Today, we will:

- Do some review example problems – predicting the mean using the *t* PDF
- Finish reviewing the pdf module: **Other PDFs – the Chi-squared PDF**
- Do an example problem – chi-squared PDF

Example: Estimating the population mean

Given: Five measurements are made of the force required to break an expensive component (too costly to do more):

312.2, 320.6, 315.5, 314.1, and 319.6 lbf

The sample mean and sample standard deviation of these data are 316.4 and 3.592 lbf.

(*a*) To do: Estimate the population mean and its confidence interval for 95% confidence level. **[See also Excel and Matlab solutions for this problem on the course website.]**

Solution: Use student's t PDF for these kinds of problems ☆

- df = degrees of freedom = n-1 = 5-1 = 4 → df = 4
- @ 95% confidence level, c = 0.95 → $\alpha = 1 - c = 1 - 0.95 = 0.05$ → $\alpha = 0.05$
- TABLE OF CRITICAL t VALUES → @ 95% confidence & df = 4, read $t_{\alpha/2} = 2.7765$ = critical t-value

OR → in EXCEL, use =TINV(0.05, 4) → get $t_{\alpha/2} = 2.7765$ ✓
(α = 0.05, df = 4)

- So, $\boxed{\mu = \bar{x} \pm t_{\alpha/2}\frac{S}{\sqrt{n}}} = 316.4 \pm (2.7765)\frac{3.592}{\sqrt{5}}$

CAUTION: Use n here, not df

Note: Do not confuse this with $x = \bar{x} \pm 2S$, which is different

$\boxed{\mu = 316.4 \pm 4.46 \text{ lbf}}$

This is NOT 2S

To 95% confidence level

- OR, in terms of range, $(316.4 - 4.46) < \mu \le (316.4 + 4.46)$, i.e.,

[We are 95% confident that μ lies in this range] → $\boxed{311.9 < \mu \le 320.9}$

NOTICE: μ_{min} < smallest data point & μ_{max} > largest data point! → Why? n is so small, need wide conf. interval

(*b*) To do: If we were to repeat for 98% confidence level, would the confidence interval be narrower, the same, or wider than the confidence interval for 95% confidence level?

Solution: Answer = $\boxed{\text{Wider}}$ Why? To be more confident, we need to have an even wider confidence interval!

[e.g. To be 100% confident of μ, we would need $-\infty < \mu < \infty$!! (infinitely wide)]

(c) To do: Estimate how many measurements would need to be taken to reduce the confidence interval to ±2.0 lbf [while maintaining a 95% confidence level].

Solution:

• Use same eq. → $\mu = \bar{x} \pm t_{\alpha/2}\frac{S}{\sqrt{n}}$ → Now let's force this to equal the desired 2.0 lbf

• Solve for n → $t_{\alpha/2}\frac{S}{\sqrt{n}} = 2.0$ → $\boxed{n = \left(\frac{S \cdot t_{\alpha/2}}{2.0}\right)^2}$ Eq. (1)

• But what is $t_{\alpha/2}$? We don't know because as n changes, $t_{\alpha/2}$ also changes (as seen in the critical t table).

• We have 2 unknowns (n & $t_{\alpha/2}$) & 1 eq. (Eq. 1).

WHAT TO DO? Answer → get second "equation" from the table

We need to iterate. Here is a good procedure that converges rapidly:

	Guess n	df = n−1	TABLE → $t_{\alpha/2}$ @ this n & 95% confidence	Eq. (1) → calculate new n
First guess →	10	9	2.2622	16.507
round to integer	17	16	2.1199	14.49
	14	13	2.1604	15.06
	15	14	2.1448	14.84 → 15

CONVERGED TO n = 15

ANSWER: We require approx. 15 data points in order to predict μ to ± 2.0 lbf and 95% confidence level ☆

☆ Note: This is still only an approximation because we use the original S. As n changes, S will also change. But this is the best we can do.

Example: Estimating population mean and standard deviation

Given: A quality control engineer pulls 10 resistors at random from an assembly line that makes 10-kΩ resistors, and measures each resistance. The measurements are given here:

Measurement number	Resistance, kΩ
1	10.10
2	10.08
3	10.11
4	10.09
5	10.07
6	10.05
7	10.12
8	10.11
9	10.08
10	10.06

sample mean = 10.087 kΩ
sample standard deviation = 0.02312 kΩ

(*a*) To do: Estimate the population mean and its confidence interval for 95% confidence level.

(*b*) To do: Estimate the population standard deviation and its confidence interval for 95% confidence level.

(*c*) To do: The company guidelines specify that the population standard deviation be less than 0.35% of the mean to 98% confidence. Estimate how many *additional* resistors the quality control engineer needs to pull off the assembly line to measure.

Solution:

[THIS PART (a) IS REVIEW]

(a) $\bar{x} = 10.087$

$S = 0.02312$

$n = 10 \rightarrow df = n-1 = 9 \rightarrow \underline{df = 9}$

$\boxed{\text{Use } t \text{ pdf}}$

@ 95% confidence level, $C = 0.95$, $\alpha = 1 - C = 0.05$, $\alpha/2 = 0.025$

$t_{\alpha/2} = 2.2622$ (from TABLE, critical t values @ df=9, 95%)

$$\mu = \bar{x} \pm t_{\alpha/2} \frac{S}{\sqrt{n}} \rightarrow \boxed{\mu = 10.087 \pm 0.017 \text{ k}\Omega}$$

(b) $\boxed{\text{Use } \chi^2 \text{ PDF}}$ @ $df = 9$, $\alpha/2 = 0.025$

$1 - \alpha/2 = 0.975$

$\chi^2_{\alpha/2} = 19.0228$ $\chi^2_{1-\alpha/2} = 2.7004$

- Use t PDF for estimating μ range.
- Use χ^2 PDF for estimating σ range.

• Lower estimate for σ : $\sigma_{low} = \sqrt{\dfrac{(n-1)\,S^2}{\chi^2_{\alpha/2}}} = \underline{0.0159}$

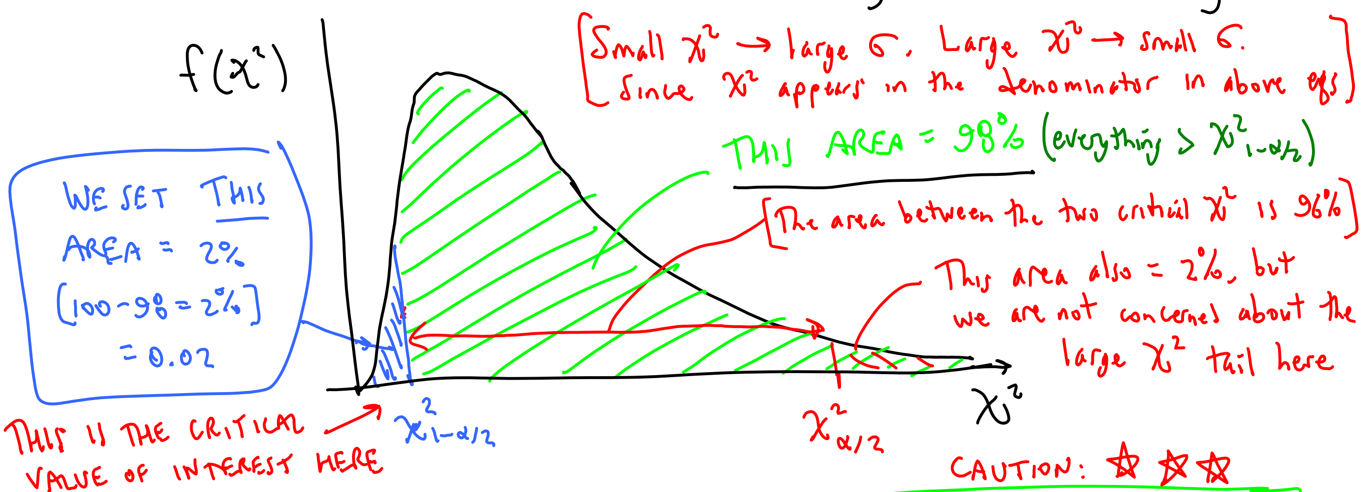
• Higher estimate for σ : $\sigma_{high} = \sqrt{\dfrac{(n-1)\,S^2}{\chi^2_{1-\alpha/2}}} = \underline{0.0422}$

Ans: $\boxed{0.0159 < \sigma < 0.0422 \ \ k\Omega}$

(c) Estimate how many additional data points for a given maximum σ.

• 0.35% of the mean $= \dfrac{0.35}{100}(10.087\,k\Omega) = \underline{0.035304\ k\Omega.}$

• For 98% confidence for one tail only [the tail on the left is the only one that concerns us]



• I.e., $\alpha/2 = 1 - 0.98 = 0.02 \rightarrow \underline{\underline{\alpha = 0.04}}$
$1 - \alpha/2 = 1 - 0.02 = 0.98$

CAUTION: ☆☆☆

We are concerned with only one side, so be careful of factor of 2

Note: Small χ^2 leads to large σ & large χ^2 leads to small σ since χ^2 appears in the denominator of our eq. for the range of σ

$$\left[(n-1)\frac{S^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le (n-1)\frac{S^2}{\chi^2_{1-\alpha/2}}\right]$$

WE ARE CONCERNED ONLY WITH THE UPPER LIMIT OF σ

[Don't care about small σ]

So, we set $\boxed{\sigma_{high} = 0.035304 = \sqrt{\frac{(n-1)\,S^2}{\chi^2_{1-\alpha/2}}}}$ (1)

Specified upper limit

To solve – need to iterate

Two Eq's — Eq (1) & Table or CHIINV macro
Two unknowns — n (or $df = n-1$) & $\chi^2_{1-\alpha/2}$

Iteration: Try $n = 15$, $df = 14 \rightarrow \chi^2_{1-\alpha/2} = 5.368$ (either interpolate from the table @ $1-\alpha/2 = 0.98$ OR use Excel $\boxed{\text{CHIINV}(0.98, 14)}$)

Eq (1) $\rightarrow$ $\sigma_{high} = \sqrt{\frac{(14)(0.02312)^2}{5.368}} = 0.0373$

Too HIGH (we want it to be < 0.0353)

Try $n = 25 \rightarrow df = 24 \rightarrow \chi^2_{1-\alpha/2} = 11.9918$

$\sigma_{high} = \sqrt{\frac{(24)(0.02312)^2}{11.9918}} = 0.0327$

Too LOW (we only need < 0.0353)

After a few more iterations, we get $n = 19$

So, we need to take $19 - 10 =$ $\boxed{\text{9 more data points}}$

FINAL ANSWER

NOTE: THIS IS STILL JUST AN ESTIMATE SINCE S & $\bar{x}$ will change as we take more measurements, but this is the best we can do with the data we have.