

**Today, we will:** *Reminder: Always text Cimbala before class (37607) or log into PE*

- Do some example problems – basic electronics and dimensional analysis
- Review the pdf module: **Errors and Calibration** and do some example problems
- Review the pdf module: **Basic Statistics** and do some example problems

### Example: Basic electronics

**Given:** The circuit shown.

**To do:**

(a) Calculate the equivalent resistance of this circuit.

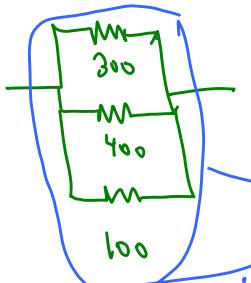
(b) Calculate the total current through this circuit.

(c) Calculate the power used by this circuit.

**Solution:**

(a) • First do the top 2 in series  
 $R_{\text{top}} = 100 + 200 = 300 \Omega$

• Now have



= 3 resistors in parallel

$$R_{\text{eq}} = \frac{1}{\frac{1}{300} + \frac{1}{400} + \frac{1}{600}} = 133.33\ldots \Omega$$

So,

$$R_{\text{equivalent}} = 133.33\ldots \Omega$$

The equivalent circuit is



(b) Ohm's Law:  $\Delta V = IR \rightarrow I = \frac{\Delta V}{R_{\text{eq}}}$  here, so

[Note: 3 sig. digits in answer since  $10.0 = 3$  sig. fig.]

$$I = \frac{\Delta V}{R_{\text{eq}}} = \frac{(10.0 - 0.0) \text{ V}}{133.33\ldots \Omega} \left( \frac{\text{A} \cdot \Omega}{\text{V}} \right) = 0.0750 \text{ A}$$

$$\text{or } I = 75.0 \text{ mA}$$

(Ohm's law)

[Unity conversion factors]

$$(c) \dot{W} = (\Delta V) I = (IR) I = I^2 R$$

$$\dot{W} = \frac{(10.0 \text{ V})^2}{133.33\ldots \Omega} \left( \frac{\text{W}}{\text{V} \cdot \text{A}} \right) \left( \frac{\text{A} \cdot \Omega}{\text{V}} \right)$$

$$\text{or, } \dot{W} = \Delta V (I) = \Delta V \left( \frac{\Delta V}{R} \right) = \frac{\Delta V^2}{R}$$

$$\dot{W} = 0.0750 \text{ W}$$

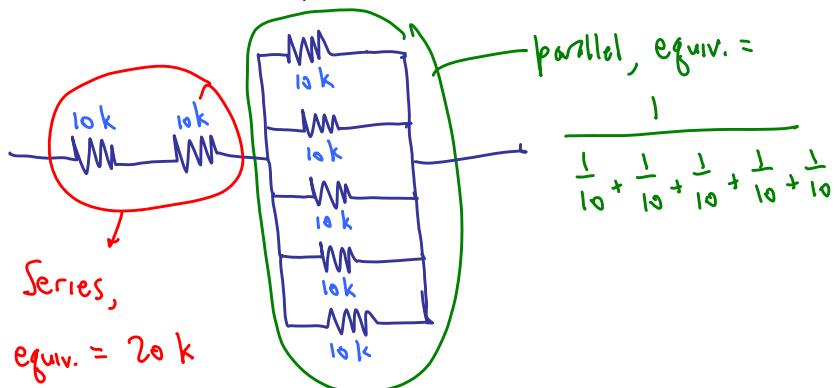
### Example: Basic electronics

**Given:** A bunch of  $10\text{ k}\Omega$  resistors is available in the lab. We have need for a resistance of  $22\text{ k}\Omega$  for a circuit we are building on a breadboard.

**To do:** Figure out how to create the required resistance from the available resistors.

**Solution:**

We use trial & error, playing with series & parallel combinations.  
Here is one option (there may be more):



### Example: Basic electronics

**Given:** A voltage signal has a DC offset. We want to remove the DC offset so that the frequency content of the AC (fluctuating) component can be analyzed.

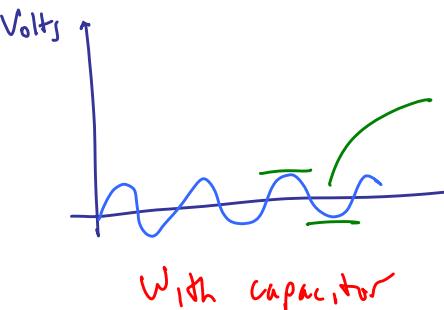
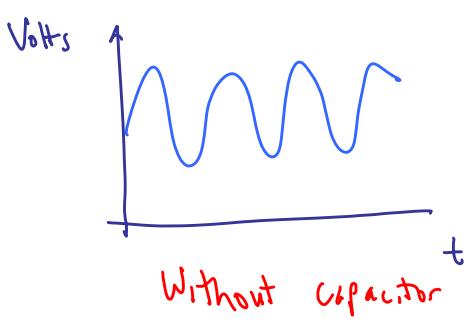
**To do:** Figure out how to remove the DC offset with a *single* electronic component (i.e., one resistor *only*, or one capacitor *only*, or one inductor *only*, etc.).

**Solution:**

See the Electronics Review notes on the course website.

A capacitor impedes DC signals. Basically, a DC voltage will build up a charge on the capacitor, but current does not flow through.

AC (fluctuating) voltages pass through, but are impeded slightly.



Notice - the amplitude also decreases, but we removed the DC component!



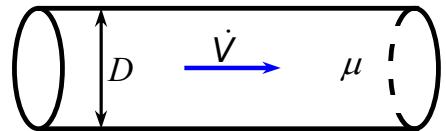
Answer:

Capacitor

This is a crude high-pass filter!

### Example: Dimensional analysis – pipe flow

**Given:** Consider fully developed laminar flow through a very long round tube. Volume flow rate  $\dot{V}$  is a function of the tube's inner diameter  $D$ , fluid viscosity  $\mu$ , and axial pressure gradient  $dP/dx$ .  
 (we treat  $dP/dx$  as one variable here)



$$\text{Pressure gradient} = dP/dx$$

**To do:** If  $D$  is doubled, holding  $\mu$  and  $dP/dx$  fixed, by what factor does  $\dot{V}$  change?

**Solution:** Use dimensional analysis to generate a nondimensional functional relationship.

Step 1:  $\dot{V} = f_{nc}(D, \mu, \frac{dP}{dx})$

Step 2:  $\left\{\frac{\dot{V}}{L^3}\right\} \quad \left\{L\right\} \quad \left\{\frac{m}{L \cdot t}\right\} \quad \left\{\frac{m}{L^2 \cdot t^2}\right\}$

$\left\{\frac{F/A}{L}\right\} = \left\{\frac{m \cdot L}{t^2 \cdot L^2 \cdot t^2}\right\}$

$n=4$

Step 3: Pick  $j=3$  since  $m, L, t$  appear in dimensions. Expect  $k=n-j=4-3=1$   $\Pi$ 's

Step 4: Pick 3 repeating variables since  $j=3 \rightarrow$  only choice is  $D, \mu, \frac{dP}{dx}$ .

Step 5:  $\Pi_1 = \dot{V} D^a \mu^b \left(\frac{dP}{dx}\right)^c$  → Force this  $\Pi$  to be dimensionless by solving for the exponents as follows:

Dimensionless  $\downarrow$

$\left\{m^0 L^0 t^0\right\} = \left\{\frac{L^3}{t}\right\} \left\{L\right\} \left\{\frac{m}{L \cdot t}\right\}^b \left\{\frac{m}{L^2 \cdot t^2}\right\}^c$

Algebra:

$$\begin{aligned} m: \quad 0 &= b + c \\ L: \quad 0 &= 3 + a - b - 2c \\ t: \quad 0 &= -1 - b - 2c \end{aligned}$$

Solve simultaneously any way you can.

Result:  $a=-4, b=1, c=-1$

So,  $\Pi_1 = \dot{V} D^{-4} \mu^1 \left(\frac{dP}{dx}\right)^{-1}$

$$\Pi_1 = \frac{\dot{V} \mu}{D^4 \left(\frac{dP}{dx}\right)}$$

Note: The constant cannot be found by dimensional analysis

Step 6:  $\Pi_1 = f_{nc}(\text{nothing}) \therefore \Pi_1 = \text{constant}$

Since there is only one  $\Pi$  in this problem

$$\dot{V} = \text{constant} \frac{D^4 \left(\frac{dP}{dx}\right)}{\mu}$$

ANSWER → 16

**Answer:** If  $D$  is doubled, holding  $\mu$  and  $dP/dx$  fixed,  $\dot{V}$  goes up by a factor of 16.

If  $D$  doubled,  $\dot{V} \uparrow$  by factor of  $2^4 = 16$

We got this without knowing any fluid mechanics!

## Example: Errors and calibration

Given:

- The actual (true) voltage is 4.6020 V.
- 256 voltage readings are taken, and the average voltage reading is 4.6015 V.

To do: (a) Calculate the systematic (bias) error and the mean bias error for this set of measurements. (b) Calculate the random (precision) error of a reading that is 4.6010 V.

Solution:

$$(a) \text{ Bias error} = \text{Systematic error} = \text{Avg. - true value} = 4.6015 - 4.6020 \checkmark = -0.0005 \text{ V}$$

$$\text{MBE} = \text{Bias error/true value} = -0.0005 / 4.6020 = -0.0001086 \approx -0.011\%$$

$$(b) \text{ Random error of this one reading} = \text{reading - average of all readings} = 4.6010 - 4.6015 = -0.0005 \text{ V}$$

## Example: Errors and calibration

Given:

- The actual (true) temperature is 22.100°C.
- Six thermometer readings are taken: 22.15, 22.22, 22.09, 22.21, 22.18, and 22.24°C.

To do: Calculate the mean, systematic error, and mean bias error for this set of data, and calculate the accuracy error (inaccuracy) and the precision error for each measurement.

$T_i$ (°C)	Inaccuracy = $T_i - T_{\text{true}}$ (°C)	Precision error = $T_i - \bar{T}$ (°C)
22.15	= 22.15 - 22.100 = 0.05	= 22.15 - 22.182 = -0.0317 ≈ -0.03
22.22	0.12	0.04
22.09	-0.01	[THIS DECIMAL PLACE IS THE PLACE FOR THE LEAST SIGNIFICANT DIGIT IN THE ANSWERS] -0.09
22.21	0.11	0.03
22.18	0.08	-0.00
22.24	0.14	-0.06

Solution: Average of all readings = Sample Mean = 22.181666 = 22.182 °C

$$\cdot \text{ Systematic error} = \text{Avg. - true value} = \frac{22.181667}{-22.100} = 0.08167$$

Keeping significant digits in mind,

$$\boxed{\text{Systematic error} = 0.082 \text{ °C}}$$

• Precision error → See table. Precision error = reading - avg. of all readings

$$\cdot \text{ MBE} = \frac{\text{Systematic error}}{\text{true value}} = \frac{0.08167}{22.110} = 0.003695$$

$$\boxed{\text{MBE} = 0.37\% \quad (\text{to 2 sig. digits} = \text{best we can do})}$$

[See also Excel spreadsheet on the website for this same problem]

★ ↗ STUDY THE EXCEL AND/OR MATLAB FILES