Today, we will:
- Continue our discussion about ***leakage***, why it occurs, and ***what to do about it***
- Review the pdf module: **Windowing with FFTs** and do an example problem

• Why is there leakage in frequency spectra?

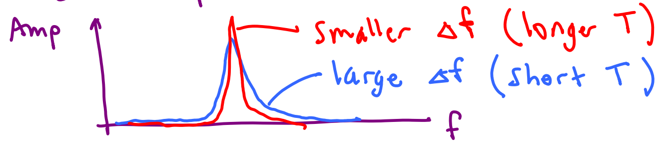
Answer: Since we sample data for some arbitrary time T, which in general has nothing to do with the signal, the first data point and the last data point are at different phases and/or voltages

[we start and stop the data acquisition at different phases in the signal]

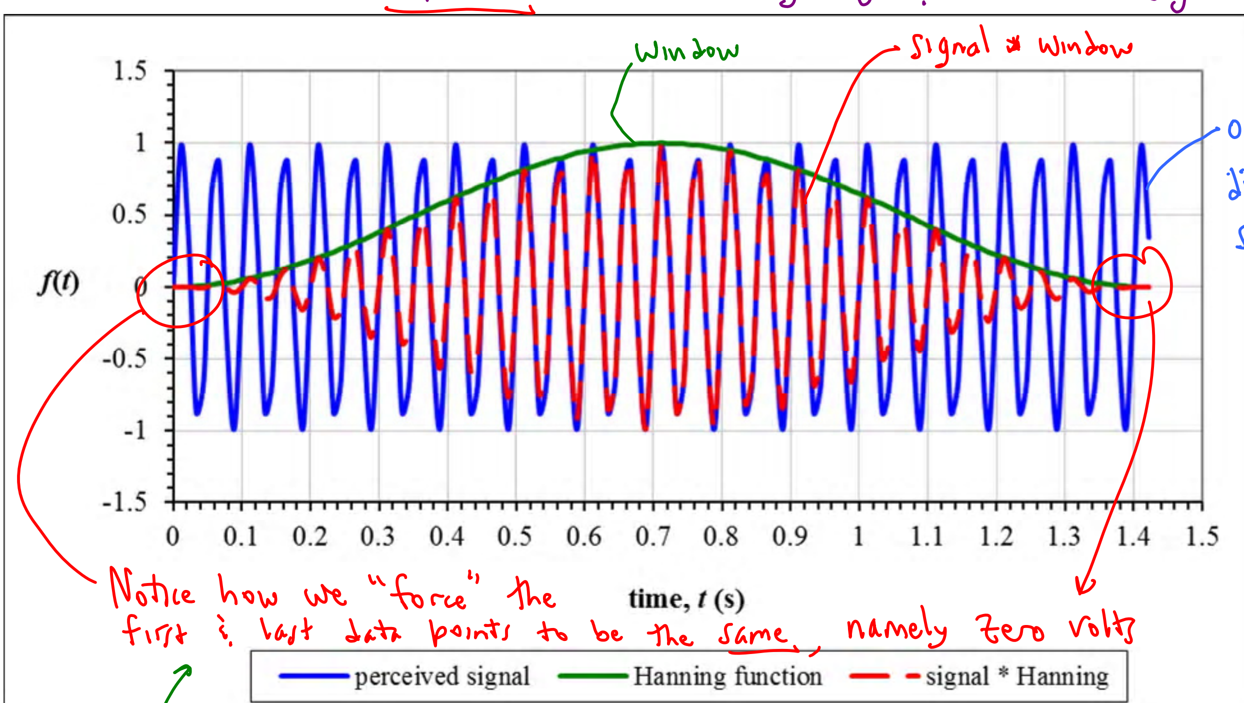
• How to reduce leakage?

Answer: There are two ways to reduce (not eliminate) leakage

(1) Increase T → This reduces Δf, the frequency resolution, which makes the spikes narrower (improved frequency resolution)



(2) Windowing → Consider the middle portion of the signal as more important than the beginning & end of the signal



original digital signal

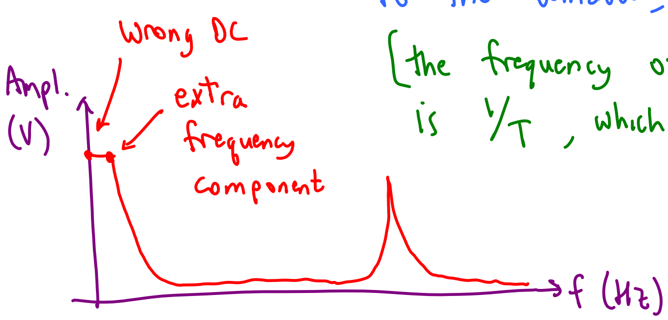
Notice how we "force" the first & last data points to be the same, namely zero volts

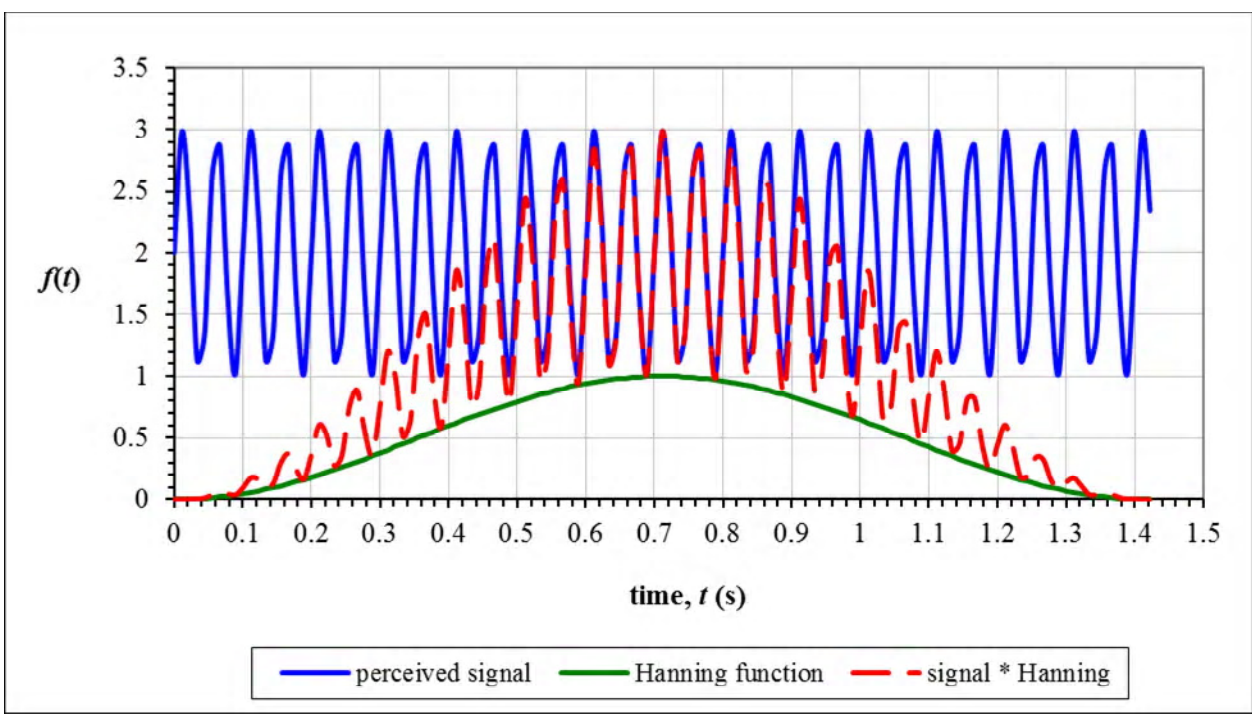
Windowing reduces leakage since now the first & last data pts are the same (zero volts)

• Signal with a DC component

- We have to be more careful when there is a non-zero DC component
- If we simply multiply the digital signal by the windowing function, it distorts the signal by forcing the first & last pt. to be zero. (see dashed red line on plot below)
- Problems: (1) Wrong DC value → the average value of red curve is not the same as the average value of blue curve.

  (2) Introduce another long period, low frequency wave due to the window, which is not physical
  [the frequency of this wave (green window function) is $1/T$, which is the same as Δf]





How to correct?

- Subtract off the mean value (DC offset)
- Multiply by Hanning window & do FFT as previously
- Add the mean value (DC offset) back in at the end (@ f = 0)

Example: FFTs & windowing

Given: Voltage data are acquired with a digital data acquisition system at a sampling frequency of 100 Hz for a total sampling time of 3 s. (real data, not fake data)

To do: If **Excel is used for the FFT**, calculate the following:

- Number of *useful* data points for calculation of a frequency spectrum
- The folding frequency of the spectrum
- The frequency resolution of the spectrum
- Generate a frequency spectrum, using the Hanning window
- Summarize the voltage signal – i.e., DC offset, frequency component(s), and their amplitudes

Solution: ↙ STUDY THIS EXAMPLE

[For plots and solution to this example problem, see the Excel file on the course website called "FFT_Example_data_with_window.xls". It is also solved using Matlab.]

- $N_{total} = T \cdot f_s = (3\text{ s})(100\text{ Hz}) = 300$ pts

  But → Excel is limited to powers of 2 in its FFT → closest = 256

- So, use $\boxed{N = 256 \text{ pts}}$ = # useful data points for calculation of the FFT.

  [For the frequency spectrum, we have half of this → 128 (actually 129) useful data points on the frequency spectrum plot]

- $f_{folding} = \frac{f_s}{2} = \frac{100\text{ Hz}}{2} = \boxed{50\text{ Hz} = f_{folding}}$ ← we plot the frequency spectrum from 0 Hz (DC) to 50 Hz (folding)

- $\Delta f = \frac{1}{T}$, $T = \frac{N}{f_s} \rightarrow \Delta f = \frac{f_s}{N} = \frac{100\text{ Hz}}{256} = \boxed{0.3906\text{ Hz} = \Delta f}$

[In class we will plug in the data & perform the frequency analysis including the Hanning Window, and properly accounting for the DC offset]

Summary: (See Excel on Website)

- DC offset ≈ 4.6 V
- 2 frequency components: $f_1 \approx 15.6$ Hz, Ampl. $\gtrsim$ 1.79 V
  $f_2 \approx 46.9$ Hz, Ampl. $\gtrsim$ 0.72 V
- random noise at all f's