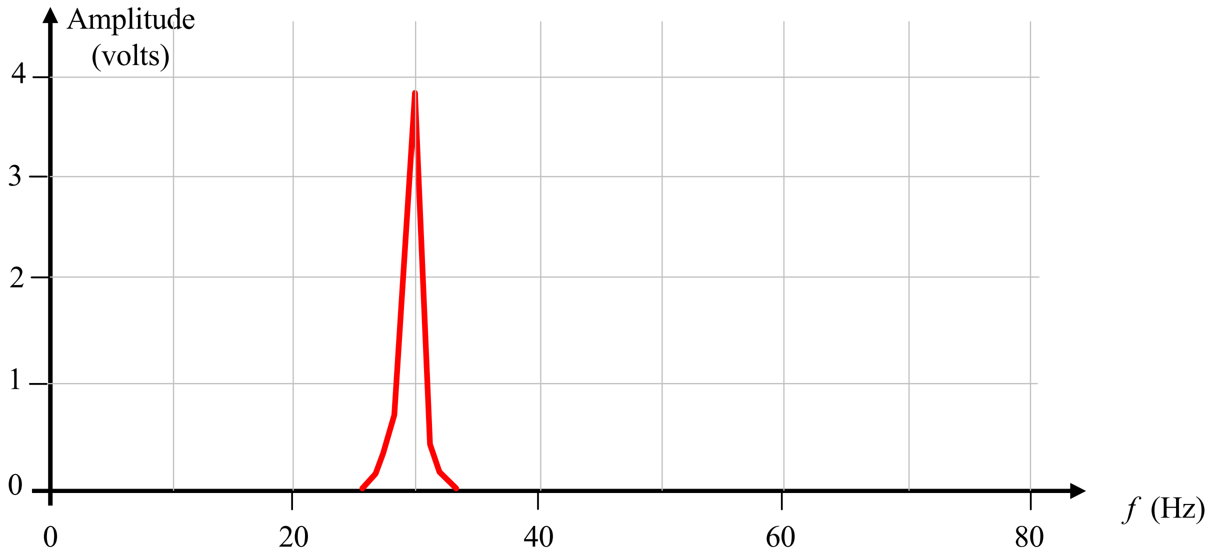
Today, we will:

- Reinforce the analogy between Fourier series and FFTs
- Review the pdf module: **How to Analyze the Frequency Content of a Signal**
- Do some example problems – analyzing the frequency content of a signal

Example: Analyzing the frequency content of a signal using an FFT

Given: A voltage signal is sampled at sampling frequency f_s = 160 Hz. The resulting frequency spectrum is shown below:



We conclude that there is a frequency component of about 30 Hz, with an amplitude of about 4 volts. But how can we be sure? What if the 30 Hz peak is *aliased* from some higher frequency?

E.g., What if actual $f = 130$ Hz? → $2(130) = 260$; f_s is only 160, ∴ aliasing

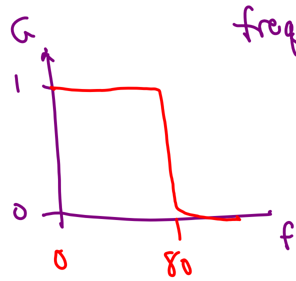
$$f_a = \left|f - f_s * \mathrm{NINT}(f/f_s)\right| = \left|130 - 160 * \mathrm{NINT}(\underbrace{130/160}_{0.8125})\right| = \left|130 - 160 * 1\right| = 30 \text{ Hz}$$

OR, suppose actual $f = 350$ Hz → $f_a = \left|350 - 160 * \mathrm{NINT}(\underbrace{350/160}_{2.1875})\right| = \left|350 - 160 * 2\right| = 30 \text{ Hz}$

☆ There is no way to tell from this one spectrum whether the 30 Hz is real or aliased. [A real freq. of 130, 190, 350, ... would produce the same spectrum!]

To do: What should we do to determine if this signal *really* has a frequency component at 30 Hz?

Solution: (1) Use an anti-aliasing filter (a low-pass filter that cuts off all frequency components > 80 Hz – then aliasing is impossible!)



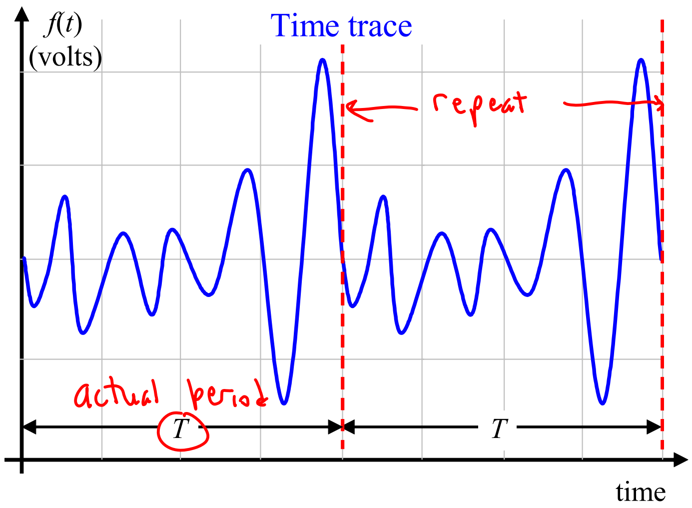
(2) Sample again, but at a different sampling frequency f_s

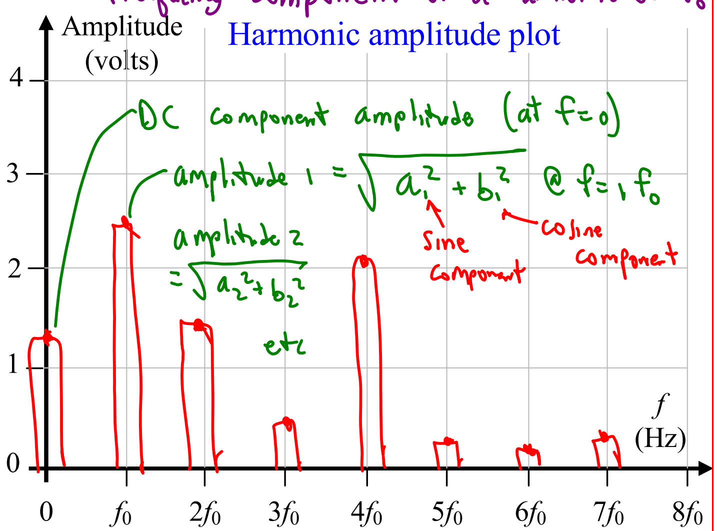
[repeat at a third f_s if necessary.]

Analogy between Fourier series and FFT (or DFT):

Fourier Series:

- T = known = actual period of the function $f(t)$ → ∴ T is very significant
- $\boxed{f_0 = \frac{1}{T}}$ = fundamental frequency → f_0 is also very significant since T is significant
- Each amplitude on the harmonic amplitude plot represents the amplitude of the frequency component of a harmonic of f_0





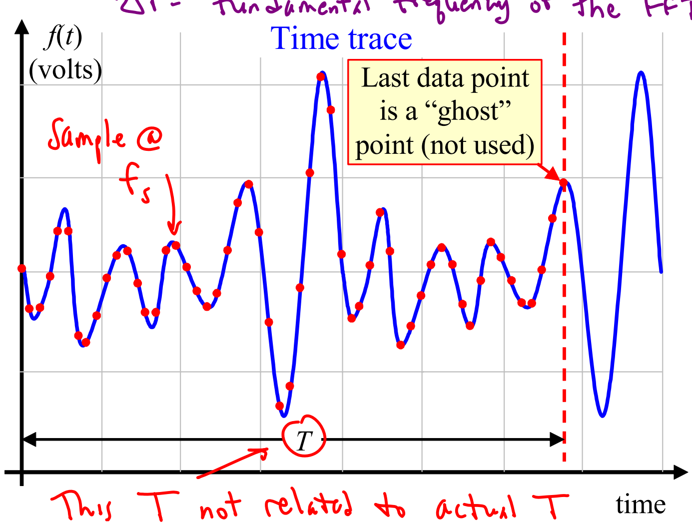
Bottom line: The harmonic amplitude plot shows us the frequency content of the periodic function

FFT:

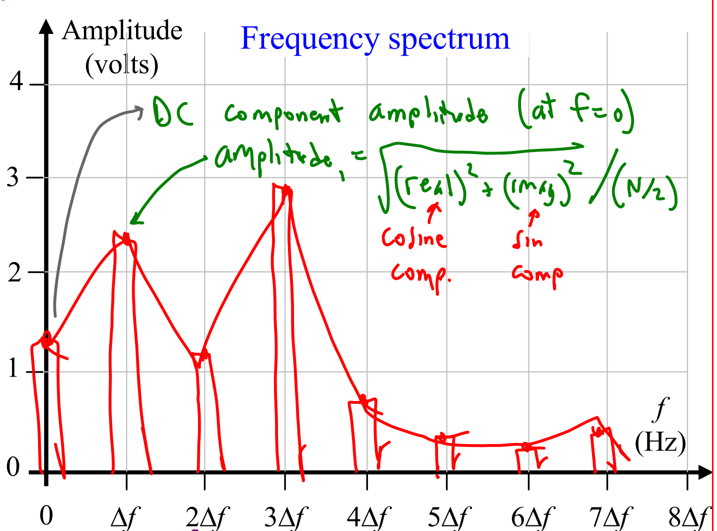
- T is arbitrary – has nothing to do with actual period of signal [may not even be an overall period]

So, T is not significant & $\boxed{\Delta f = \frac{1}{T}}$ is also arbitrary, not significant.

- Δf = "fundamental frequency of the FFT"



This T not related to actual T of the signal



$\boxed{\text{Max} = \frac{f_s}{2}}$

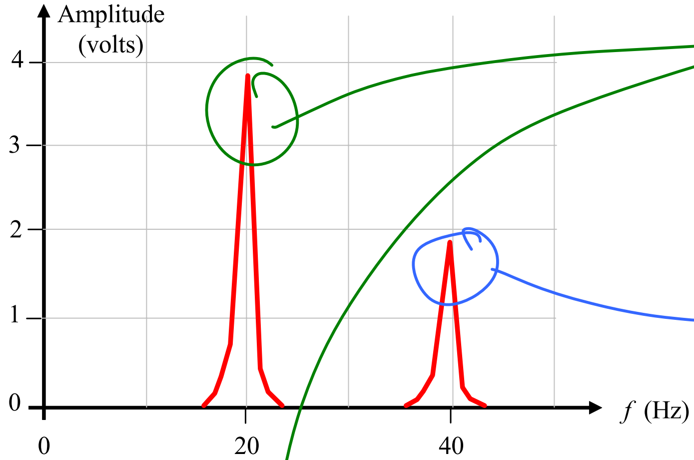
Δf is analogous to f_0 of Fourier Series, but Δf is arbitrary, whereas f_0 is significant

Bottom line: The frequency spectrum shows the frequency content of the signal

Example: FFTs

Given: A voltage signal is sampled at two different sampling frequencies, 100 Hz and 160 Hz. The resulting frequency spectra are shown below:

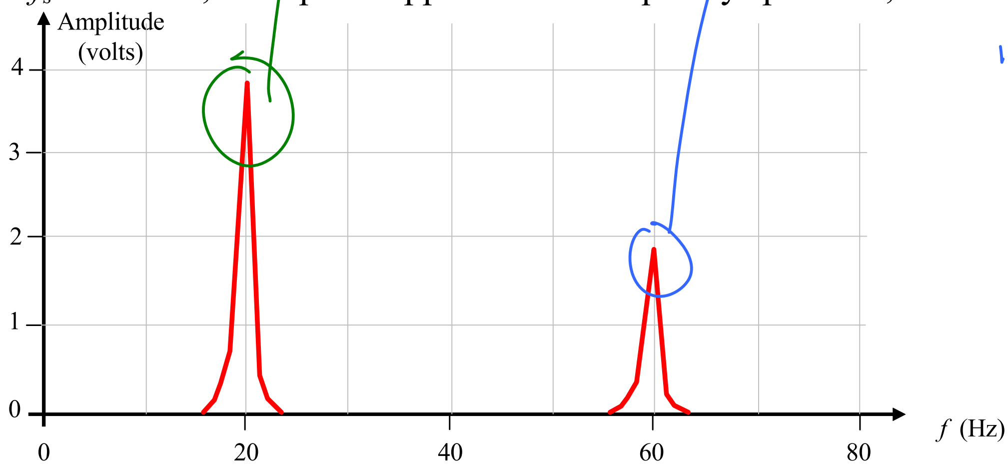
- At f_s = 100 Hz, two spikes appear in the frequency spectrum, at 20 and 40 Hz.



The component at 20 Hz & ≈ 3.8 V is probably real since it appears in both spectra

These two have the same amplitude, but different frequencies, so we are sure that aliasing is occurring.

- At f_s = 160 Hz, two spikes appear in the frequency spectrum, at 20 and 60 Hz.



To do: Determine the frequencies most likely to actually be in the signal.

Solution:

- We "trust" the second one more since f_s is larger (less likely to alias)
- Assume that the 60 Hz spike is real.
- Consistency check → if f = 60 Hz & f_s = 100 Hz,

$$f_a = \left|60 - 100 * \mathrm{NINT}\left(\underbrace{60/100}_{(0.6)}\right)\right| = |60 - 100*1| = \underline{40 \text{ Hz}}$$

- Since everything is consistent, we write our final answer:

$$\boxed{\begin{array}{l} \bullet \approx 20 \text{ Hz component w/ amplitude} \approx 4 \text{ V} \\ \bullet \approx 60 \text{ Hz component w/ amplitude} \approx 2 \text{ V} \end{array}}$$

[cannot know f or ampl. precisely due to leakage]

- To verify further, we would choose an even larger f_s
   - Do not pick an integer multiple of 100 or 160

     200, 400 ✗ ✗   320, 480 ✗ ✗
   - Do not pick a sum or difference of 100 & 160

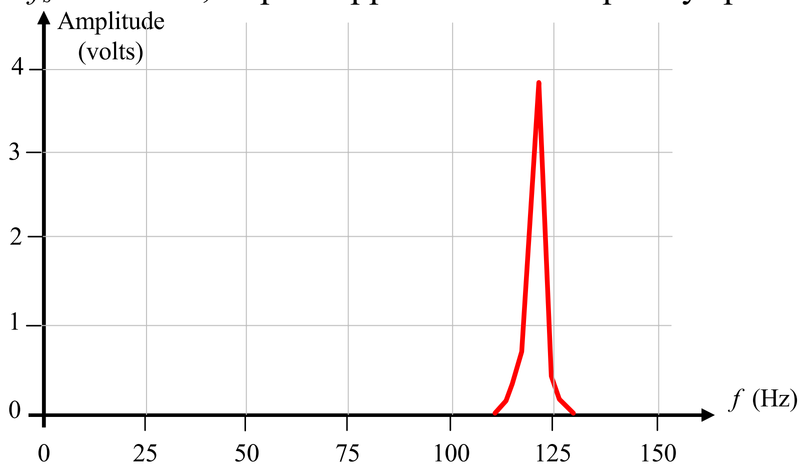
     $100 + 160 = 260$ ✗

     or any multiple of that (520 ✗)
   - A good choice might be 195, 275, etc.

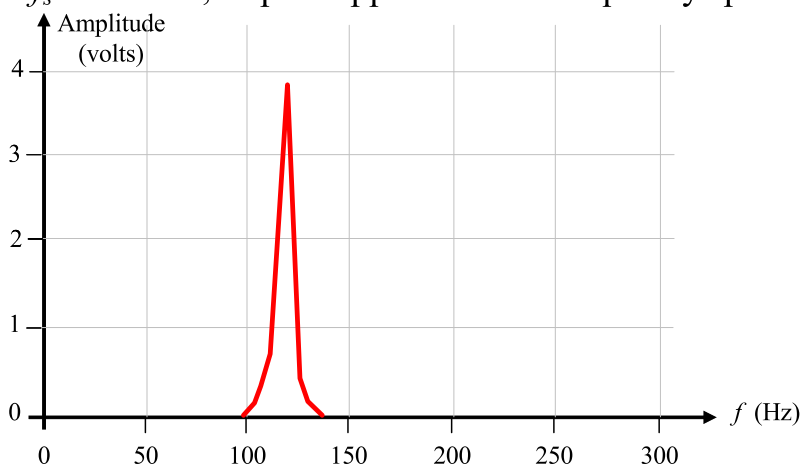
Example: FFTs

Given: A voltage signal is sampled at two different sampling frequencies, 300 Hz and 600 Hz. The resulting frequency spectra are shown below:

- At f_s = 300 Hz, a spike appears in the frequency spectrum at 120 Hz.



- At f_s = 600 Hz, a spike appears in the frequency spectrum, at 120 Hz.



To do: Determine the frequency most likely to actually be in the signal.

Solution:

- Assume that $f = 120$ Hz is correct. This is consistent with both spectra, & $2f = 240$ Hz is within the Nyquist sampling freq. for both cases
- But, 600 Hz is an integer multiple of 300 Hz [600 = 2*300]
  So, 600 Hz was not a good choice for the second f_s
- There may be other frequencies that alias to 120 Hz for both cases

  Example → $f = 480$ Hz

  Check consistency:

@ $f = 480$ Hz, & $f_s = 300$ Hz,

$$f_a = \left|480 - 300 * \text{NINT}\left(\underset{1.6}{\frac{480}{300}}\right)\right| = |480 - 300 * 2| = \underline{120 \text{ Hz}}$$

@ $f = 480$ Hz & $f_s = 600$ Hz,

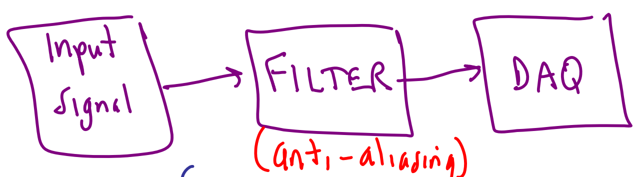
$$f_a = \left|480 - 600 * \text{NINT}\left(\underset{0.8}{480/600}\right)\right| = |480 - 600 * 1| = \underline{120 \text{ Hz}}$$

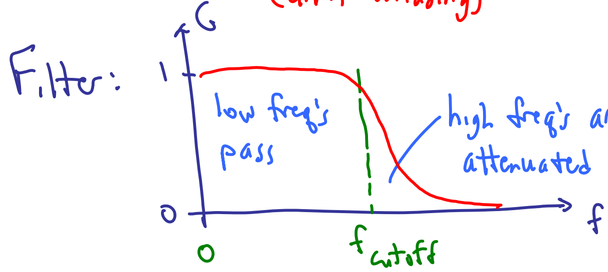
• Conclusion — The actual signal is just as likely to have a frequency component at 480 Hz as at 120 Hz!

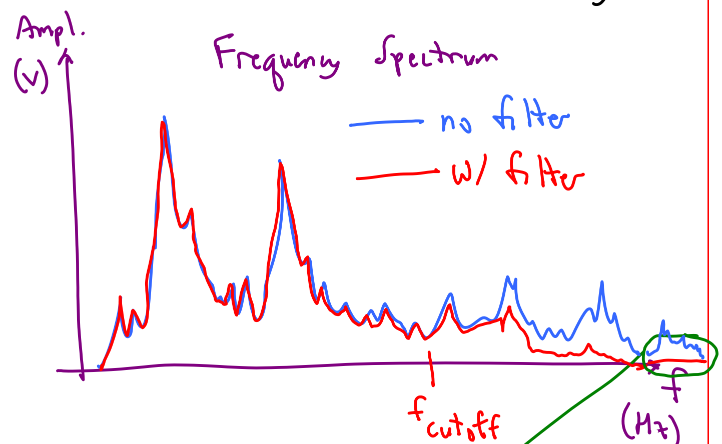
Bottom line — When sampling at a second frequency to check for aliasing, do not us f_s = integer multiple of original f_s

---

Another way to avoid aliasing errors is to use an anti-aliasing filter (a low-pass filter to remove high frequencies that could alias)







[That is why we call it an anti-aliasing filter!]

The anti-aliasing filter removes all frequency content at high frequencies and therefore these high frequencies cannot alias back to our spectrum

[Note: Set f_{cutoff} high enough so that we do not attenuate important frequencies in the signal!]