Today, we will:

- Do a couple review example problems – outliers, data pairs
- Review the pdf module: **Experimental Uncertainty Analysis**
- Do some example problems – experimental uncertainty analysis

Example: Outliers – data pairs

Given: Twenty data points of (x,y) pairs are measured.

Data point	x	y
1	0	3.7
2	0.1	4.2
3	0.2	5.1
4	0.3	6.6
5	0.4	7.4
6	0.5	8.9
7	0.6	10.4
8	0.7	10.9
9	0.8	11.9
10	0.9	11.5
11	1	12.2
12	1.1	14.7
13	1.2	15.3
14	1.3	16.8
15	1.4	17.2
16	1.5	5.6
17	1.6	19.5
18	1.7	4.5
19	1.8	21.3
20	1.9	22.5

★ Study this file, posted on course website

To do: Eliminate any "official" outliers, one at a time.

Solution:

See Excel spreadsheet on the website called Example_Outliers_data_pairs.xls – I will show in class how to do the analysis in Excel.

- This problem turns out to have several outliers, but we must remove them one at a time!
- Note: The method to remove outliers assumes that all deviations are purely random. We would not use this method for data that have real (repeatable) deviations. ★

Example: Outliers – data pair measurements

Given: Omar takes 12 data pair measurements – temperature as a function of pressure. He performs a regression analysis and plots the data and the least-squares fit. One of the data pairs looks a little suspect, so he performs the standard statistical technique to determine if this data pair is an official outlier:

- The standard error (from the regression analysis) is 0.0244°C. $= S_{yx}$
- The residual of the suspect data pair is 0.0468°C, and is the maximum residual.
- A plot of standardized residual vs. pressure reveals that the standardized residual of the suspect data pair is ***inconsistent*** with its neighbors.

To do: Is this data pair an official outlier or not?

Solution:

- Recall, residual = deviation or error between the fitted value of y (we call it Y) and the actual value of y (we call it y)

$$\boxed{e_i = Y_i - y_i}$$ for each data pair (x_i, y_i)

residual, fitted value, actual data point

- $$\boxed{\text{Standardized residual} = \frac{e_i}{S_{yx}}}$$ [nondimensional — here $\frac{°C}{°C}$]

- So, $\frac{e_i}{S_{yx}} = \frac{0.0468\ °C}{0.0244\ °C} = \underline{\underline{1.918}}$

- Is this data point an official outlier?

Recall → must satisfy 2 criteria to be an outlier:

- $|e_i / S_{yx}| > 2$ ← No
- e_i / S_{yx} is inconsistent with its neighbors ← YES

- Since we do not meet both criteria, this is not an outlier

$$\boxed{\text{Answer: NOT AN OUTLIER – CANNOT REMOVE}}$$

Example: Experimental uncertainty analysis

Given: The cutoff frequency of a simple first-order filter is $\omega = \frac{1}{RC}$, where

- ω = radian frequency (radians/s)
- R = resistance, measured to be $R = 12\underline{0}0\ \Omega \pm 5\%$ (to 95% confidence)
- C = capacitance, measured to be $C = 0.100\ \mu\text{F} \pm 1\%$ (to 95% confidence)

To do: Predict ω to 95% confidence in standard engineering format, and with the proper number of significant digits.

Solution:

• First calculate the predicted mean value of ω, $\bar{\omega} = \frac{1}{\bar{R}\bar{C}}$

$$\bar{\omega} = \frac{1}{(1200\ \Omega)(0.100 \times 10^{-6}\ \text{F})}\underbrace{\left(\frac{\text{F}\cdot\Omega}{\text{s}}\right)}_{\text{(unity conversion factor)}} = \underline{\underline{8333.33\ \frac{\text{rad}}{\text{s}} = \bar{\omega}}}$$

• Component uncertainties: $u_R = \pm 5\% (1200\ \Omega) = \pm 60\ \Omega$

$u_C = \pm 1\% (0.100\ \mu\text{F}) = \pm 0.001\ \mu\text{F}$

THIS STEP IS UNNECESSARY IN THIS PROBLEM, AS WE SHALL SEE

• Calculate the predicted uncertainty

Does this eq. qualify as the simple exponent type? YES, $\omega = R^{-1}C^{-1}$

So,
$$\frac{u_\omega}{\omega} = \left[\left(a_R \frac{u_R}{R}\right)^2 + \left(a_C \frac{u_C}{C}\right)^2\right]^{1/2} \leftarrow \text{RSS uncertainty}$$

exp. = −1 $\quad \frac{u_R}{R} = 5\% \quad$ exp = −1 $\quad \frac{u_C}{C} = 1\%$

$$\frac{u_\omega}{\omega} = \left\{[(-1)(0.05)]^2 + [(-1)(0.01)]^2\right\}^{1/2} = 0.05099$$

• Finally, $u_\omega = \frac{u_\omega}{\omega}\omega = (0.05099)\left(8333.33\ \frac{\text{rad}}{\text{s}}\right) = 424.918\ \frac{\text{rad}}{\text{s}}$

Final answer: $\boxed{\omega = 83\underline{3}0 \pm 42\underline{5}\ \text{rad/s}}$ to 95% confidence (to 3 sig. digits)

Example: Experimental uncertainty analysis

Example

Given: Quantities A and B are measured, 200 times each:

- The sample mean and sample standard deviation for A are $\bar{A} = 5.20$ and $S_A = 0.252$.
- The sample mean and sample standard deviation for B are $\bar{B} = 22.32$ and $S_B = 1.05$.
- Quantity C is *not* measured directly, but it is *calculated* as $\boxed{C = A^2 + 3B}$.

To do: Report C in standard engineering format.

Solution:

$2S_A = 0.504$

$2S_B = 2.10$

• First, calculate $\bar{C} = \bar{A}^2 + 3\bar{B} = \underline{94.0 = \bar{C}}$

• Now write A & B in standard engineering format (to 95% confidence level)

[Recall, 95% confidence $\approx \pm 2\sigma \approx \pm 2S$]

$$\text{So, } A = \bar{A} \pm u_A = 5.20 \pm 0.504$$

$$B = \bar{B} \pm u_B = 22.32 \pm 2.10$$

These are the component uncertainties, and we use them to construct the uncertainty of C, u_C

• $\underline{\text{METHOD A}}$: The simple (exponent equation) form:

$$C = A^{2} + 3B^{1} \qquad a_A = 2 \qquad a_B = 1 \qquad \frac{u_C}{C} = \sqrt{\left(a_A \frac{u_A}{A}\right)^2 + \left(a_B \frac{u_B}{B}\right)^2}$$

$$\frac{u_C}{C} = \sqrt{\left(2\,\frac{0.504}{5.20}\right)^2 + \left(1\,\frac{2.10}{22.32}\right)^2} = 0.2155$$

• So, $u_C = \frac{u_C}{C} \cdot C = (0.2155)(94.0) = 20.257 \approx \underline{20.3 = u_C}$

& thus $\boxed{C = 94.0 \pm 20.3}$ ✗ WRONG!

This is not correct, because $C = A^2 + 3B$ is $\underline{\underline{\text{not}}}$ of the "simple" exponent form. Why?

The plus sign!

[$C = 3A^2B$ would be okay, but $C = A^2 + 3B$ is $\underline{\text{not}}$ okay]

☆ WE $\underline{\text{MUST}}$ USE THE GENERAL RSS EQ. WITH DERIVATIVES ☆

· METHOD B

$$C = A^2 + 3B \longrightarrow \begin{cases} \dfrac{\partial C}{\partial A} = 2A & \text{(treating B like a constant)} \\[2ex] \dfrac{\partial C}{\partial B} = 3 & \text{(treating A like a constant)} \end{cases}$$

These are partial derivatives

· General RSS uncertainty equation: RSS: (root of the sum of the squares)

$$u_c = u_{RSS} = \sqrt{\left(u_A \frac{\partial C}{\partial A}\right)^2 + \left(u_B \frac{\partial C}{\partial B}\right)^2}$$

$$u_c = \sqrt{\left[(0.504)(2)(5.20)\right]^2 + \left[(2.10)(3)\right]^2} = 8.195$$

· Finally $C = \bar{C} \pm u_c$

$$\boxed{C = 94.0 \pm 8.20}$$

(to 95% confidence)

✓ This answer is correct!

Notice that the uncertainty here differs greatly from the one we (erroneously) calculated using the simple equation.

★ BOTTOM LINE: Be careful — Do not use the simple formula unless the equation is of the simple exponent form. $\longrightarrow C = \text{Constant} \cdot A^a B^b \cdots$

[Any ⊕, ⊖, sin( ), cos( ), $e^{(\,)}$, etc. makes the equation be not of the simple form.]

The general RSS eq. (with derivatives) always works.

Example: Experimental uncertainty analysis

Given: Jan uses a thermocouple to measure temperature. She performs a regression analysis on temperature T (units = ^{o}C) as a function of thermocouple voltage V (units = mV). The best-fit straight line is $T = b + aV$, where $b = -0.8152^{o}$C and $a = 23.182^{o}$C/mV. She takes her voltage data with a digital data acquisition system that has an uncertainty of +/-0.124 mV.

To do: When the voltage reading is $V = 5.520$ mV, calculate the temperature with its appropriate uncertainties.

Solution:

- We calculate: $\overline{T} = b + a\overline{V} = \left(-0.8152^{\circ}\text{C}\right) + \left(23.182\dfrac{^{\circ}\text{C}}{\text{mV}}\right)\left(5.520\text{ mV}\right) = 127.149^{\circ}\text{C}$
- This equation for T is *not* of the simple exponent type, because of the plus sign. So, we *cannot* use the simpler RSS equation. We *must* use the general RSS equation with derivatives,

$$u_T = \sqrt{\sum_{i=1}^{N}\left(u_{x_i}\frac{\partial T}{\partial x_i}\right)^2}$$

- Here, there is only one independent variable, since $T = T(V)$ [can use total derivative]
- So, $u_T = \sqrt{\left(u_V \dfrac{\partial T}{\partial V}\right)^2} = u_V \dfrac{\partial T}{\partial V} = u_V \dfrac{dT}{dV}$

[Here, N=1 (one variable), so no summation required]

- But $T = b + aV \rightarrow \dfrac{\partial T}{\partial V} = a$

$\therefore$ $u_T = u_V a = (0.124\text{ mV})\left(23.182\dfrac{^{\circ}\text{C}}{\text{mV}}\right) = \underline{\underline{2.87457^{\circ}\text{C}}}$

- Finally, $\boxed{T = \overline{T} \pm u_T = 127.1 \pm 2.87\ ^{\circ}\text{C}}$

This is how Jan should report the temperature

☆ Note: For a linear equation with one component, $R = b + ax$,

$\underline{\underline{u_R = a\,u_x}}$. [Intercept b does nothing to the uncertainty – all b does is shift the R(x) line up or down]

Example: Experimental uncertainty analysis

Given: In a fluid mechanics experiment, the change in pressure is $\Delta P = 1.06 \rho V^2$, where

- ΔP = change in pressure (N/m^2, which is the same units as pascals, Pa)
- ρ = density, measured to be $\rho = 660. \pm 15.0\ kg/m^3$ (to 95% confidence)
- V = velocity, measured to be $V = 1.52 \pm 0.028$ m/s (to 95% confidence)

To do: Write ΔP at these values of ρ and V in standard engineering format.

Solution:

Do this one on your own for practice

Answer: $\boxed{\Delta P = 16\underline{2}0 \pm 70.\ N/m^2}$