

**Today, we will:**

- Do a review example problem – Experimental design and Taguchi arrays
- Review the pdf module: **Taguchi Orthogonal Arrays**
- Review the pdf module: **Response Surface Methodology (RSM)**, and do examples

**Example - Continued from last time.**

**Given:** A toy gun that shoots Nerf bullets is being designed. The engineers want to maximize the distance traveled by the bullet. Three parameters are to be varied:

- $a$  = spring constant
- $b$  = weight of the bullet
- $c$  = diameter of the bullet

The engineers decide to test 4 levels for each of these 3 parameters.

**(a) To do:** Calculate how many runs are required for a full-factorial analysis.

**Solution:**

$$N = L^P = 4^3 = \boxed{64}$$

**(b) To do:** Design a Taguchi array such that each level of each parameter appears twice.

**Solution:**

We need 8 runs since we have four levels, and we want each level of each parameter to appear twice; thus,  $N = 4 \times 2 = \boxed{8}$  for a fractional factorial design array.

We come up with the following Taguchi design array:

run #	$a$	$b$	$c$
1	1	1	1
2	1	2	4
3	2	3	3
4	2	4	2
5	3	1	2
6	3	2	3
7	4	3	1
8	4	4	4

**Q.** Is this a proper Taguchi array?

**A.** Yes – it meets both criteria

- Each level of each parameter appears the same # of times (2 times)
- No unnecessary repeats

**Q.** Is this an orthogonal array?

**A.** **No** → For an orthogonal array, for each level of each parameter, all levels of all the other parameters

must appear at least once. [E.g.  $b=2 \rightarrow c=4, 3$ . missing  $c=1 \& c=2$ .]

[See the orthogonal array for this case ( $L=4, P=3$ ) on the course website]

(c) To do: Now that the Taguchi design array is set up, experiments are performed, with the following results. Calculate the level averages, and determine which levels are optimum.

run #	a	b	c	$X$ (m)
1	(1)	1	1	5.5 $\leftarrow x_1$
2	(1)	2	(4)	6.2 $\leftarrow x_2$
3	2	(3)	3	6.0 $\leftarrow x_3$
4	2	4	2	7.3 $\leftarrow x_4$
5	3	1	2	7.6 $\leftarrow x_5$
6	3	2	3	8.1 $\leftarrow x_6$
7	4	(3)	(1)	9.7 $\leftarrow x_7$
8	4	4	(4)	7.2 $\leftarrow x_8$

e.g.  
 $\bar{X}_{c_4} = (x_2 + x_8)/2$

Solution: [See also Excel file on the course website – Example\_Taguchi\_array.xls]

Q. How many level averages do we need?

A. We have 4 levels of each parameter }  $\therefore$  Need  $4 \times 3 = 12$  level averages  
 We have 3 parameters

Examples:

$\bar{X}_{a_1} = (x_1 + x_2)/2 = (5.5 + 6.2)/2 = 5.85 = \bar{X}_{a_1}$ (color coded)	$\bar{X}_{b_3} = (x_3 + x_7)/2 = (6.0 + 9.7)/2 = 7.85 = \bar{X}_{b_3}$
	$\bar{X}_{c_4} = (x_2 + x_8)/2 = (6.2 + 7.2)/2 = 6.70 = \bar{X}_{c_4}$

: etc. Calculate all 12 level averages

See Excel file → Bottom line to maximize  $X$ ,

- Use level 4 of parameter a [largest spring constant]
- Use level 3 of parameter b [medium weight]
- Use level 1 of parameter c [smallest diameter]

(d) To do: Recommend follow-up experiments if only two parameters can be tested.

Solution: Note: A confirmation experiment is not necessary since this is run 7!

Since C has the smallest effect on  $X$ , we test only a : b

- Test larger values of a (Spring constant)
- Test b around level 3 (Weight)

## Example: Experimental design using RSM

**Given:** Cory uses RSM to increase the performance (speed) of a photocopier machine. He changes three parameters simultaneously, but in no particular pattern. He takes enough data to determine the direction of steepest ascent. Here are the variables:

- $a$ , the tension on a belt drive, varied from 400 to 550 N.
- $b$ , the volume flow rate of cooling air supplied, varied from 1.8 to 4.5 L/min.
- $c$ , the voltage supplied to a component, varied from 20 to 35 V.

**To do:**

(a) Calculate the *range* of coded variable  $x_2$  corresponding to physical variable  $b$ .

Ans: -1 to 1

[All coded variables are dimensionless ; range from -1 to 1]

(b) Calculate coded variable  $x_1$  corresponding to physical variable  $a = 475$  N.

**Solution:**

$$a_{\text{mid}} = \frac{400 + 550}{2} = \underline{\underline{475}}$$

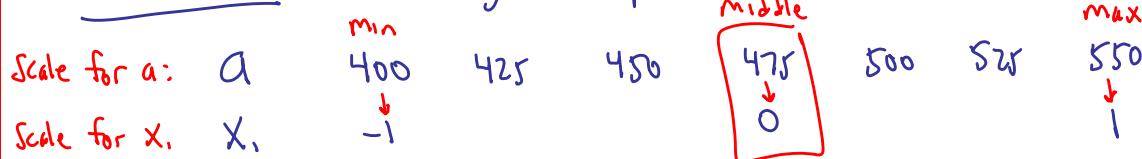
$$a_{\text{range}} = 550 - 400 = \underline{\underline{150}}$$

Method A → Use the equation

$$x_1 = 2 \left( \frac{a - a_{\text{mid}}}{a_{\text{range}}} \right) = 2 \left( \frac{475 - 475}{150} \right) = \boxed{0}$$

Answer

Method B → Linearly interpolate



OR, linearly interpolate like this:

$$\begin{array}{cc} \begin{matrix} a \\ \text{Min} \rightarrow 400 \end{matrix} & \begin{matrix} x_1 \\ -1 \end{matrix} \\ \text{Our value} \rightarrow 475 & \boxed{\quad} \\ \begin{matrix} \text{Max} \rightarrow 550 \\ 1 \end{matrix} & \end{array} \rightarrow \text{Linear interpolation yields } \boxed{x_1 = 0}$$