Today, we will:

- Do a review example problem – experimental uncertainty analysis
- Review the pdf module: **Experimental Design**
- Do some example problems – experimental design (Taguchi design arrays)

Example: Experimental uncertainty analysis

Given: In fluid mechanics class, we learn that the change in pressure as you go down in a liquid (hydrostatic pressure change) is $\Delta P = \rho g h$, where

- ΔP = change in pressure (N/m^2, which is the same units as pascals, Pa)
- ρ = density ρ = 1000.0 kg/m^3 ± 0.02% (measured to 95% confidence)
- g = gravitational acceleration g = 9.807 m/s^2 (a known constant)
- h = change in liquid depth h = 3.45 ± 0.03 m (measured to 95% confidence)

To do: Write ΔP at these values of ρ, g, and h in standard engineering format.

Solution:

- Write the equation as $\boxed{\Delta P = g\,\rho^1 h^1}$ → This is of the simple exponent form $[R = \text{Constant}\; x_1^{a_1} x_2^{a_2} \cdots]$

[ΔP = 33834.2 N/m^2]

So, we can use the simpler formula.

However, lets do it using the general formula, for practice.

(You should try it the other way on your own — should get the same answer.)

- $\Delta P = g \rho h$ → $\dfrac{\partial P}{\partial \rho} = gh$ $\dfrac{\partial P}{\partial h} = g\rho$

[We treat g as a constant, not a component variable]

- RSS uncertainty:

$$U_{\Delta P} = \sqrt{\left(u_h \frac{\partial P}{\partial h}\right)^2 + \left(u_\rho \frac{\partial P}{\partial \rho}\right)^2}$$

$\frac{\partial P}{\partial h} = g\rho$, $\frac{\partial P}{\partial \rho} = gh$

u_h = 0.03 m (given)

u_ρ = 0.02% (1000 kg/m^3) = 0.200 kg/m^3

- Plug in numbers and units: ΔP = 33830 ± 294 N/m^2

or, to proper sig. digits, $\boxed{\Delta P = 33800 \pm 290 \text{ N/m}^2 \text{ to 95\% confidence}}$

Example – Taguchi array

Given: Susan proposes this experimental design array for 3 parameters and 4 levels for each parameter, choosing to test each level of each parameter *twice*.

Run #	*a*	*b*	*c*	*X*
1	1	1	1	X_1
2	1	2	3	X_2
3	2	3	4	X_3
4	2	4	2	X_4
5	3	1	2	X_5
6	3	2	3	X_6
7	4	3	1	X_7
8	4	4	4	X_8

To do: Explain (be *specific*) why this array is *not* a proper (optimized) Taguchi array. **How would you fix it?**

Solution:

Problem: Unnecessary repeats!

- This is a partial-factorial array.
- Fix by swapping the 2 & 3 for parameter c in rows 5 & 6 as shown here

[Now everything is proper]

5	3	1	3	X_5
6	3	2	2	X_6

- NOTE: This is not an orthogonal array, since there are not enough runs.

E.g., when a=1, we test b = 1 & 2 (we do not test b=3 or 4 when a=1)

Example – Taguchi array

Given: Josh proposes this experimental design array for 3 parameters and 4 levels for each parameter, choosing to test each level of each parameter *twice*.

Run #	*a*	*b*	*c*	*X*
1	1	1	1	X_1
2	1	2	3	X_2
3	2	3	3	X_3
4	2	4	2	X_4
5	3	1	2	X_5
6	3	2	4	X_6
7	4	3	2	X_7
8	4	4	4	X_8

To do: Explain (be *specific*) why this array is *not* a proper (optimized) Taguchi array. **How would you fix it?**

Solution:

- Problem: There are 3 level 2's for parameter c, but only 1 level 1 for parameter c.
- Fix easily by changing this 2 to a 1, then everything is fine
- Note, we have 8 runs here. A full factorial array would require $N = L^P = 4^3 = 64$ runs!
- Look up the orthogonal array for L=4 & P=3 → Need 16 runs

[The array shown here is not orthogonal]

Example: Experimental design using Taguchi arrays

Given: Bryan uses a Taguchi design array with 3 parameters and 4 levels for each parameter, choosing to test each level of each parameter *twice*. The design array (a valid Taguchi array) is shown here, along with the experimental results of the test.

Taguchi, $P=3$, $L=4$				
Run #	***a***	***b***	***c***	***X***
1	1	1	1	*1.65*
2	1	2	4	*1.71*
3	2	3	3	*1.85*
4	2	4	2	*1.71*
5	3	1	2	*1.79*
6	3	2	3	*1.76*
7	4	3	1	*1.75*
8	4	4	4	*1.68*

To do: Calculate level average $\bar{X}_{b3}$ to three significant digits.

Solution: $\bar{X}_{b_3} = (1.85 + 1.75)/2 = \boxed{1.80}$

(average of the two times that b = 3)

extra: $\bar{X}_{c_4} = (1.71 + 1.68)/2 = 1.695 \approx \boxed{1.70}$

(to 3 digits)

Example: Experimental design using Taguchi arrays

Given: A toy gun that shoots Nerf bullets is being designed. The engineers want to maximize the distance traveled by the bullet. Three parameters are to be varied:

- *a* = spring constant
- *b* = weight of the bullet
- *c* = diameter of the bullet

The engineers decide to test 4 levels for each of these 3 parameters.

(*a*) To do: Calculate how many runs are required for a full-factorial analysis.

$N = L^P = 4^3 = \boxed{64 \text{ runs required}}$

(*b*) To do: Design a Taguchi array such that each level of each parameter appears twice.

- How many runs will we need? $4 \times 2 = 8$

[This array will still not be orthogonal – would require 16 runs]

- Generate a Taguchi partial factorial, non-orthogonal array: (This is only one of many possible proper arrays)

Run #	a	b	c
1	1	1	1
2	1	2	4
3	2	3	3
4	2	4	2
5	3	1	2
6	3	2	3
7	4	3	1
8	4	4	4

- Columns a & b are trivial.
- Column c needs some thought – like a Sudoku puzzle!