Today, we will: Start a new topic – **Signal Conditioning (filters, amplifiers, etc.)** These are analog devices

- Do a review example problem – anti-aliasing filter application & FFT
- Review the pdf module: **Filters**, and do some example problems

Signal → Filter → DAQ

Example: FFT with an anti-aliasing filter

Given: A signal contains: Analog

- f_1 A 200 Hz periodic component at 5.0 V amplitude (the desired signal).
- f_2 A 700 Hz periodic component, also at 5.0 V amplitude (undesired noise).

We low-pass filter the signal to attenuate the 700 Hz component. The filter reduces the amplitude of the 700 Hz component of the signal by a factor of five, but it does not significantly affect the 200 Hz component. → $G = 1$ @ 200 Hz → $G = \frac{1}{5} = 0.2$ @ 700 Hz

We sample the signal digitally at 1000 Hz, taking 512 data points.

(*a*) To do: Calculate the frequency resolution. → Δf is not affected by the filter!

$$\Delta f = \frac{1}{T}, \quad T = \frac{N}{f_s} \rightarrow \Delta f = \frac{f_s}{N} = \frac{1000 \text{ pts/s}}{512 \text{ pts}} = \boxed{1.953 \text{ Hz} = \Delta f}$$

(*b*) To do: Sketch what the frequency spectrum will look like.

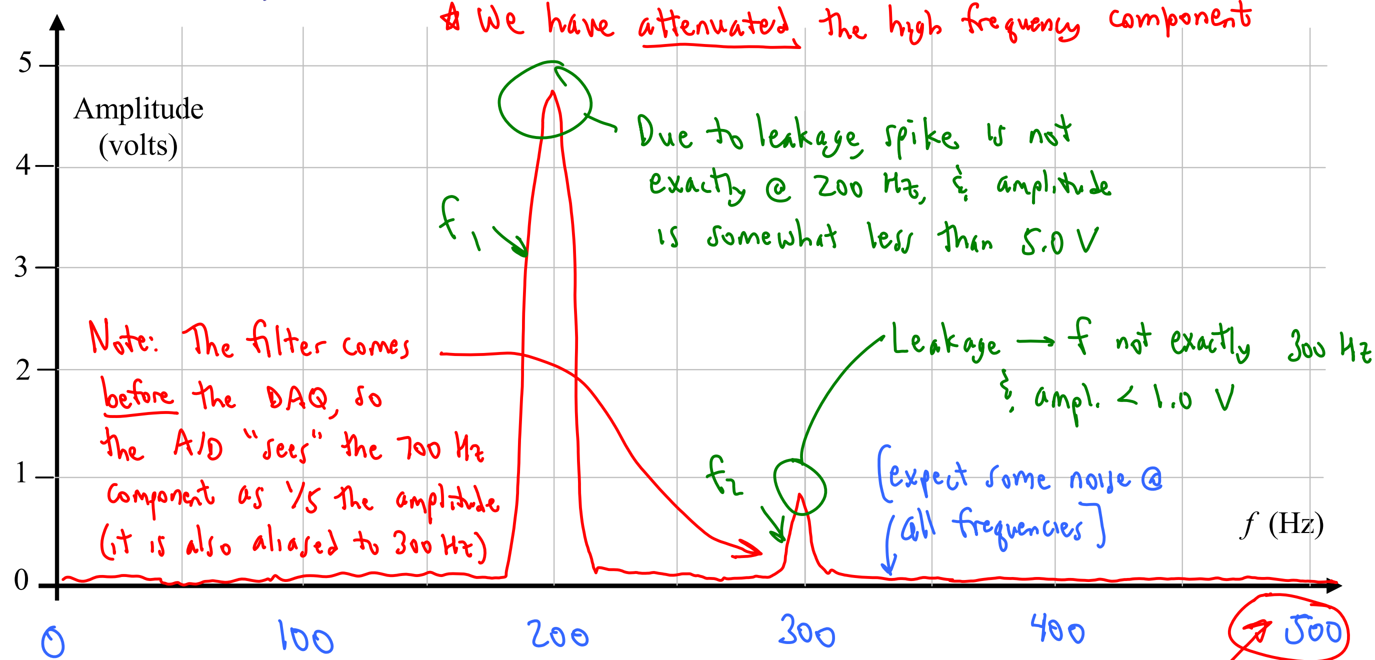
Solution: · Check for aliasing → @ $f_1 = 200$ Hz, no aliasing since $f_s = 1000 \text{ Hz} > 2f_1$

@ $f_2 = 700$ Hz, yes aliasing since $f_s = 1000 \text{ Hz} < 2f_2$

@ $f_2 = 700$ Hz, $f_a = |700 - 1000 * \text{NINT}(700/1000)| = 300$ Hz

- At $f_1 = 200$ Hz, Amplitude = 5.0 V & $G_{filter} \approx 1$ → final ampl. ≈ 5.0 V
- At $f_2 = 700$ Hz, Amplitude = 5.0 V & $G_{filter} = \frac{1}{5}$ → final ampl. ≈ 1.0 V

★ We have attenuated the high frequency component



[Since $f_s = 1000$ Hz, we plot the spectrum to $f_{folding} = 500$ Hz, due to the Nyquist criterion]

Example: Filters

Given: A voltage signal contains useful data up to about 1000 Hz. There is also some unwanted noise at frequencies greater than 3000 Hz. We want to use a low-pass filter so that the noise is attenuated by at least 95%. We plan to use a cutoff frequency of 2000 Hz so that there is minimal attenuation of the 1000 Hz component of the signal.

To do: Calculate the required order of the low-pass filter.

Solution:

- Butterworth low-pass filter of order n:

$$\boxed{G = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{cutoff}}\right)^{2n}}}} \quad (1)$$

- To achieve 95% attenuation means that $|V|_{out} = 0.05\,|V_{in}|$

i.e., $\boxed{G = 0.05}$ since $G = \frac{|V_{out}|}{|V_{in}|}$

[Be careful with the wording: "95% attenuation" means $G = 0.05$ NOT $G = 0.95$]

- Solve Eq. (1) for unknown filter order n:

$$1 + \left(\frac{f}{f_{cutoff}}\right)^{2n} = \frac{1}{G^2} \longrightarrow \left(\frac{f}{f_{cutoff}}\right)^{2n} = \frac{1}{G^2} - 1$$

take ln of both sides $\longrightarrow$ $2n \ln\left(\frac{f}{f_{cutoff}}\right) = \ln\left(\frac{1}{G^2} - 1\right)$

[recall, $\ln(a^b) = b\ln(a)$]

So, $$n = \frac{\ln\left(\frac{1}{G^2} - 1\right)}{2\ln(f/f_{cutoff})}$$

USE: $f = 3000$ Hz = unwanted noise

$f_{cutoff} = 2000$ Hz = specified

OR, $$n = \frac{\ln\left(\frac{1}{0.05^2} - 1\right)}{2\ln(3000/2000)} = 7.3853$$

Can't have a Butterworth filter with $n = 7.39$, so ...

$\boxed{\text{Answer: } n = 8}$

We need an 8^{th}-order filter

Example: Filters and digital data acquisition

Given: Vibrations around 90 Hz with an amplitude of about 5 V are measured with a DAQ. Data are sampled at f_s = 500 Hz (exceeding the Nyquist criterion). Unfortunately, there is also some electronic interference noise at 3600 Hz, with an amplitude of about 1 V.

To do: Sketch the frequency spectrum that you would expect to see for two cases:

- as is (no filter)
- with a 4th-order low-pass anti-aliasing filter set to a cutoff frequency of 200 Hz

Solution:

• First check for aliasing: Signal → $f = 90$ Hz, $2f = 180$ Hz, $f_s = 500$ Hz

∴ NO ALIASING

Noise → $f = 3600$ Hz, $2f = 7200$ Hz

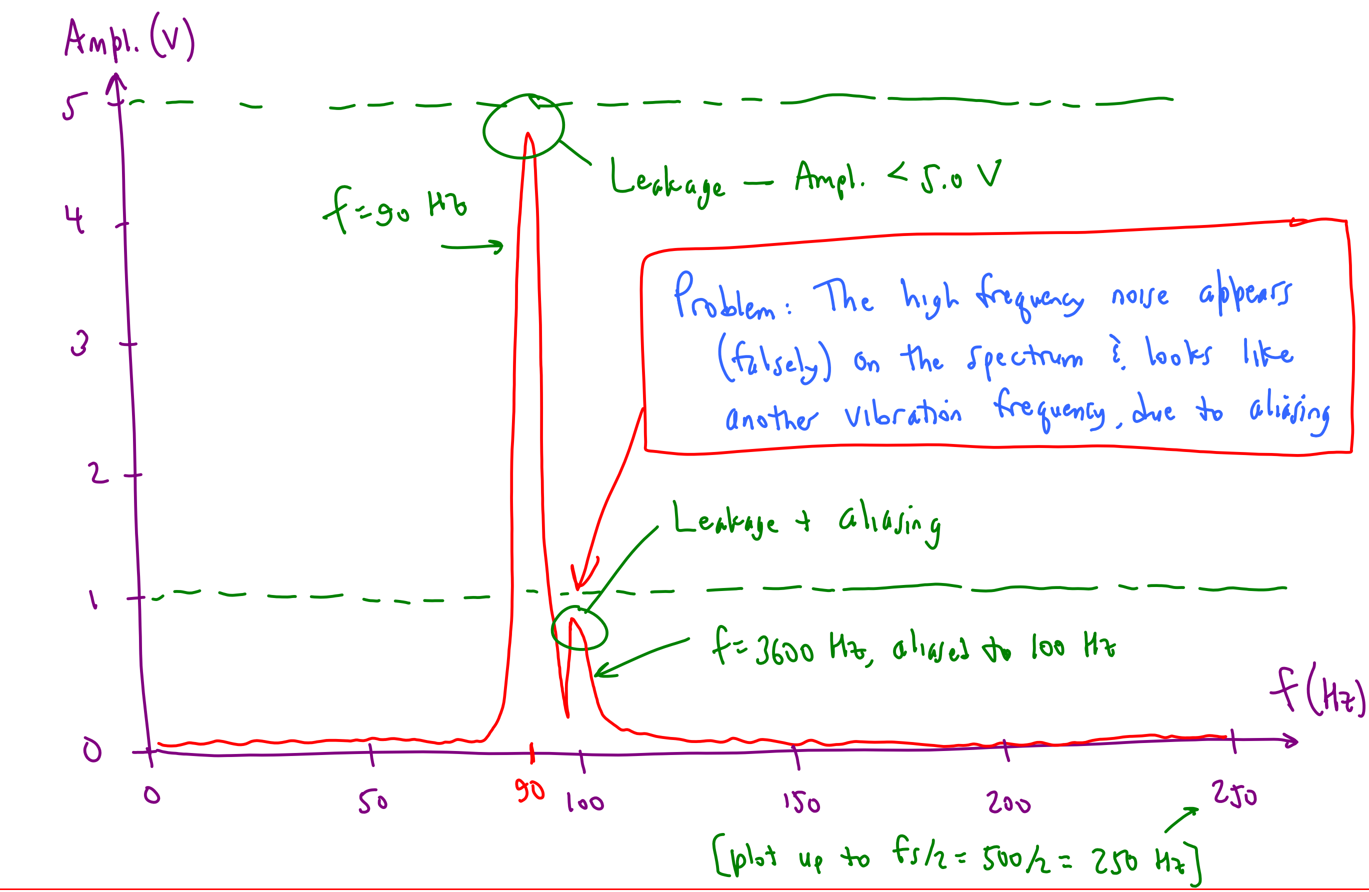
@ $f_s = 500$ Hz, definitely aliasing.

Calc $f_a = |f - f_s * \text{NINT}(f/f_s)| = |3600 - 500 * \text{NINT}(\underbrace{3600/500}_{7.2})|$

$f_a = |3600 - 500 * 7| = 100$ Hz

The noise will alias to $f = 100$ Hz

• Frequency Spectrum w/o a filter



- Repeat with 4th-order low-pass filter (Butterworth)

$$f_{cutoff} = 200 \text{ Hz}$$
$$n = 4 \quad (\text{filter order})$$

- Signal $\rightarrow$ $f = 90$ Hz, $G = \dfrac{1}{\sqrt{1+\left(\dfrac{f}{f_{cutoff}}\right)^{2n}}} = \dfrac{1}{\sqrt{1+\left(\dfrac{90}{200}\right)^{8}}} = \underline{0.9992}$

We see that the filter does not affect the 90 Hz signal significantly

- Noise $\rightarrow$ $f = 3600$ Hz,

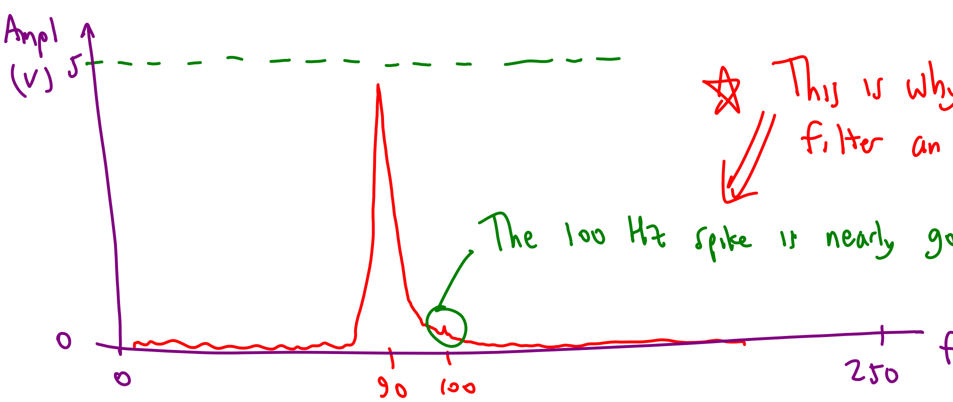
$$G = \frac{1}{\sqrt{1+\left(\frac{3600}{200}\right)^{8}}} = 0.00000953$$

(almost zero!)

Careful! Do not use the aliasing freq. of 100 Hz here $\rightarrow$ the filter comes before the DAQ (filter is analog)

[The noise is almost completely removed by the low-pass filter]

- Spectrum with filter:

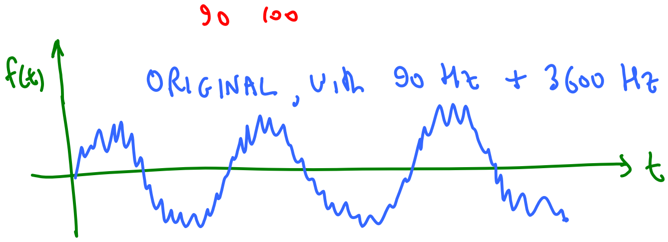


The 100 Hz spike is nearly gone!

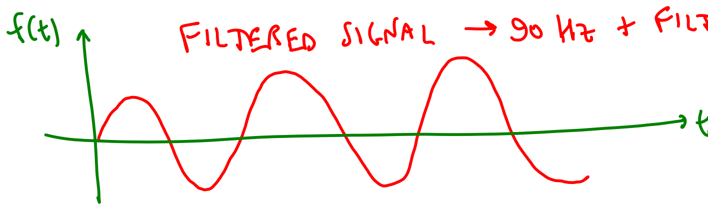
☆ This is why we call a low-pass filter an anti-aliasing filter!

- Time trace:

ORIGINAL, with 90 Hz + 3600 Hz (w/o filter)



FILTERED SIGNAL $\rightarrow$ 90 Hz + FILTERED 3600 Hz (w/ filter)



[The filter has removed the high frequency noise]