Today, we will:

- Do a review example problem – digital data acquisition & aliasing
- Review the pdf module: **Fourier Transforms, DFTs, and FFTs** and do some examples

Example: Digital data acquisition

Given: Andy collects data with a digital data acquisition system that is 14-bit and has a range of -5 to 5 V. He samples at a sampling frequency of 200 Hz.

(***a***) **To do**: Calculate the quantization error in millivolts.

Solution: Quantization error $= \pm \frac{V_{max} - V_{min}}{2^{N+1}} = \pm \frac{5-(-5)\ \text{V}}{2^{14+1}} \left(\frac{1000\ \text{mV}}{\text{V}}\right) = \boxed{\pm 0.3052\ \text{mV}}$

[the resolution is twice this, $\Delta V = 0.6104$ mV]

(***b***) **To do**: For each case, will Andy's signal be clipped? Is there any aliasing? If so, what frequencies will he see (perceive)?

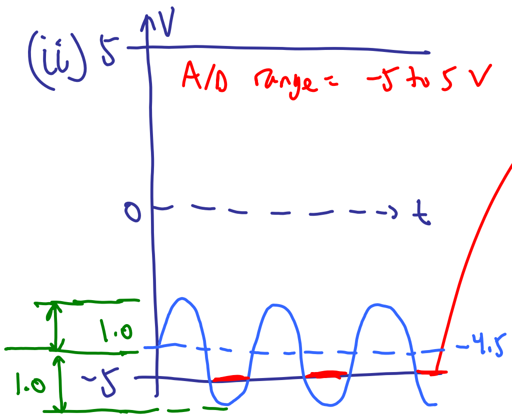
(i) Signal has a frequency of 40 Hz with a range of –3 to 3 V.
(ii) Signal has a frequency of 120 Hz with DC offset = –4.5 V and amplitude = 1.0 V.
(iii) Signal is $f(t) = 3.5\sin(700\pi t) + 1.0$ V.

Solution:

(i) • **No clipping** since –3 to 3 is in the range of A/D

• Check Nyquist → $f = 40$ Hz, $2f = 80$ Hz, $f_s = 200 > 80$ **No aliasing**

(ii)



Will clip the lower parts of the signal!

Yes, there will be clipping

• Check Nyquist → $f = 120$, $2f = 240$
$f_s = 200 < 2f$
∴ **Yes, aliasing**

• Predict aliasing frequency: $f_a = |f - f_s * \text{NINT}(f/f_s)|$
$= |120 - 200 * \text{NINT}(120/200)| = |120 - 200*1| = 80$ (where $120/200 = 0.6$)

Will alias at 80 Hz

[& will see additional peaks due to the clipping]

(iii) $f = 350$ Hz [$2\pi f t = 700\pi t$]

• Range $= 1.0 - 3.5 = -2.5$ to $1.0 + 3.5 = 4.5$ } **NO CLIPPING**

• check Nyquist → **Yes, aliasing**

We predict $\boxed{f_a = 50\ \text{Hz}}$

[try it on your own to get this]

Example: DFTs and FFTs

Given: Voltage data are acquired with a digital data acquisition system. A DFT (or FFT) is performed, and a frequency spectrum plot is generated.

To do: Which of the following has the better frequency resolution?

Case a: Data are sampled at f_s = 100 Hz, and 512 data points are taken.
Case b: Data are sampled at f_s = 200 Hz, and 256 data points are taken.

$\Delta f = 1/T$

Solution:

(a) → $f_s = 100$ Hz, $N = 512$ pts } T = sampling time $= \frac{N}{f_s} = \frac{512 \text{ pts}}{100 \text{ pts/s}} = 5.12 \text{ s} = T$ → $\Delta f = 0.1953$ Hz

(b) → $f_s = 200$ Hz, $N = 256$ pts } $T = \frac{N}{f_s} = \frac{256 \text{ pts}}{200 \text{ pts/s}} = 1.28 \text{ s} = T$ → $\Delta f = 0.7812$ Hz

Answer: Case (a) is better because T is longer & ∴ Δf is smaller (better frequency resolution)

Example: DFTs and FFTs

Given: A signal contains frequencies up to 500 Hz. Voltage data are acquired with a digital data acquisition system at f_s = 1000 Hz to avoid aliasing. 2048 data points are taken, a DFT or FFT is performed, and a frequency spectrum plot is generated.

To do: Calculate the following:

- The total sampling time
- The folding frequency of the resulting frequency spectrum
- The frequency resolution of the resulting frequency spectrum

Solution:

- $T = \frac{N}{f_s} = \frac{2048 \text{ pts}}{1000 \text{ pts/s}} = 2.048 \text{ s} = T$
- $f_{folding} = \frac{f_s}{2} = \frac{1000 \text{ Hz}}{2} = 500 \text{ Hz} = f_{folding}$
- $\Delta f = \frac{1}{T} = \frac{1}{2.048 \text{ s}} = 0.4883 \text{ Hz} = \Delta f$ OR, $\Delta f = \frac{1}{T} = \frac{f_s}{N} = \frac{1000 \text{ pts/s}}{2048 \text{ pts}} = 0.4883$ Hz ✓

Note: To improve the frequency resolution of the FFT, we need to make Δf smaller. But $\Delta f = \frac{1}{T} = \frac{f_s}{N}$ ★ The only way to decrease Δf is to sample for a longer time

So... Improve resolution by: (1) Sample more data points (increase N)
(2) decrease f_s (smaller sampling frequency) ✗

★ But choice 2 is not good because we would have aliasing!