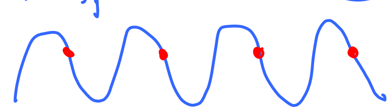
Today, we will:
- Do some review example problems – digital data acquisition and aliasing
- Review the pdf module: **Signal Reconstruction**, and do an example problem
- Discuss the pdf module: **Spectral Analysis and Fourier Series**, and begin to discuss the **Fast Fourier Transform**.

Example: Digital data acquisition

Given: Consider each of the following scenarios, with signal frequency f and digital sampling frequency f_s.

To do: In each case, determine if there is aliasing, and calculate the *perceived* frequency inferred from the acquired data.

Note: The general formula for perceived frequency is $f_{\text{perceived}} = \left| f - f_s \cdot \text{NINT}\left(\frac{f}{f_s}\right) \right|$.

	f (Hz)	f_s (Hz)	Aliasing? (Yes or No)	$f_{\text{perceived}}$ (Hz)
(a)	600	2000	$2f = 1200$ $f_s > 2f$, ∴ No (no aliasing)	Since no aliasing, $f_{\text{perceived}} = f_{\text{signal}} = 600$ Hz OR, use formula: 0.3 $f_{\text{perceived}} = \left\|600 - 2000 * \text{NINT}\left(\frac{600}{2000}\right)\right\| = \|600-0\| = 600$ ✓
(b)	70	90	$2f = 140$ $f_s < 2f$ ∴ YES	$\|70 - 90 * \text{NINT}(70/90)\| = \|70 - 90\| = 20$ (0.7777..) 1
(c)	700	700	$2f = 1400$ $f_s < 2f$ ∴ YES	1 $\|700 - 700 * \text{NINT}(700/700)\|$ 0 $= \|700 - 700\| = 0$ looks like DC! 
(d)	80	70	$2f = 160$ $f_s < 2f$ ∴ YES	1.14 $\|80 - 70 * \text{NINT}(80/70)\|$ 10 $= \|80 - 70 * 1\|$ $= 10$
(e)	600	320	$2f = 1200$ $f_s < 2f$ ∴ YES	$\|600 - 320 * \text{NINT}(600/320)\|$ 40 (1.875) $= \|600 - 320 * 2\| = 40$
(f)	380	250	$2f = 760$ $f_s < 2f$ ∴ YES	$\|380 - 250 * \text{NINT}(380/250)\|$ 120 (1.52) $= \|380 - 250 * 2\| = 120$

Example: Digital data acquisition

Given: A voltage signal from a segment of music contains frequency components at 150, 350, and 700 Hz. It also contains some electronic noise at 60 Hz. We sample the data digitally at a sampling frequency of 400 Hz.

To do: Is there any aliasing? If so, what frequencies will we see (perceive)?

Solution:

Check Nyquist criterion for each frequency in the signal:

f (Hz)	$2f$ (Hz)	f_s (Hz)	Aliasing?
60	120	400	No $f_s > 2f$
150	300	400	No $f_s > 2f$
350	700	400	YES $f_s < 2f$
700	1400	400	YES $f_s < 2f$

∴ The two lower frequencies will not alias (will be perceived correctly), but the two higher frequencies will be aliased because we are not sampling at high enough sampling frequency.

• Now calculate the perceived frequencies. Use $\boxed{f_{perceived} = \left| f - f_s * NINT\left(\frac{f}{f_s}\right) \right|}$

f (Hz)	f_s (Hz)	$f_{perceived}$ (Hz)
60	400	60 (No aliasing)
150	400	150 (No aliasing)
350	400	$f_a = f_{perceived} = \left|350 - 400 * NINT(350/400)\right|$ (0.875) $= \left|350 - 400 * 1\right| = 50$ Hz
700	400	$f_a = f_{perceived} = \left|700 - 400 * NINT(700/400)\right|$ (1.75) $= \left|700 - 400 * 2\right| = 100$ Hz

☆ NOTICE HOW ALIASING HAS CAUSED CONFUSION!

	(analog) ORIGINAL SIGNAL		(digital) PERCEIVED SIGNAL
original (analog) signal had	60 Hz	→	60 Hz
	150 "	→	150 "
	350 "	→	50 "
	700 "	→	100 "

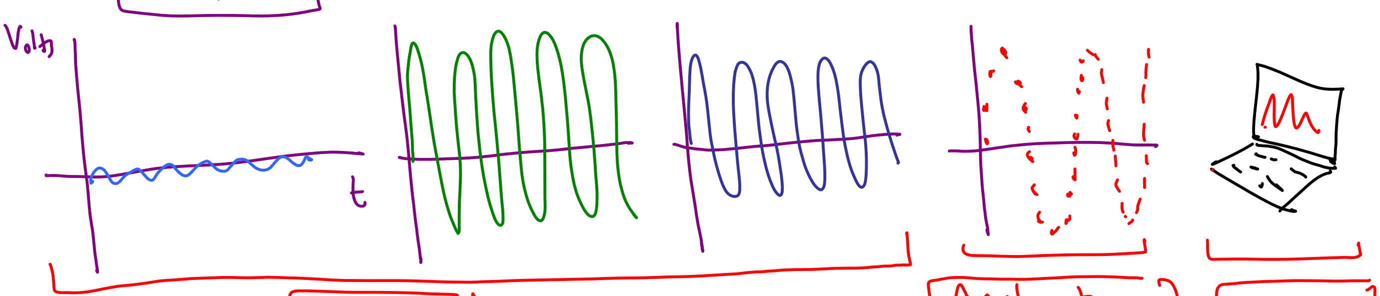
These two are aliased!

(The music would sound really strange due to this shift in two of the frequencies)

☆ Bottom line — always sample at $f_s > 2f_{max}$ to avoid aliasing ! (satisfy the Nyquist criterion) ☆

Typical setup in a lab

transducer (usually a small voltage output) → amplifier (amplifies the signal) → filter (attenuates certain frequencies) → A/D Converter → memory card or computer, etc



ANALOG

Analog to digital conversion

DIGITAL

☆ Three potential problems when doing digital data acquisition:

1) clipping (V exceeds range of A/D)
2) poor resolution (not using many bins of the A/D)
3) aliasing (not sampling fast enough)

Example: Fourier analysis

Given: A voltage signal is of the form: $f(t) = 5.20 + 1.50\sin(18.0\pi t)$ $\omega_o = 18.0\pi$

(*a*) To do: Calculate the fundamental frequency in Hz and the fundamental period in s.

Solution:

Recall the general eq. for a sine wave, $f(t) = C + A\sin(\omega_o t) = C + A\sin(2\pi f_o t)$

$\boxed{\omega_o = 2\pi f_o}$

So, $2\pi f_o t = 18.0\pi t \rightarrow f_o = \frac{18.0}{2} = \underline{9.00 \text{ Hz}}$

Also, T = fundamental period $= \frac{1}{f_o} = \frac{1}{9.00\ 1/s} = 0.1111$ s

$\boxed{f_o = 9.00 \text{ Hz} \quad T = 0.111 \text{ s}}$ ☆

(*b*) To do: Calculate the Fourier coefficients.

Solution:

Two ways to do this

- Long way → Use the eq's for the Fourier coefficients & integrate (lots of math & integration – try it on your own for fun.)
- Short way → Recognize that $f(t)$ is already in the same form as a Fourier sine series!!

Fourier sine series: $f(t) = C_o + \sum_{n=1}^{\infty} a_n \sin(n\omega_o t)$

(when $n=1$) a_1

Our function: $f(t) = 5.20 + 1.50\sin(1(18\pi)t)$

So … without much work at all, we conclude:

☆ $\boxed{C_o = 5.20,\ a_1 = 1.50,\ \text{all other Fourier coefficients} = 0}$

[this is kind of a trick question]