Today, we will:
- Do a review example problem – RSM
- Review the pdf module: **Hypothesis Testing and do some example problems**

Example: Response surface methodology

Given: We want to optimize the performance (maximum power P in units of horsepower) of a gasoline engine for a car. We vary three parameters simultaneously, and measure the engine's output power for each run with a dynamometer:
- a = spark plug gap (inches)
- b = spark timing (degrees before TDC – top dead center)
- c = fuel:air mixture (# of turns of the set screw that controls fuel:air mixture)

***a* (inches)**	***b* (degrees)**	***c* (# turns)**	***P* (hp)**
0.030	10	0	156
0.030	9	-0.25	160
0.029	9	0	162
0.029	11	0.25	159
0.031	11	-0.25	154
0.031	10	0.25	158

(a) To do: Calculate the coded variable x_b for the second row of data.

Solution: $\left.\begin{array}{l} b_{range} = 11-9=2 \\ b_{mid} = (9+11)/2 = 10 \end{array}\right\}$ $x_b = 2\left(\frac{b-b_{mid}}{b_{range}}\right) = 2\left(\frac{9-10}{2}\right) = -1 \rightarrow \boxed{x_2 = -1 \text{ @ } b=9}$

$[x_2 = x_b]$

OR, just look & see that 9 is b_{min}! So,

b:	$\boxed{9}$	10	11
x_b:	$\boxed{-1}$	0	1

(b) To do: Determine the direction of steepest ascent in terms of physical variables.

Solution: Solution done in Excel – see file on course website: **sample_car_engine_RSM.xls**.

Result

$$\boxed{\begin{array}{l} \Delta a = -0.0007 \text{ in.} \\ \Delta b = -1 \text{ degrees} \\ \Delta c = 0.1 \text{ turns} \end{array}}$$

(rounded off to 1 sig. digit to make it easier to run the tests)

See Excel file online

Example: Response surface methodology

Given: We want to optimize the efficiency η (%) of a machine, where η depends on three parameters *a*, *b*, and *c*. We run several cases, varying *a*, *b*, and *c* simultaneously around the current operating point:

Current operating point →

a	*b*	*c*	η (%)
15	4	0.2	71.5
16	6	0.5	80.6
15	5	0.3	74.4
14	6	0.4	67.7
16	5	0.2	80.3
14	4	0.5	64.6

To do: Determine the direction of steepest ascent. [I used Excel for convenience; file not on website – try it on your own for practice.]

Solution:

- Convert to coded variables x_1, x_2, and x_3.
- Use regression analysis to determine the direction of steepest ascent.

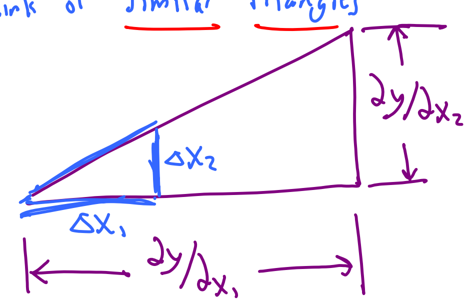
Answer: $\boxed{\left(\frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2}, \frac{\partial y}{\partial x_3}\right) = (6.604,\ 1.576,\ -0.505)}$.

- Pick *one* of the increments (Δx_1, Δx_2, or Δx_3), and then calculate the other two. I picked $\Delta x_2 = 0.5$.

$$\Delta x_1 = \Delta x_2 \frac{\partial y/\partial x_1}{\partial y/\partial x_2} = (0.50)\frac{6.604}{1.576} = 2.095$$

$$\Delta x_3 = \Delta x_2 \frac{\partial y/\partial x_3}{\partial y/\partial x_2} = (0.50)\frac{-0.505}{1.576} = -0.1602$$

Think of similar triangles



Answer: $\boxed{\Delta x_1 = 2.1 \qquad \Delta x_3 = -0.16}$

(rounded to 2 sig. digits)

- Convert these coded variable increments back to physical variables, Δa, Δb, and Δc.

$$\Delta a = \Delta x_1 \frac{a_{range}}{2} = (2.095)\frac{2}{2} = 2.095$$

Similarly for Δb & Δc [do on your own for practice]

Answer: $\boxed{\Delta a = 2.1 \qquad \Delta b = 0.50 \qquad \Delta c = -0.024}$ (rounded to 2 sig. digits)

- Final step: You may round off (for convenience) and *march* in the direction of steepest ascent, varying *a*, *b*, and *c* *simultaneously*. [You cannot actually *do* this, since you don't have the experiment.]

Example: Hypothesis testing

Given: A manufacturer claims that a strut can hold at least 100 kg before failing. We perform 10 tests to failure, and record the load:

- $n = 10$ (number of data points in the sample)
- $\bar{x} = 101.4$ kg (sample mean)
- $S = 2.3$ kg (sample standard deviation)

To do: Determine if we should accept or reject the manufacturer's claim.

Solution:

- This is a (one-sided two-sided) *t*-test.
- The null hypothesis: μ_0 = the manufacturer's claim → $\mu_0 = 100$ kg
- The "side" of the null hypothesis: Least likely scenario is that $\boxed{\mu < \mu_0}$ since our $\bar{x}$ is $> \mu_0$
- The alternative hypothesis: $\boxed{\mu > \mu_0}$ (opposite)
- The critical *t* statistic:

$$@\ \mu = \mu_0,\quad t_{crit} = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{101.4 - 100}{2.3/\sqrt{10}} = \boxed{1.9248 = t_{crit}}$$

- The *p*-value:

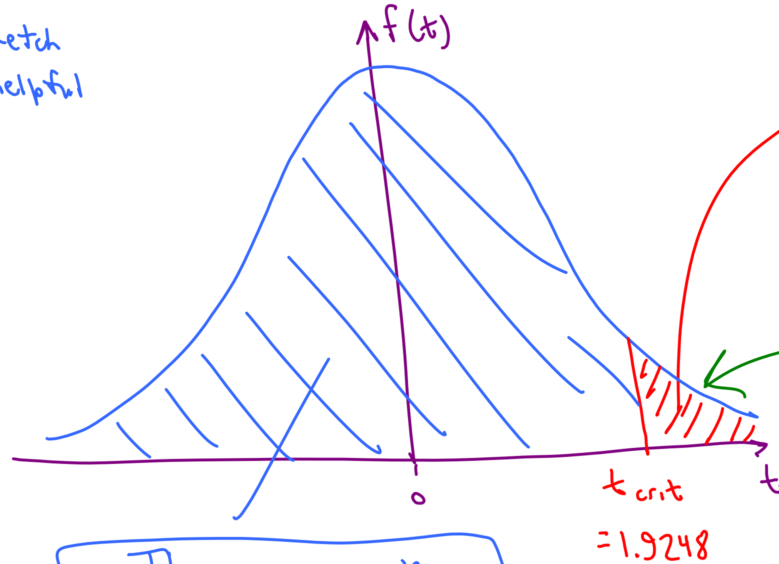
Look up in table @ $t_{crit} = 1.9248$, $df = 10 - 1 = 9$, # tails = 1 → $\boxed{p = 0.04319}$

[one sided] → # tails = 1

OR, in Excel, =TDIST(1.9248, 9, 1) = 0.04319 ✓
(t_{crit}, df, # tails)

- Interpretation and conclusion:

A sketch is helpful



The p-value is this area of the tail = 0.04319 ≈ 4.32%

This area, the p-value, represents the probability that the null hypothesis is true, i.e., that the manufacturer's claim is false

This area is the probability that the null hypothesis is false, i.e., that the manufacturer's claim is true

- The probability that the null hypothesis is true (actual $\mu < \mu_0$) is 4.32% THIS IS THE LEAST-LIKELY SCENARIO, that $\mu < 100$ kg

[This is least likely since our test data show that $\bar{x} > \mu_0$]
(101.4 > 100 kg)

- So, the opposite of this → The probability that the null hypothesis is <u>not</u> <u>true</u> is 100 − 4.32 = <u>95.68%</u>

- Thus, we are 95.7% confident that the manufacturer's claim is true

Final answer – we accept the manufacturer's claim since we are confident to > 95% (to standard engineering confidence)

In a journal paper, I would write:
"The claim is accepted to more than 95% confidence ($p = 0.043$)"

Note: There is a 4.3% chance that the actual population mean μ is less than 100 kg (the claim). How? We may by chance, have picked 10 struts to test that were the <u>stronger</u> ones, compared to the real mean.

Bottom line: Since $p < 0.05$, we accept the manufacturer's claim