \dot{W} T^a \omega^b$

$$\{\Pi_1\} = \left\{\left(\frac{mL^2}{t^3}\right)\left(\frac{mL^2}{t^2}\right)^a\left(\frac{1}{t}\right)^b\right\} = \{m^0 L^0 t^0\}$$

Now equate exponents: mass: $m^1 m^a = m^0 \rightarrow m^{1+a} = m^0 \rightarrow 1+a=0$

Solve: $\boxed{a=-1}$

Length: $L^2 L^{2a} = L^0 \rightarrow L^{2+2a} = L^0 \rightarrow 2+2a=0$

Solve: $\boxed{a=-1}$

[fortunately, $a=-1$ for both calculations. If they disagreed, we would suspect an error either in the problem setup or in our algebra.]

Time: $t^{-3} t^{-2a} t^{-b} = t^0 \rightarrow t^{-3-2a-b} = t^0 \rightarrow -3-2a-b=0$

Plug in $a=-1$ & solve for $b \rightarrow \boxed{b=-1}$

~~Step 6:~~ So, $\Pi_1 = \dot{W} T^a \omega^b = \dot{W} T^{-1} \omega^{-1} \rightarrow \boxed{\Pi_1 = \dfrac{\dot{W}}{\omega T}}$

Step 6: (write functional form) $\Pi_1 = \text{fnc}(\Pi_2, \Pi_3, \ldots)$

Here, however, there is only one Pi! So, $\Pi_1 = \text{fnc}(\text{nothing})$

$\therefore \Pi_1 = \text{constant}$

So, ... $\Pi_1 = \dfrac{\dot{W}}{\omega T} = \text{constant} \rightarrow \boxed{\dot{W} = \text{const} \cdot \omega T}$

This is our final answer

Note: We stop here. We cannot determine the constant unless we do an experiment or some physics

BUT, we get the correct equation for $\dot{W}$ to within a constant without having to know any physics!

[From physics, recall that $\dot{W} = \omega T$, so the constant turns out to equal 1]

Never underestimate the power of dimensional analysis!