Today, we will:

- Do some review example problems – The Gaussian PDF
- Review the pdf module: **The Central Limit Theorem (CLT)** and do some examples
- Review the first half of the pdf module: **Other PDFs – the Student's *t* PDF**

Example: Review and probability

Given: The temperature of an ice bath is measured numerous times with a digital thermometer. The *true* temperature of the ice bath is 0.0000°C.

- The sample mean temperature is $\bar{T} = -0.0125$°C
- The sample standard deviation of all the readings is 0.0341°C
- We assume that the precision errors in the readings are purely random

(*a*) To do: Write T in standard engineering format, T = -0.0125 ± $\boxed{}$ °C .

(*b*) To do: Calculate the bias error (also called systematic error).

(*c*) To do: Calculate the probability that any random reading is greater than 0°C.

Solution:

(a) $T = \bar{T} \pm 2\sigma$ "two sigma" = standard engineering confidence level $\approx 95\%$

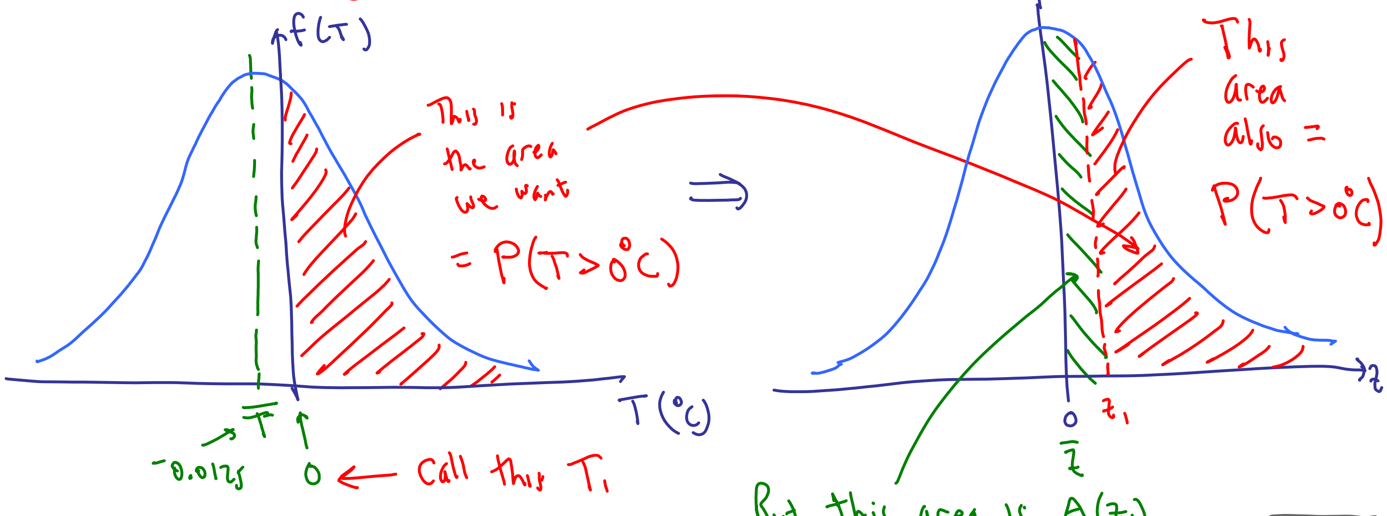
So, $T \approx \bar{T} \pm 2S = -0.0125 \pm 2(0.0341) = \boxed{-0.0125 \pm 0.0682\ ^\circ C}$

(b) Bias error $= \underbrace{\text{avg of all readings}}_{\text{sample mean}} - \underbrace{\text{true value}}_{\text{given}}$

$= -0.0125 - 0.0000\ ^\circ C = \boxed{-0.0125\ ^\circ C}$

(c) Since errors are random, we approximate the distribution as Gaussian.

Sketches are extremely helpful!



$z_1 = \frac{T_1 - \bar{T}}{S} = \frac{0-(-0.0125)}{0.0341} = 0.3666 \rightarrow$ Table, $A(z_1) = 0.1430$, so $P(T > 0^\circ C) = \frac{1}{2} - A(z) = \boxed{35.7\%}$

Example: Probability – power requirement measurements

Given: Bev takes 200 measurements of the power requirement for an electronic instrument running in a steady-state mode. We assume that the precision errors are purely random. The sample mean is 35.92 W, and the sample standard deviation is 0.60$_0$ W.

To do:

(***a***) Considering the proper number of significant digits, show how Bev should write the power in standard engineering format (95% confidence level), i.e.,

$P = 35.92 \pm$ $\boxed{}$ W

(***b***) Calculate the percentage of the readings that are expected to be less than 35.92 W.

(***c***) Calculate the percentage of the readings that are expected to be greater than 37.12 W.

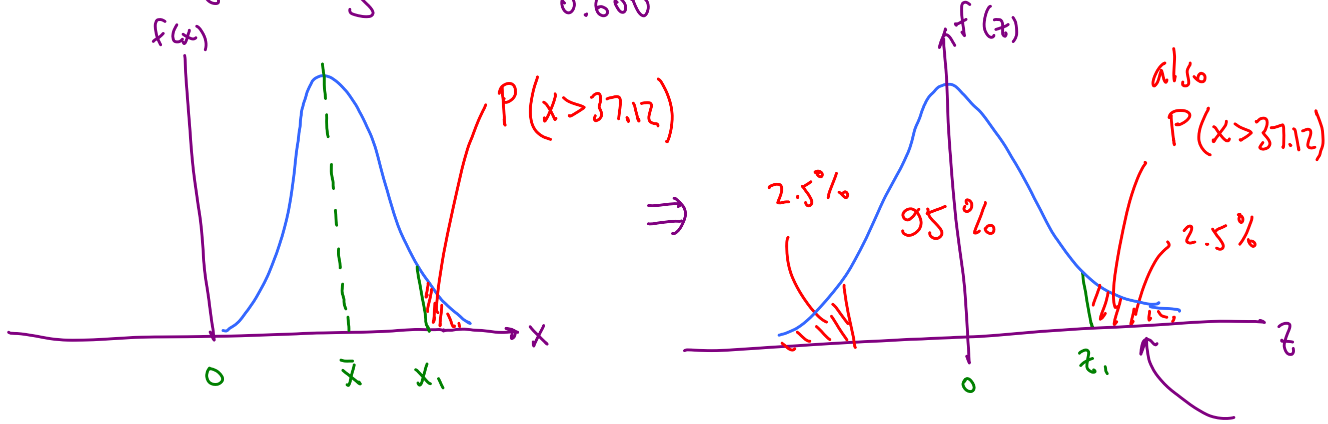
(***d***) Estimate the number of readings that are expected to be greater than 37.12 W.

Solution: (a) Use $2\sigma \approx 2S$ = standard 95% engr. confidence level

$$P = 35.92 \pm 2(0.600\text{ W}) = \boxed{35.92 \pm 1.20 \text{ W}}$$

(b) [trick question] → Since mean is 35.92, and the median of a Gaussian pdf is equal to the mean, half of the readings are smaller than 35.92 & half are larger. Answer $\boxed{50\%}$

(c) $$z_1 = \frac{x_1 - \mu}{\sigma} \approx \frac{x_1 - \bar{x}}{S} = \frac{37.12 - 35.92}{0.600} = 2.00$$



Note: Since $z_1 = 2.00$ or "2 sigma", we know that the tail area is $\approx 2.5\%$

So, $\boxed{\text{the approximate answer is } 2.5\%}$

OR, more precisely, @ $z_1 = 2.00$, $A(z_1) = 0.47725$, so $P(x>37.12) = \frac{1}{2} - A(z_1) = 0.02275$

So, $\boxed{\text{the more precise answer is } 2.28\%}$

(d) Number $= 0.02275\,(200 \text{ readings}) = 4.550$ readings $\approx \boxed{5 \text{ readings}}$

OR, approximately $0.025(200) = 5.00 \approx \boxed{5 \text{ readings}}$

Example: Estimating population standard deviation

Given: A company produces resistors by the thousands, and Mark is in charge of quality control.

- He picks 20 resistors at random as sample 1, and calculates the mean, $\bar{x}_1$.
- He picks 20 *other* resistors as sample 2, and calculates the mean, $\bar{x}_2$. …
- Mark continues to do this until sample 25, and calculates the mean, $\bar{x}_{25}$.

The *average* of all the means is $\overline{(\bar{x}_1)} = (\bar{x}_1 + \bar{x}_2 + ... + \bar{x}_{25})/25 = 8.235\ \text{k}\Omega$.

The *standard deviation* of all the means is $\sigma_{\bar{x}} \approx S_{\bar{x}} = 0.282\ \text{k}\Omega$.

CAREFUL: Do not confuse n & N

To do: Estimate the **population standard deviation, σ**.

Solution:

- Use Central Limit Theorem (CLT) with $n = 20$, $N = 25$

- From CLT, $S_{\bar{x}} = \dfrac{S_{overall}}{\sqrt{n}} \approx \dfrac{\sigma}{\sqrt{n}} \rightarrow \sigma \approx S_{\bar{x}}\sqrt{n}$

$= (0.282\ \text{k}\Omega)\sqrt{20}$

[Notice that σ is much larger than $S_{\bar{x}}$] $\rightarrow$ or $\boxed{\sigma \approx 1.26\ \text{k}\Omega}$

Example: Estimating population standard deviation

Given: Ron takes 50 pressure measurements, and repeats this 19 more times, for a total of 20 samples of 50 data points each. He calculates the sample mean for each set (sample) of 50 measurements. The standard deviation of the 20 sample means is 0.150 kPa.

To do: Estimate the population standard deviation of all the measurements (in units of kPa to 3 significant digits).

Solution:

Use CLT with $n = 50$, $N = 20$

[recall, n = # data pts per sample
N = # samples]

$\sigma_{\bar{x}} \approx S_{\bar{x}} = 0.150\ \text{kPa}$

CLT says: $\sigma_{\bar{x}} \approx \dfrac{\sigma}{\sqrt{n}} \rightarrow \sigma \approx \sigma_{\bar{x}}\sqrt{n} \approx S_{\bar{x}}\sqrt{n}$

$= (0.150\ \text{kPa})\sqrt{50} = 1.06\ \text{kPa}$

So, we predict $\boxed{\sigma \approx 1.06\ \text{kPa}}$

Example: Estimating the population mean

Given: A company produces resistors by the thousands, and Gerry is in charge of quality control. He picks 20 resistors at random as a sample, and calculates the sample mean $\bar{x} = 8.240\ \text{k}\Omega$ and sample standard deviation $S = 0.314\ \text{k}\Omega$.

To do:

(*a*) Estimate the **population mean** and the **confidence interval of the population mean** (as a ± value) to standard 95% confidence level.

(*b*) Repeat for 99% confidence level. *Do you expect the confidence interval to be wider or narrower?*

Solution:

(a) Use t-PDF & $\boxed{\mu = \bar{x} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}}$ (1)

[$t_{\alpha/2}$ is the critical value – found in the table]

• From the student's t PDF table of critical values, @ df = n-1 = 20-1 = 19, & at 95% confidence level, look up $\boxed{t_{\alpha/2} = 2.0930}$

[or, in Excel, use = TINV(0.05, 19) = 2.0930 ✓

α = 0.05 for C = 0.95   df

• So, Eq (1) becomes

$$\mu = 8.240 \pm 2.0930 \frac{0.314}{\sqrt{20}}$$

or

$$\boxed{\mu = 8.240 \pm 0.147\ \text{k}\Omega}$$

(b) To 99% confidence level, we expect confidence interval to be <u>wider</u>

[to be <u>more</u> confident, we need to have a wider range of values.]

E.g. in limit, to be 100% confident, I would need a range of $-\infty$ to ∞ !!

• Re-do numbers @ C = 0.99 (99% confidence) → α = 1 - 0.99 = 0.01, & df = 19

Table or EXCEL → $\underline{t_{\alpha/2} = 2.8609}$

$$\therefore \mu = \bar{x} \pm t_{\alpha/2} \frac{S}{\sqrt{n}} = 8.240 \pm 2.8609 \frac{0.314}{\sqrt{20}} = \boxed{8.240 \pm 0.201\ \text{k}\Omega}$$

(yes, the confidence interval is indeed <u>wider</u>!)